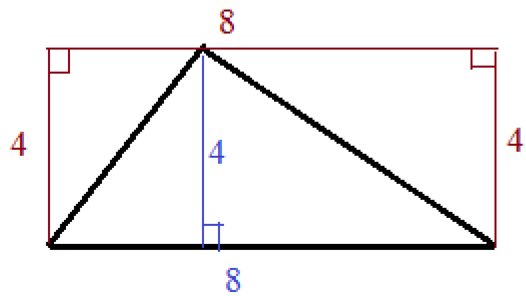
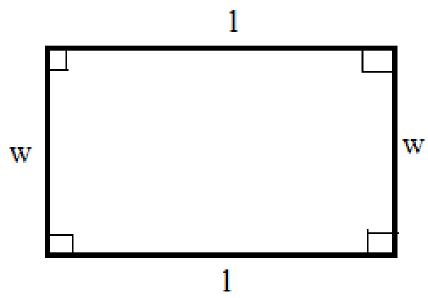
Area and Perimeter of Polygons 1



Includes formulas, examples, and practice questions
(w/answers)

Area and Perimeter of a Rectangle

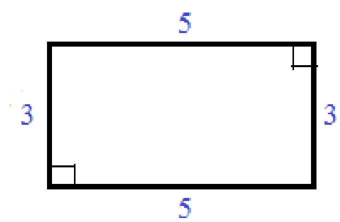


Area = lw

Perimeter = 2l + 2w

l = length
w = width

Example:

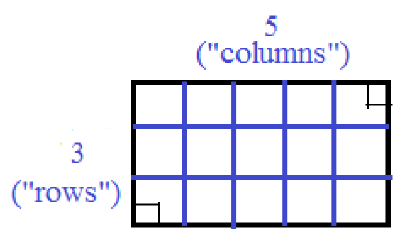


Area = lw = (5)(3) = 15 square units

Perimeter = 2l + 2w = 2(5) + 2(3) = 16 units

Observations: the perimeter is simply adding up every side -- 3 + 5 + 3 + 5

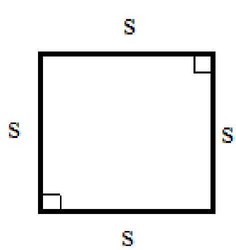
the area is the number of 1 unit boxes --



15 total
square units

Area and Perimeter of a Square

A square is a special type of rectangle. So, using substitution:

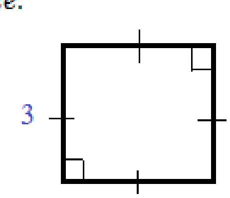


Area = s^2

Perimeter = 4s

s = side (length)

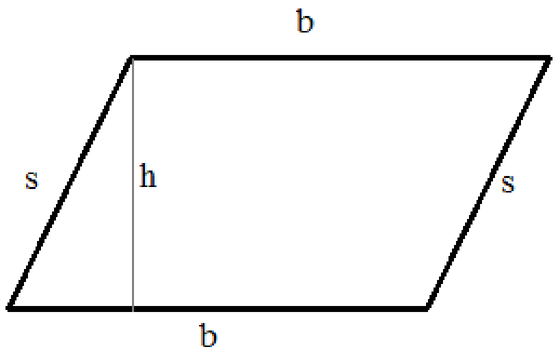
Example:



Area = $(s)^2$ = (s)(s) = 9 square units

Perimeter = 4(s) = 12 units

Area and Perimeter of a Parallelogram

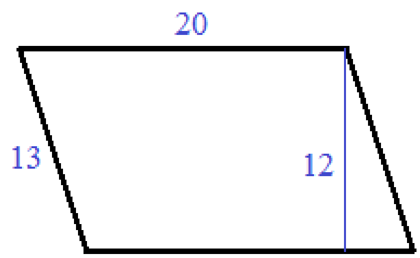


Area = bh

Perimeter = s + b + s + b = 2(b + s)

s = side
b = base
h = height

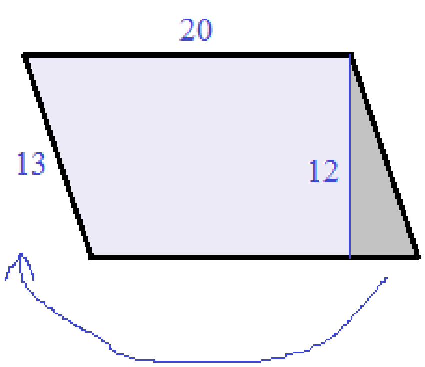
Example:

