Composite Functions Topics

Practice Exercises (with Solutions)

Topics include interpreting graphs, tables, inverses, domain, average rate of change, and more.

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Composite Functions

Finding the domain:

The domain is the set of independent values that are defined in a function.

When finding the domain of composite functions, you must find the domain of the first function AND the composite function.

Example: $f(x) = \frac{1}{x+2}$ $g(x) = \frac{x-1}{x+5}$ What is the domain of $g(f(x))$?

Method 1: Find composite function, then determine domain

The composite function is $\dfrac{\left(\frac{1}{x+2}\right) - 1}{\left(\frac{1}{x+2}\right) + 5} = \dfrac{\left(\frac{1}{x+2}\right) - \left(\frac{x+2}{x+2}\right)}{\left(\frac{1}{x+2}\right) + \left(\frac{5x+10}{x+2}\right)} = \dfrac{\frac{-x-1}{x+2}}{\frac{5x+11}{x+2}} = \dfrac{-x-1}{5x+11}$

$x = -2$ $x = -11/5$

So, the domain is all real numbers except

$x \neq -2$ or $x \neq -11/5$

Method 2: Find domain of 1st function, then identify elements that would conflict with 2nd function

The first function is $f(x) = \frac{1}{x+2}$ so x cannot equal −2

then, the second function is $g(x) = \frac{x-1}{x+5}$ so, x cannot equal -5..

***So, when is $f(x) = -5$?

$-5 = \frac{1}{x+2}$ $-5x - 10 = 1$ $-5x = 11$ $x = -11/5$

Therefore, x cannot be -2 because it's undefined in $f(x)$...
And,

x cannot be -11/5, because $f(-11/5) = -5$
and, −5 is undefined in $g(x)$

Example: $f(x) = \frac{1}{x+2}$ $g(x) = \frac{x-1}{x+5}$ What is the domain of $f(g(x))$?

Domain of $g(x)$ is all reals except x = -5

Domain of $f(x)$ is all reals except x = -2

So, when is $g(x) = -2$?

$-2 = \frac{x-1}{x+5}$

$-2x - 10 = x - 1$

$-9 = 3x$

$-3 = x$

Therefore, the domain is

all real numbers where $x \neq -3, -5$

$f(g(x)) = \dfrac{1}{\frac{x-1}{x+5} + 2}$ $x = -5$

$= \dfrac{1}{\frac{x-1}{x+5} + \frac{2(x+5)}{(x+5)}}$

$= \dfrac{x+5}{3x+9}$ $x = -3$

Composite Functions Topics

I. Components of Functions

Split the following into 2 (or more) components.

Example: $h(x) = (x + 3)^2$

If $h(x) = (f \circ g)(x)$, what are $f(x)$ and $g(x)$?

$f(x) = x^2 \quad g(x) = (x + 3)$

because $f(g(x)) = (x + 3)^2$

Note: $g(x) = x^2 \quad f(x) = (x + 3)$
is NOT correct!

$h(x) = (f \circ g)(x)$ Determine possible functions $f(x)$ and $g(x)$:

a) $h(x) = \dfrac{1}{x^2 + 1}$

b) $h(x) = \sqrt{x} + 1$

c) $h(x) = \sqrt{x + 1}$

d) $h(x) = \sqrt{2x + 1}$

e) $h(x) = (3x + 9)^5$

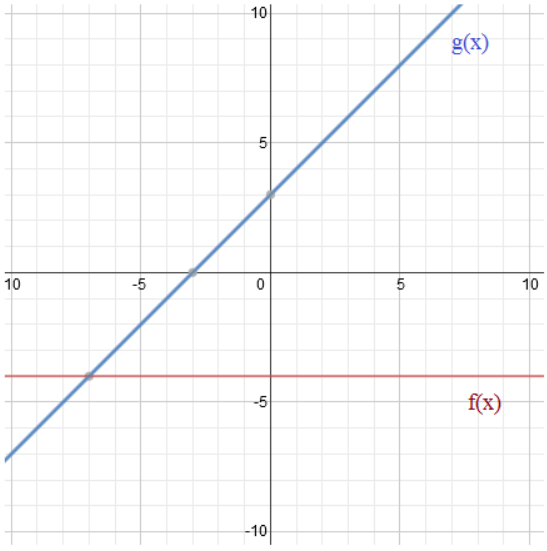
f) $h(x) = \sin^4 x$

$p(t) = (f \circ g \circ h)(t)$ Determine possible functions $f(t)$, $g(t)$, and $h(t)$