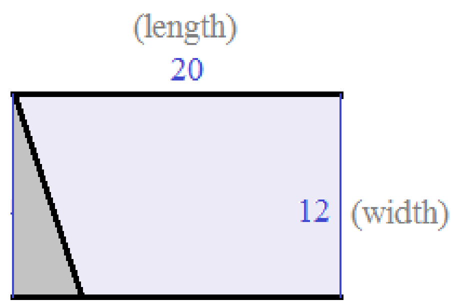


Area = bh = 20(12) = 240 square units

Perimeter = 2(b + s) = 66 units

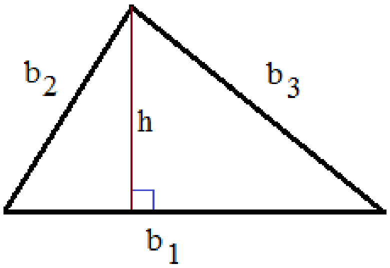
Observation: To verify the area of a parallelogram, transform the figure into a rectangle!





Area of rectangle is lw = 20(12) = 240 square units

Area and Perimeter of a Triangle

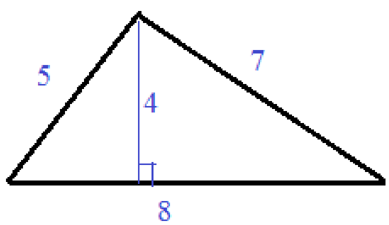


$$\text{Area} = \frac{1}{2}bh$$

$$\text{Perimeter} = b_1 + b_2 + b_3$$

b = base
h = height

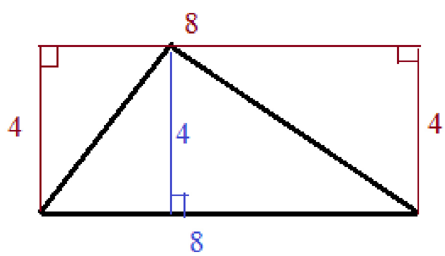
Example:



$$\text{Area} = \frac{1}{2}bh = \frac{1}{2}(8)(4) = 16 \text{ square units}$$

Perimeter = sum of the sides =
$8 + 5 + 7 = 20$ units

Observation: A triangle is one-half of a rectangle, so the triangle's area is one-half!

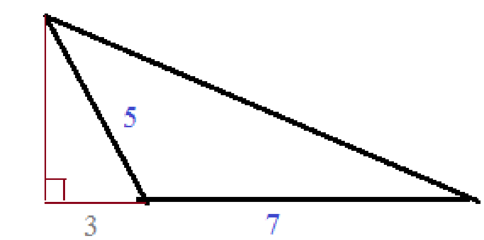


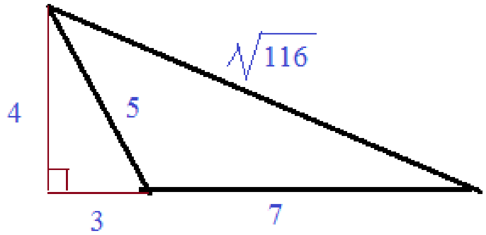
Area of rectangle = bh (i.e. length x width)
= 32

Area of triangle = 16

Example:

"Obtuse Triangle"





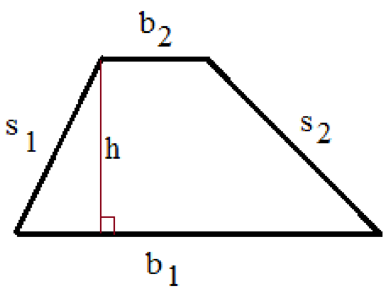
(approx.)

$$\text{Perimeter} = 7 + 5 + \sqrt{116} = 22.77$$

$$\text{Area} = \frac{1}{2}bh = \frac{1}{2}(7)(4)$$

$$= 14 \text{ sq. units}$$

Area and Perimeter of a Trapezoid

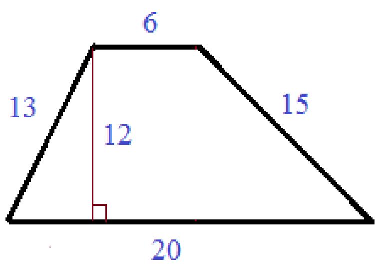


$$\text{Area} = \frac{1}{2}(b_1 + b_2)\,h$$

$$\text{Perimeter} = b_1 + b_2 + s_1 + s_2$$

b = base
s = side
h = height

Example:

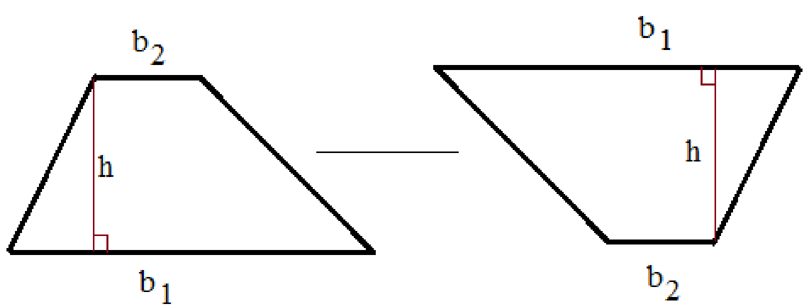


Perimeter = 20 + 13 + 6 + 15 = 54 units

$$\text{Area} = \frac{1}{2}(b_1 + b_2)h$$

$$= \frac{1}{2}(20 + 6)(12) = 156 \text{ square units}$$

Observation: a trapezoid is one-half of a parallelogram



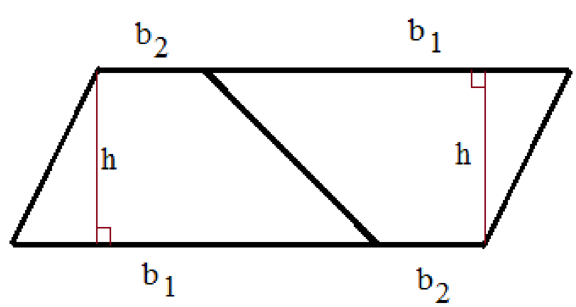
When we double the trapezoid, we get a parallelogram.

area of this parallelogram:

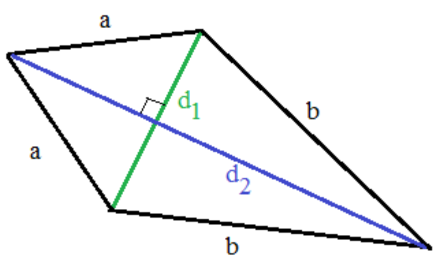
$$(b_1 + b_2)(h)$$

area of each trapezoid:

$$\frac{(b_1 + b_2)(h)}{2}$$



Area and Perimeter of a Kite

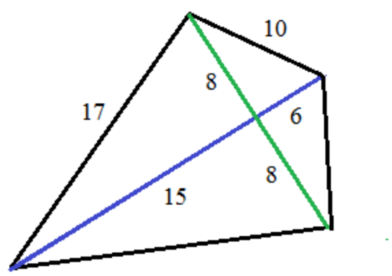


$$\text{Area} = \frac{1}{2} d_1 d_2$$

$$\text{Perimeter} = 2a + 2b$$

a = side
b = side
d_1 = diagonal
d_2 = diagonal

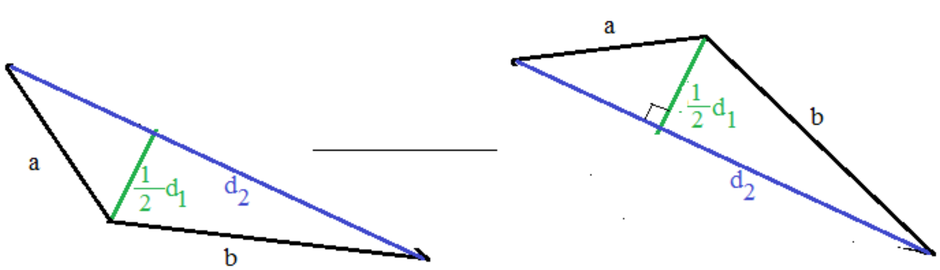
Example:



$$\text{Area} = \frac{1}{2} d_1 d_2 = \frac{1}{2}(16)(21) = 168 \text{ square units}$$

$$\text{Perimeter} = 2a + 2b = 2(10) + 2(17) = 54 \text{ units}$$

Observation: A kite is 2 congruent triangles.