g) $p(t) = \cos^2 (3t + 5)$

h) $p(t) = \log (t^2 + 1)$

II. Answer the questions for the following graph:



a) $(f + g)(3) =$

b) $(f \circ g)(3) =$

c) $(g \circ f)(3) =$

d) $(f \circ f)(1) =$

e) $g(g(4)) =$

f) $g^{-1}(3) =$

g) $f^{-1}(3) =$

h) $(f - g)(0) =$

Composite Functions Topics

III. Domain

1) For the given functions $f(x) = \sqrt{x}$

$g(x) = 2x + 3$ find the domains of the composites:

a) $f \circ g$

b) $g \circ f$

c) $f \circ f$

d) $g \circ g$

2) $f(x) = \frac{3}{x-1}$ $g(x) = \frac{2}{x}$

Find the domains:

a) $f \circ g$

b) $g \circ f$

c) $f \circ f$

d) $g \circ g$

3) $f(x) = x^2 - 16$ $g(x) = \sqrt{x}$

Find the domains:

a) $f(g(x))$

b) $g(f(x))$

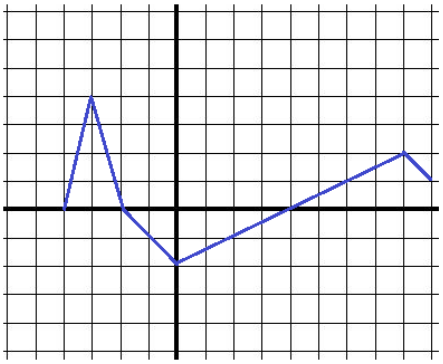
c) $f(f(x))$

d) $g(g(x))$

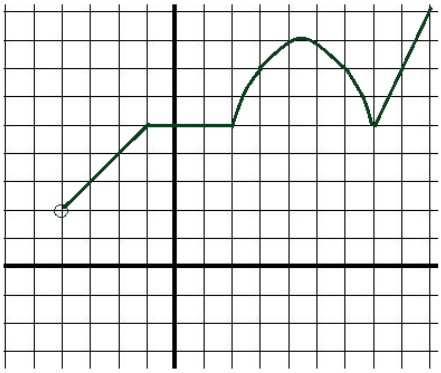
IV: Inverse and Composite Values (graph)

Composite Functions Topics

$f(x)$



$g(x)$



What value(s) of x solves each equation?

a) $f(x) = 4$

b) $f(x) = -1$

c) $g(x) + 2 = 9$

d) $f(x) \cdot g(x) = 0$

e) $f(x) + g(x) = 4$

f) $(g \circ g)(4) =$?

g) $g(f(x)) = 4$

h) $(f \circ g)(x) = 1$

i) $\left(\frac{f}{g}\right)(0) =$

V. Intrepreting values from a table

a) What is the domain of f? g?

b) What is the domain of $\frac{g}{f}$? $\frac{f}{g}$?

c) What is the domain of $f(g(x))$? $g(f(x))$?

d) $(f \circ f)(0) =$

e) $(g \circ g)(-1) =$

f) If $(f \circ g)(x) = 3$, what is x?

g) If $g(f(x)) = -4$ then what is x?

h) If $fg = 8$, what is x?

Assume the values in the table are all the elements in each function.

x	$f(x)$	$g(x)$
-4	-1	-3
-3	6	2
-2	4	0
-1	0	5
0	1	6
1	3	-1
2	3	1
3	2	4
4	-2	-4

VI. Applications

Composite Functions Topics

1) A dress size in France as it relates to the US is modeled in the function

$$s(x) = x - 32$$

And, a dress size in the US as it relates to Italy is modeled by the function

$$y(x) = 2(x + 10)$$

What is the function for the dress size in France as it relates to Italy?