Area of each triangle is

$\frac{1}{2}$(base)(height)

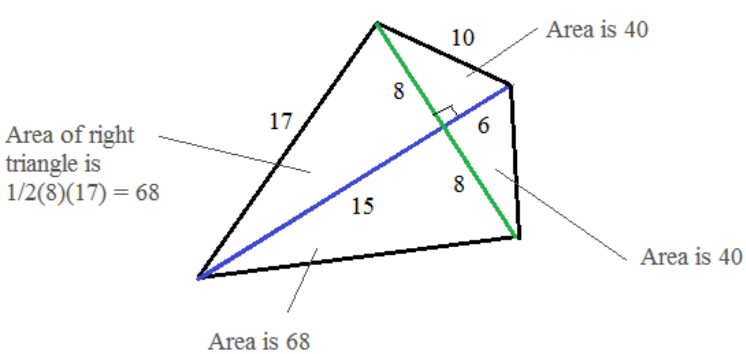
$\frac{1}{2}(d_2)(\frac{1}{2} d_1)$

$\frac{1}{4} d_1 d_2$

Area of both triangles:

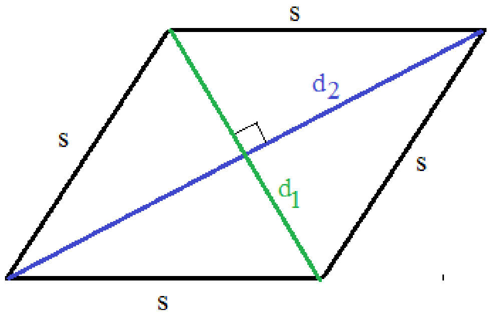
$\frac{1}{2} d_1 d_2$

Observation: Since one diagonal is a perpendicular bisector, the kite consists of 4 right triangles.



Total area: 168
square units

Area and Perimeter of a Rhombus



$$\text{Area} = \frac{1}{2}\, d_1 d_2$$

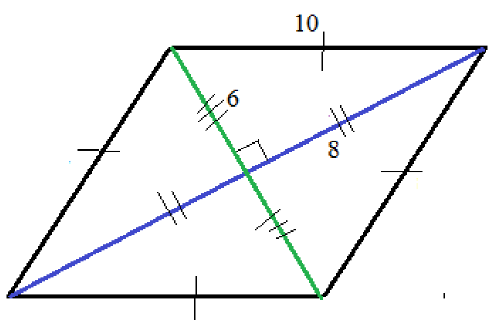
$$\text{Perimeter} = 4s$$

s = side

d_1 = diagonal

d_2 = diagonal

Example:

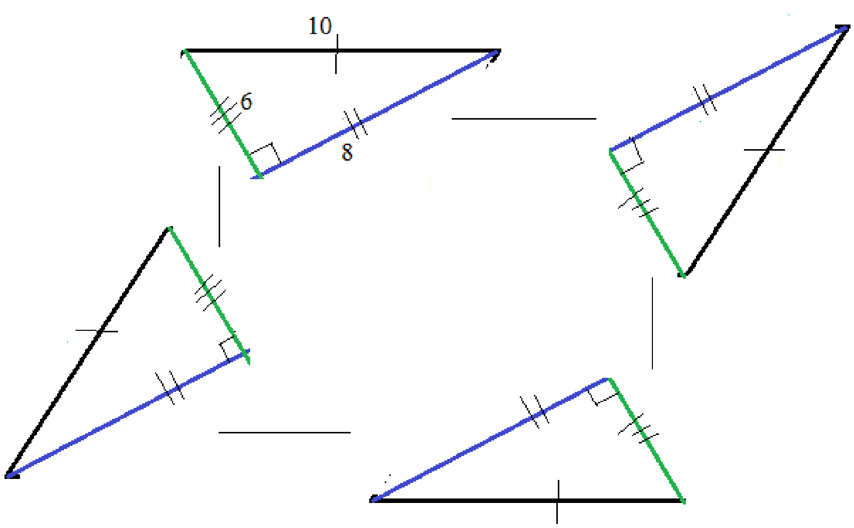


Perimeter = 4 x 10 = 40 units

$$\text{Area} = \frac{1}{2}(12)(16) = 96 \text{ square units}$$

diagonal 1 = 12
diagonal 2 = 16

Observation: Since diagonals of a rhombus are perpendicular bisectors, there are 4 congruent right triangles.