2) Using the given functions, find the Average Rates Of Change (AROC)

$\frac{f(a+h)-f(a)}{h}$ $\frac{f(x+\triangle x)-f(x)}{\triangle x}$ $\frac{f(x+h)-f(x)}{h}$

a) $f(x) = 3x + 2$

b) $g(x) = 2x^2 + x - 1$

c) $h(x) = \frac{1}{x-1}$

VII. Miscellaneous Questions

a) Find $f \circ g \circ h$

$f(x) = x^2 + 4$

$g(x) = 5x$

$h(x) = x^2 - x - 2$

b) $f(x) = -3x$

$g(x) = -x + 4$

What is $(g \circ f)(x)$?

a) $3x - 4$

b) $-3x - 4$

c) $3x + 4$

d) $3x^2 + 4$

e) $-3x^2 + 4$

Composite Functions Topics

c) $f(x) = 2x + 1$

$g(x) = x^2$

For what values of x does $(f \circ g)(x) = (g \circ f)(x)$?

d) $f(x) = 3x + 8$

If $f(f(x)) = 23$, what is x ?

e) Given: $f(x) = (x - 6)(x - 4)$

$g(x) = x + 1$

When is $g(f(x)) = 0$?

f) $f(x) = x^2 - 4$ $g(x) = \sqrt{3x}$

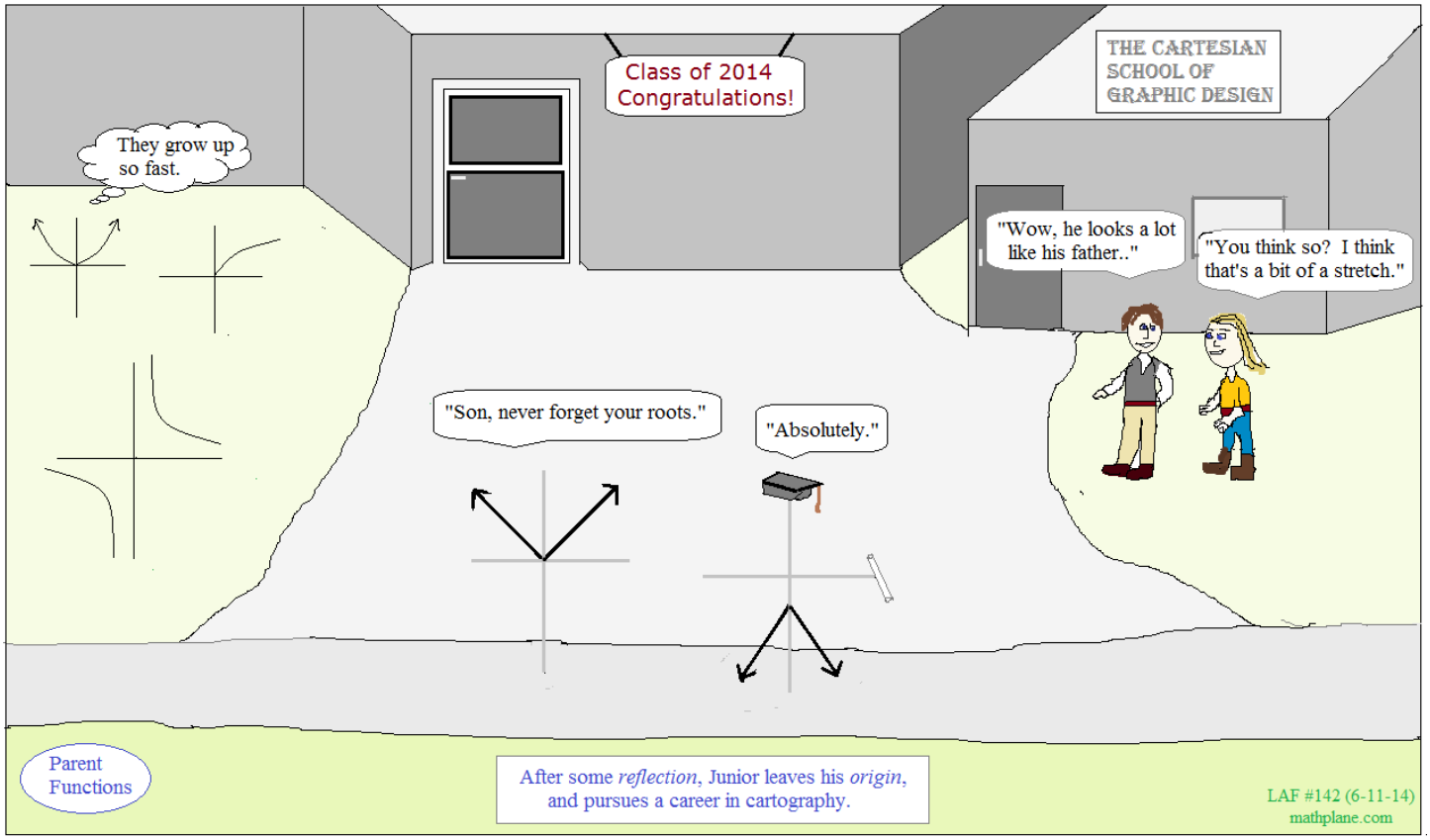
Find and compare the domain of $(f \circ g)(x)$ and $(g \circ f)(x)$...

g) $f(x) = \sqrt{x + 4}$

$g(x) = \frac{3}{x}$

Find $(f \circ g)(x)$ and its domain...

$(g \circ f)(x)$ and its domain


Class of 2014
Congratulations!
THE CARTESIAN SCHOOL OF GRAPHIC DESIGN
They grow up so fast.
"Wow, he looks a lot like his father.."
"You think so? I think that's a bit of a stretch."
"Son, never forget your roots."
"Absolutely."
Parent Functions
After some *reflection*, Junior leaves his *origin*, and pursues a career in cartography.
LAF #142 (6-11-14)
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Solutions-→

I. Components of Functions

Split the following into 2 (or more) components.

Example: $h(x) = (x + 3)^2$

If $h(x) = (f \circ g)(x)$, what are $f(x)$ and $g(x)$?

$f(x) = x^2 \quad g(x) = (x + 3)$

because $f(g(x)) = (x + 3)^2$

Note: $g(x) = x^2 \quad f(x) = (x + 3)$
is NOT correct!

SOLUTIONS

Composite Functions Topics

$h(x) = (f \circ g)(x)$ Determine possible functions $f(x)$ and $g(x)$:

a) $h(x) = \dfrac{1}{x^2 + 1}$ $f(x) = \dfrac{1}{x}$ $g(x) = x^2 + 1$

b) $h(x) = \sqrt{x} + 1$ $g(x) = \sqrt{x}$ $f(x) = x + 1$

c) $h(x) = \sqrt{x + 1}$ $g(x) = x + 1$ $f(x) = \sqrt{x}$

d) $h(x) = \sqrt{2x + 1}$ $f(x) = \sqrt{x}$ $g(x) = 2x + 1$

e) $h(x) = (3x + 9)^5$ $f(x) = x^5$ $g(x) + 3x + 9$

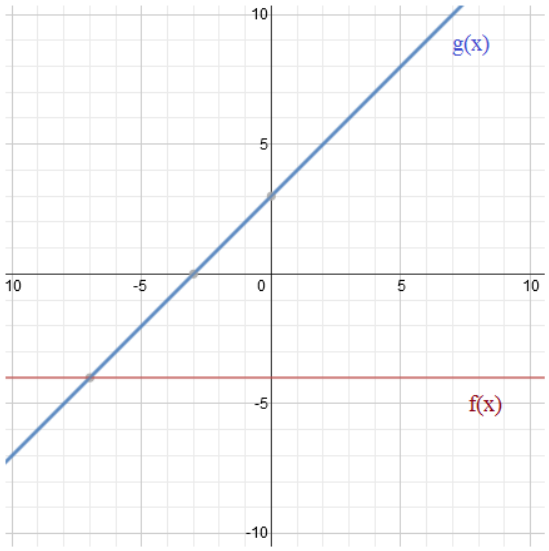
f) $h(x) = \sin^4 x$ $f(x) = x^4$ $g(x) = \sin x$

$p(t) = (f \circ g \circ h)(t)$ Determine possible functions $f(t)$, $g(t)$, and $h(t)$

g) $p(t) = \cos^2(3t + 5)$ $f(t) = t^2$ $g(t) = \cos(t)$ $h(t) = 3t + 5$

h) $p(t) = \log(t^2 + 1)$ $f(t) = \log(t)$ $g(t) = t + 1$ $h(t) = t^2$

II. Answer the questions for the following graph:



a) $(f + g)(3) =$ $f(3) + g(3) = -4 + 6 = 2$

b) $(f \circ g)(3) =$ $g(3) = 6$ and $f(6) -4$

c) $(g \circ f)(3) =$ $f(3) = -4$ and then $g(-4) = -1$

d) $(f \circ f)(1) =$ $f(1) = -4$ and then $f(-4) = -4$

e) $g(g(4)) =$ $g(4) = 7$ and then $g(7) = 10$

f) $g^{-1}(3) =$ "g of what number equals 3" ?
0 (because $g(0) = 3$)

g) $f^{-1}(3) =$ since no input into f(x) would produce 3, there is no solution ϕ

h) $(f - g)(0) =$ $f(0) - g(0) = -4 - 3 = -7$

III. Domain

Composite Functions Topics

SOLUTIONS

1) For the given functions $f(x) = \sqrt{x}$

$g(x) = 2x + 3$ find the domains of the composites:

a) $f \circ g$

b) $g \circ f$

c) $f \circ f$

d) $g \circ g$

a) First, find the domain of g.. all real numbers

Then, find the domain of $f \circ g$.. $\sqrt{2x + 3} \longrightarrow x \geq -3/2$