The area of each right triangle is

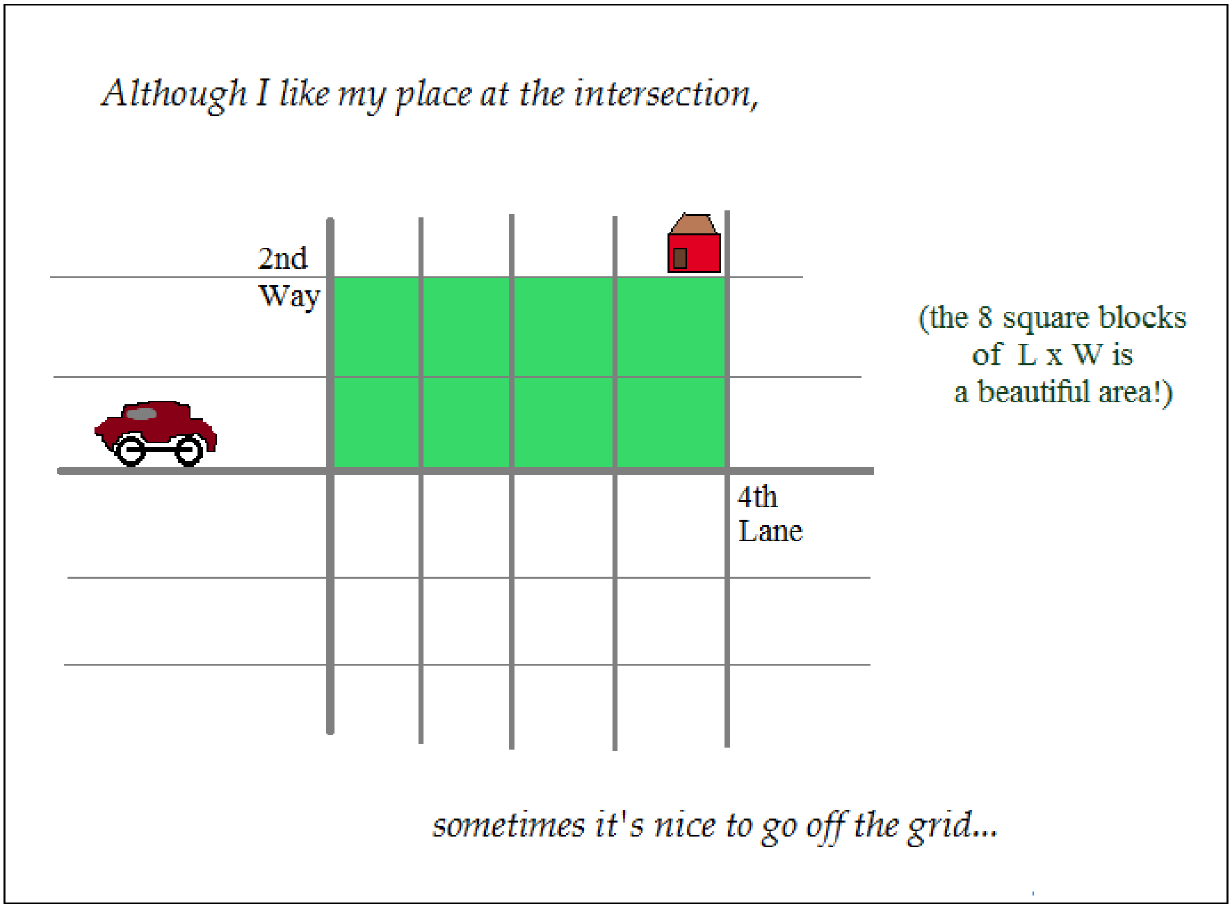
$$\frac{1}{2}\text{(base)(height)} =$$

$$\frac{1}{2}(8)(6) = 24 \text{ units}$$

Total of 4 triangles is

96 square units

Area Code



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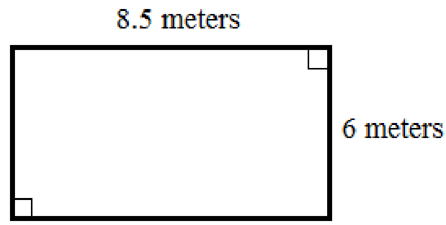
Practice Quiz (w/solutions)

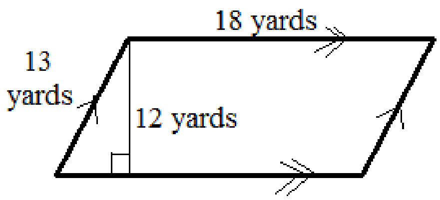
Determine the area and perimeter of each polygon:

Area and Perimeter of Polygons Quiz

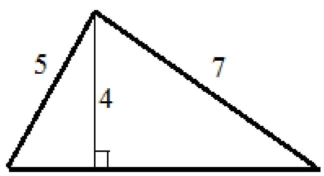
1)



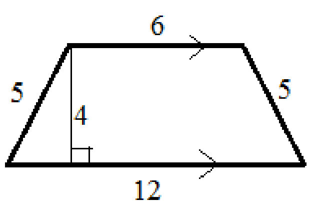
2)



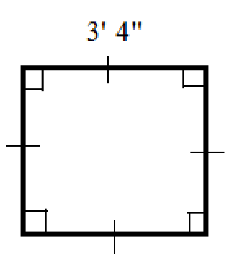
3)



4)



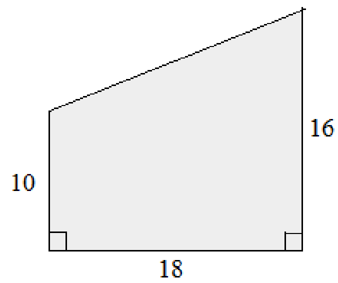
5)



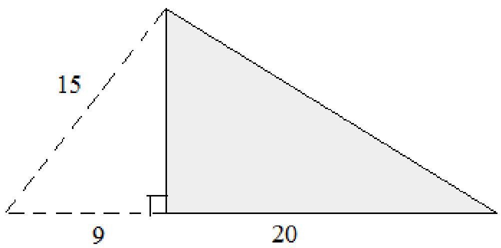
Determine the area and perimeter of each (shaded) figure:

Area and Perimeter of Polygons Quiz

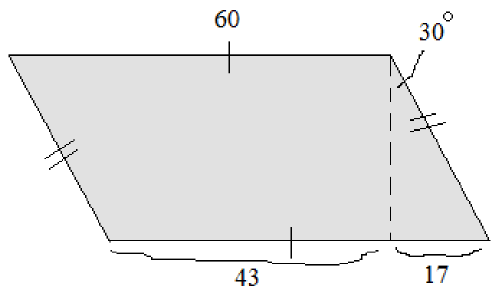
6)



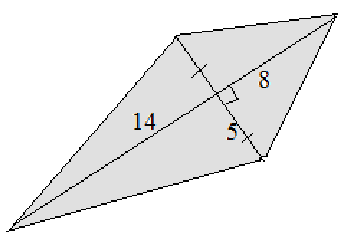
7)



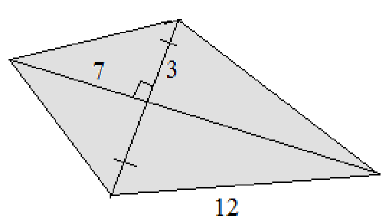
8)



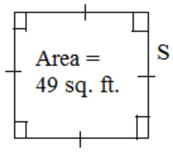
9)



10)



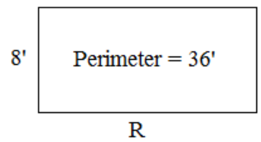
Area and Perimeter: finding parts of geometry figures