Finally, identify the intersection.. {all real} $\cap$ { $x \geq -3/2$ } = $x \geq -3/2$

b) Domain of f: $x \geq 0$

Of those numbers, all of them can go into g

$x \geq 0$

$2\sqrt{x} + 3$

c) Domain of f: $x \geq 0$

Domain of $f \circ f$: $x \geq 0$

{domain of f} $\cap$ {domain of $f \circ f$} = $x \geq 0$

d) Domain of g: all real numbers

Domain of $g \circ g$ $2(2x + 3) + 3$: all real numbers

therefore, domain is all real numbers

2) $f(x) = \frac{3}{x - 1}$ $g(x) = \frac{2}{x}$

Find the domains:

a) $f \circ g$

b) $g \circ f$

c) $f \circ f$

d) $g \circ g$

a) domain of g: all reals except x = 0

domain of $f \circ g$ $\frac{3}{\frac{2}{x} - 1}$ $x \neq 2$

all real numbers except 0 and 2

b) $x \neq 1$

domain of f: $x \neq 1$

$g(f(x)) = \frac{2}{\frac{3}{x - 1}} = \frac{2x - 2}{3}$

domain of composite: all reals

domain: all reals except 1

c) domain of f: all reals except x = 1

$\frac{3}{(1) - 1}$ is undefined

domain of $f \circ f$: all reals except x = 4

$\frac{3}{\frac{3}{x - 1} - 1}$

{x | $x \neq 1, 4$}

d) domain of g: all reals except 0

since any result from this domain will work, the domain is

{x | $x \neq 0$}

3) $f(x) = x^2 - 16$ $g(x) = \sqrt{x}$

Find the domains:

a) $f(g(x))$

b) $g(f(x))$

c) $f(f(x))$ all real numbers

d) $g(g(x))$ $x \geq 0$

a) $f(g(x))$

first, find numbers coming from $g(x)$.. $x \geq 0$

then, consider which of those numbers are permitted in $f(x)$..

all of them..

domain of $f(g(x))$: $x \geq 0$

domain of $f(g(x))$ = domain of $g(x)$ $\cap$ domain of $f(g(x))$

NOTE: When finding domain of composite, you must consider the domain of the first function as well as the composite...

$g(x) = \sqrt{x}$ $f(g(x)) = x - 16$

$x \geq 0$ $\cap$ all real

DOMAIN: $x \geq 0$

b) domain of $f(x)$ $\cap$ domain of $g(f(x))$

$f(x) = x^2 - 16$ $g(f(x)) = \sqrt{x^2 - 16}$

all real $\cap$ $x \leq -4$ $x \geq 4$

domain of the composite: $|x| \geq 4$

$g(f(x))$

$g(x)$ may only contain values that are not negative...

So, $f(x)$ must be non-negative...

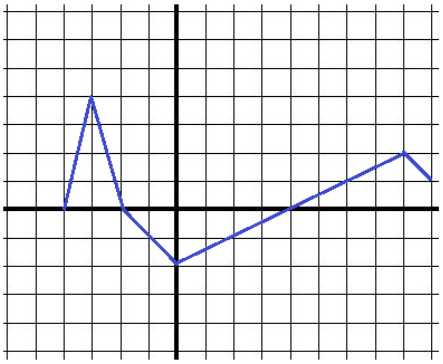
$x^2 - 16 \geq 0$ $|x| \geq 4$

IV: Inverse and Composite Values (graph)