S = __________

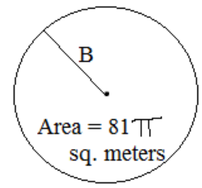
Perimeter = __________

Rectangle



R = __________

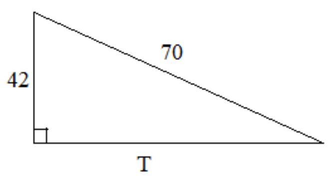
Area = __________



B = __________

Circumference = __________

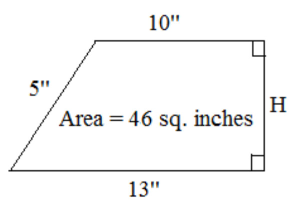
Perimeter = 168 inches



T = __________

Area = __________

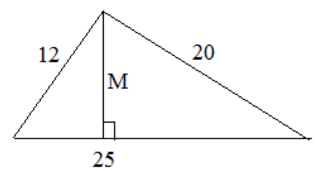
Trapezoid



H = __________

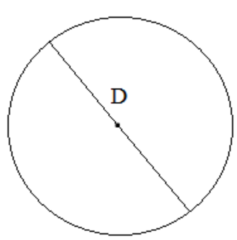
Perimeter = __________

Area = 100 sq. units



M = __________

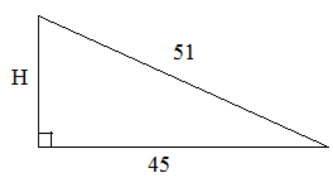
Circumference = 12 feet



(diameter) D = __________

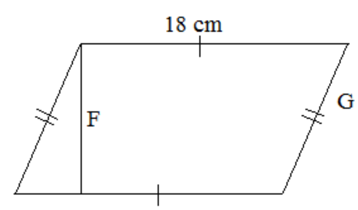
Area = __________

Area = 540 sq. meters



H = __________

Perimeter = __________



Area = 216 cm^2

Perimeter = 62 cm

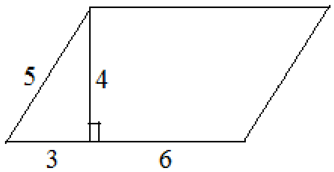
F = __________

G = __________

Answer the following questions:

Area and Perimeter of Polygons Quiz

1) What is the area of the parallelogram? The perimeter?



2) If the area of a square is 144 square feet, what is the perimeter?

3) The length of a rectangle is twice its width. If the perimeter is 66cm, what is the area?

4) What is the area of a 5-12-13 special right triangle?

**Challenge: Given: Rectangle ABCD

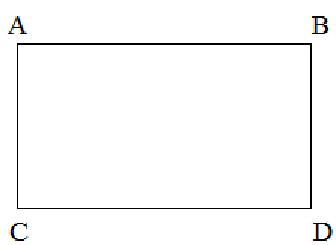
$\overline{AC} = 2x + 5$

$\overline{AD} = 6x - 1$

$\overline{BD} = 3y - 6$

$\overline{BC} = 2y + 8$

What is the perimeter of $\square$ABDC?



Area and Perimeter of Polygons Quiz

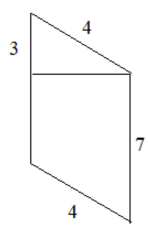
Can you answer the following?

A) The diagonals of a rhombus are 6 and 8. What is the height?

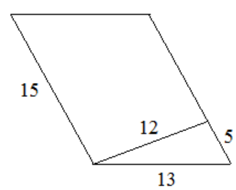
B) A trapezoid with perimeter 44 has non-parallel sides of length 8 and 10.
If the height is 5, what is the area of the trapezoid?
What is the length of each base?

C) What is the area and perimeter of each parallelogram?

1)



2)

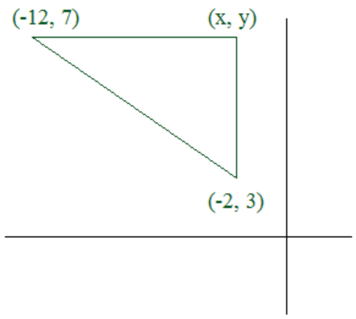


D) Find the area of a trapezoid with sides 12, 17, 40, and 25
where 12 and 40 are the bases.

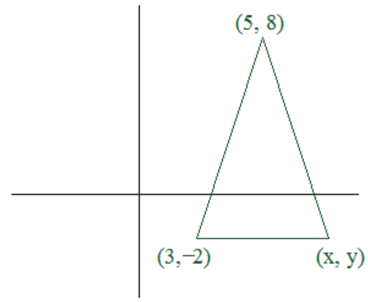
Identify the missing coordinates. Then, find the area of each figure.