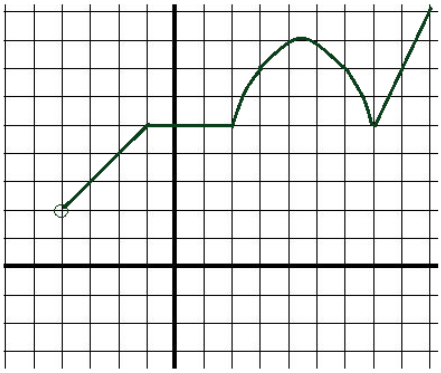
SOLUTIONS

Composite Functions Topics

$f(x)$



$g(x)$



What value(s) of x solves each equation?

a) $f(x) = 4$

when x = -3

$f^{-1}(4) = x$

$= -3$

b) $f(x) = -1$

when x = -1 or 2

$f^{-1}(-1) = x$

$= -1$ or 2

c) $g(x) + 2 = 9$

$g(x) = 7$ this occurs when x = 3 or 6

d) $f(x) \cdot g(x) = 0$

This occurs if $f(x) = 0$ or $g(x) = 0$...

x = ~~-4~~, -2, or 4

since $g(-4)$ does not exist, it is eliminated..

e) $f(x) + g(x) = 4$

answers include the interval [-2, -1], x = 2 (and, somewhere between -4 and -3)

because if x = -4, then 2. If x = -3, then 7. In between, the composite crosses 4.

f) $(g \circ g)(4) = ?$

$g(4) = 8$

then, $g(8) =$ 7

g) $g(f(x)) = 4$

$g(x) = 4$ when x is -2...

So, when is $f(x) = -2$?

this occurs when x = 0

h) $(f \circ g)(x) = 1$

since $f(x)$ must equal 1...

$g(x)$ must equal -3.75, -2.25, 6, or 9..

this occurs when

x = 9, 2.5, 6.5, 7.5

i) $\left(\frac{f}{g}\right)(0) = \frac{f(0)}{g(0)} = \frac{-2}{5}$

V. Intrepreting values from a table

a) What is the domain of f? g?

domain for each: {-4, -3, -2, -1, 0, 1, 2, 3, 4}

b) What is the domain of $\frac{g}{f}$? $\frac{f}{g}$?

all elements except -1 (because $f(-1) = 0$)

all elements except -2

c) What is the domain of $f(g(x))$? $g(f(x))$?

$g(x)$ must be -4, -3, -2, -1, 0, 1, 2, 3, or 4 to qualify for $f(x)$

therefore, domain of $f(g(x))$ is {-4, -3, -2, 1, 2, 3, 4}

d) $(f \circ f)(0) =$ $f(0) = 1$.. then, $f(1) = 3$

e) $(g \circ g)(-1) =$ $g(-1) = 5$... g(5) Does Not Exist!

f) If $(f \circ g)(x) = 3$, what is x? x = -3, 2

g) If g(f(x)) = -4 then what is x? x = -2

$f(x)$ must be -4, -3, -2, -1, 0, 1, 2, 3, or 4 to qualify for $g(x)$

therefore, domain of $g(f(x))$ is {-4, -2, -1, 0, 1, 2, 3, 4}

note: if x = -3, then $f(-3) = 6$ and, 6 is not qualify for the domain of $g(x)$

h) If fg = 8, what is x? x = 3 , 4

Assume the values in the table are all the elements in each function.

x	$f(x)$	$g(x)$
-4	-1	-3
-3	6	2
-2	4	0
-1	0	5
0	1	6
1	3	-1
2	3	1
3	2	4
4	-2	-4

VI. Applications

1) A dress size in France as it relates to the US is modeled in the function

$$s(x) = x - 32$$

And, a dress size in the US as it relates to Italy is modeled by the function

$$y(x) = 2(x + 10)$$

What is the function for the dress size in France as it relates to Italy?

2) Using the given functions, find the Average Rates Of Change (AROC)

$\frac{f(a+h)-f(a)}{h}$ $\quad$ $\frac{f(x+\triangle x)-f(x)}{\triangle x}$ $\quad$ $\frac{f(x+h)-f(x)}{h}$

a) $f(x) = 3x + 2$

$$\frac{3(a+h)+2-(3a+2)}{h}$$

$$\frac{3a+3h+2-3a-2}{h}$$

$$\frac{3h}{h} = \boxed{3}$$

b) $g(x) = 2x^2 + x - 1$

$$\frac{2(x+\triangle x)^2 + (x+\triangle x) - 1 - (2x^2 + x - 1)}{\triangle x}$$

$$\frac{2x^2 + 4x\triangle x + 2\triangle x^2 + x + \triangle x - 2x^2 - x}{\triangle x}$$

$$\frac{4x\triangle x + 2\triangle x^2 + \triangle x}{\triangle x}$$

$$\boxed{4x + 2\triangle x + 1}$$

c) $h(x) = \frac{1}{x-1}$

$$\frac{\frac{1}{(x+h)-1} - \frac{1}{x-1}}{h}$$

$$\frac{\frac{(x-1) - [(x+h)-1]}{[(x+h)-1](x-1)}}{h}$$

$$\frac{\frac{-h}{[(x+h)-1](x-1)}}{h}$$

$$\boxed{\frac{-1}{(x+h-1)(x-1)}}$$

NOTE: AROC between 2 and 5

$x = 2$, $\triangle x = 3$ → change between 2 and 5

$4x + 2\triangle x + 1$

so, AROC $= 4(2) + 2(3) + 1 = 15$

slope between (2, 9) and (5, 54) is 45/3 = 15

Average Rate Of Change (AROC)

$\frac{f(a+h)-f(a)}{h}$ or $\frac{f(a)-f(b)}{a-b}$

VII. Miscellaneous Questions

a) Find $f \circ g \circ h$

$f(x) = x^2 + 4$

$g(x) = 5x$

$h(x) = x^2 - x - 2$

$f(g(h(x)))$

working from right to left:

$g \circ h = 5(x^2 - x - 2)$

$= 5x^2 - 5x - 10$

then, find $f \circ (g \circ h)$

$(5x^2 - 5x - 10)^2 + 4$

$$\begin{array}{r} 5x^2 - 5x - 10 \\ \times \quad 5x^2 - 5x - 10 \\ \hline 25x^4 - 25x^3 - 50x^2 \\ -25x^3 + 25x^2 + 50x \\ + \quad -50x^2 + 50x + 100 \\ \hline 25x^4 - 50x^3 - 75x^2 + 100x + 100 \end{array}$$

$$\boxed{25x^4 - 50x^3 - 75x^2 + 100x + 104}$$

b) $f(x) = -3x$

$g(x) = -x + 4$

What is $(g \circ f)(x)$?

$g(f(x)) = -(-3x) + 4$

$= 3x + 4$

a) 3x - 4

b) -3x - 4

c) 3x + 4

d) $3x^2 + 4$

e) $-3x^2 + 4$

SOLUTIONS

Composite Functions Topics

If you input the size in Italy, the output is the dress size in US...

$y(x) = 2x + 20$

Then, if you input the US size, the output is the dress size in France...

$s(x) = x - 32$

$s(y(x)) = (2x + 20) - 32$

$= 2x - 12$

where x is the dress size in Italy..

SOLUTIONS

Composite Functions Topics

c) $f(x) = 2x + 1$

$g(x) = x^2$

For what values of x does $(f \circ g)(x) = (g \circ f)(x)$?

$f(g(x)) = f(x^2) = 2x^2 + 1$

$g(f(x)) = g(2x + 1) = 4x^2 + 4x + 1$

$2x^2 + 1 = 4x^2 + 4x + 1$

$2x^2 + 4x = 0$

$2x(x + 2) = 0$

$x = 0, -2$

d) $f(x) = 3x + 8$

If $f(f(x)) = 23$, what is x ?

outside function f	Inside function f
$f(x) = 23$	$f(x) = 5$
$3x + 8 = 23$	$3x + 8 = 5$
$x = 5$	$x = -1$

e) Given: $f(x) = (x - 6)(x - 4)$

$g(x) = x + 1$

When is $g(f(x)) = 0$?

So, when is $g(x) = 0$?

This occurs when $x = -1$, because $g(-1) = 0$

Now, we must find out when $f(x) = -1$...

$-1 = (x - 6)(x - 4)$

$-1 = x^2 - 10x + 24$

$(x - 5)(x - 5) = 0$

$x = 5$

f) $f(x) = x^2 - 4$ $\quad g(x) = \sqrt{3x}$

Find and compare the domain of $(f \circ g)(x)$ and $(g \circ f)(x)$...

$[0, \infty)$ $\qquad$ $(-\infty, -2] \cup [2, \infty)$

domain of g(x) ----> $x \geq 0$

$3x - 4$ <----> domain of f(g(x)) -----> all reals

then, the intersection of these domains is $x \geq 0$

domain of f(x) ----> all reals

$\sqrt{3(x^2 - 4)}$ <------> domain of g(f(x)) -----> $x \leq -2$ or $x \geq 2$

then, the intersection is $x \leq -2$ or $x \geq 2$

g) $f(x) = \sqrt{x + 4}$

$g(x) = \frac{3}{x}$

Find $(f \circ g)(x)$ and its domain...

$(g \circ f)(x)$ and its domain

$(g \circ f)(x) = g(f(x)) = \frac{3}{\sqrt{x + 4}}$

since domain of f(x) is $x \geq -4$,

and domain of $(g \circ f)(x)$ $x > -4$,

the domain is the intersection $(-4, \infty)$

$(f \circ g)(x) = f(g(x)) = \sqrt{\frac{3}{x} + 4}$

since domain of g(x) is all reals EXCEPT 0

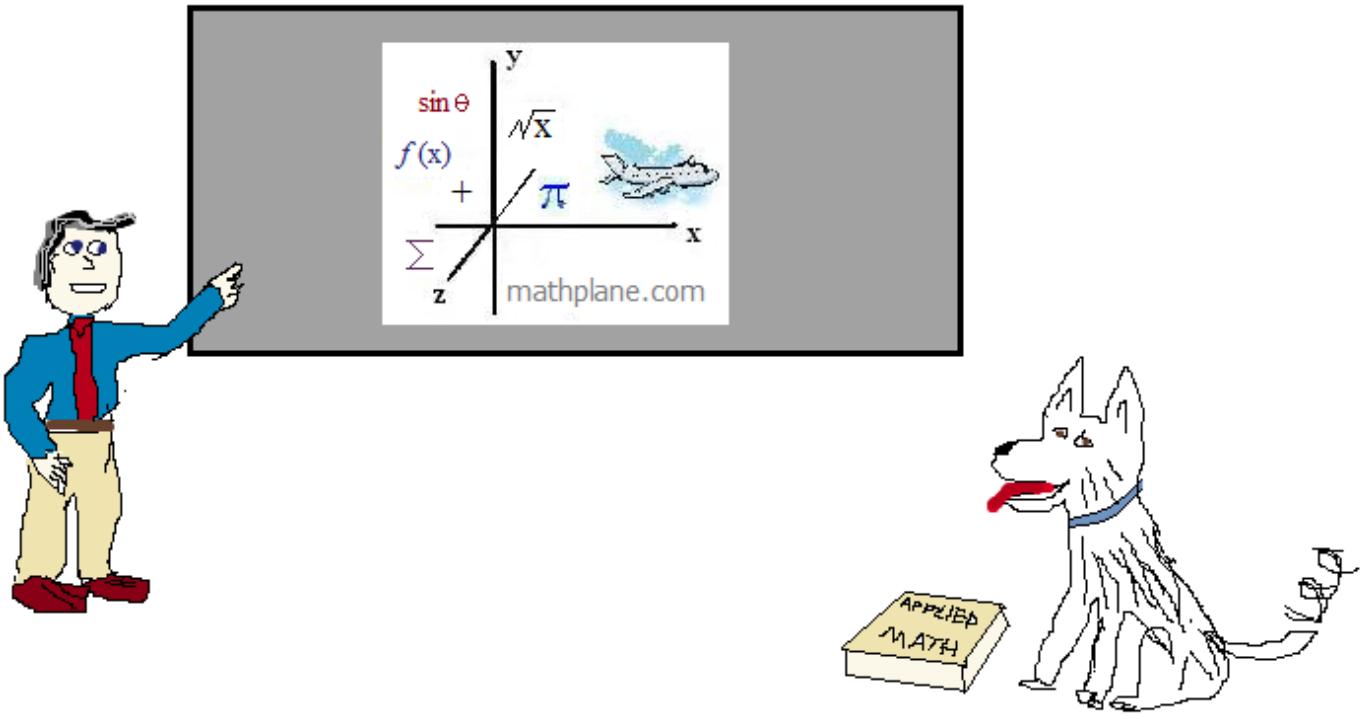
and, domain of $(f \circ g)(x)$ $\frac{3}{x} + 4 \geq 0$

domain is $(-\infty, -3/4] \cup (0, \infty)$

Thanks for visiting. (Hope it helped!)

If you have questions, suggestions, or requests, let us know.

Cheers



Also, TeachersPayTeachers, Facebook, Google+, TES, & Pinterest.

And, Mathplane *Express* for mobile at Mathplane.ORG