Quadrilaterals, Triangles, and Coordinates

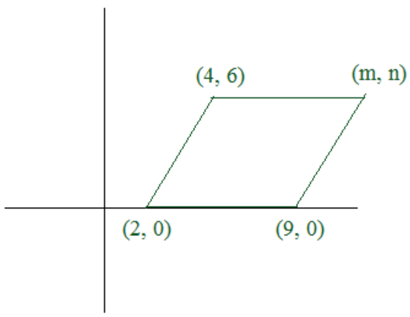
1) Right Triangle



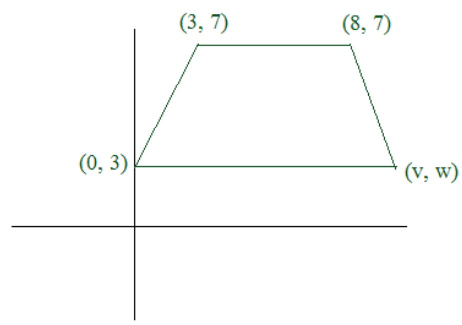
2) Isosceles Triangle



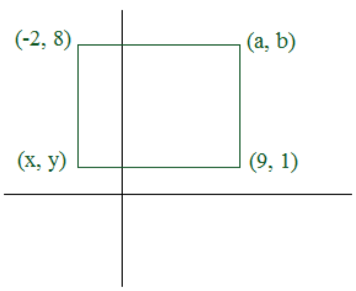
3) Parallelogram



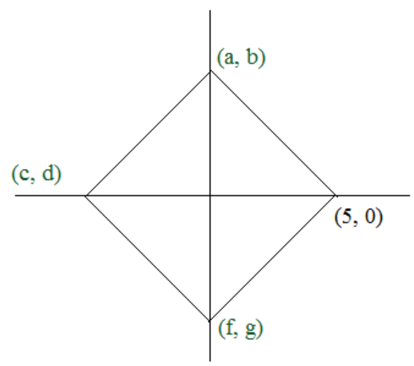
4) Isosceles Trapezoid



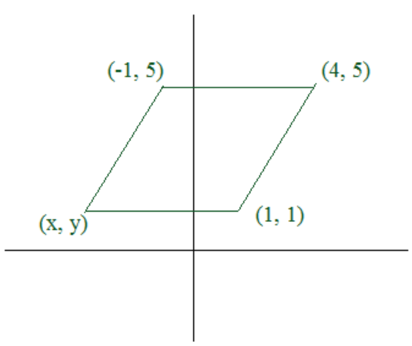
5) Rectangle



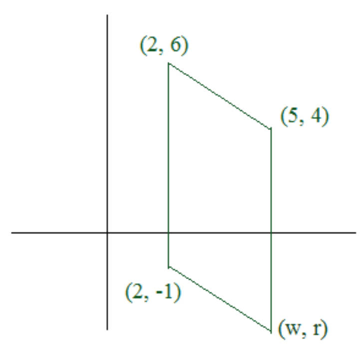
6) Square



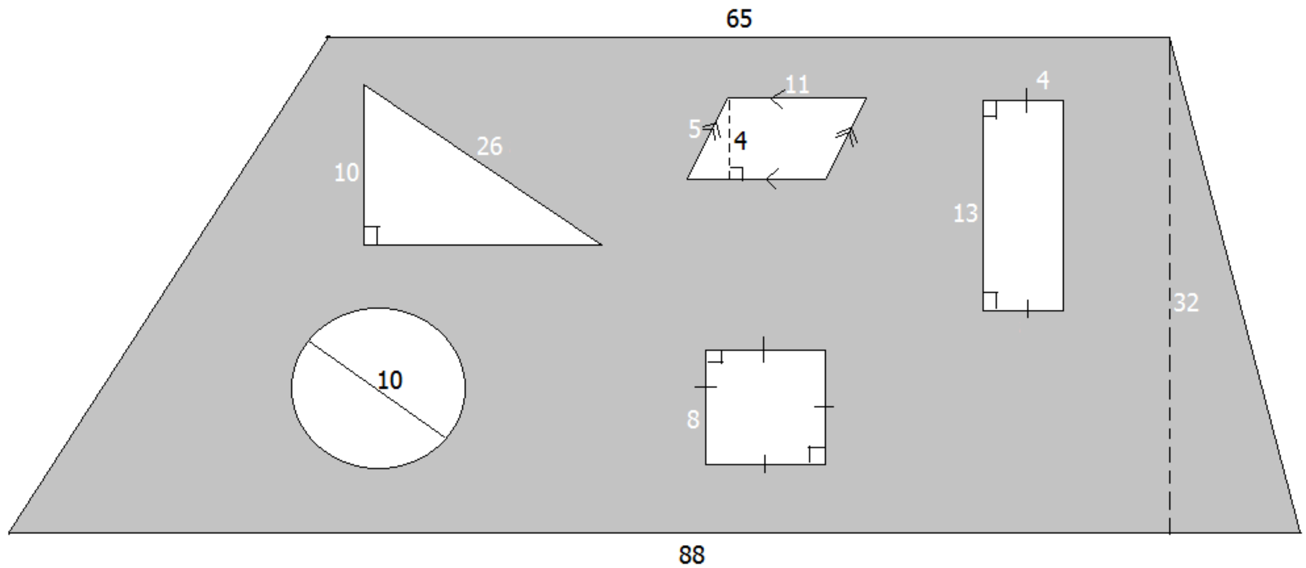
7) Rhombus

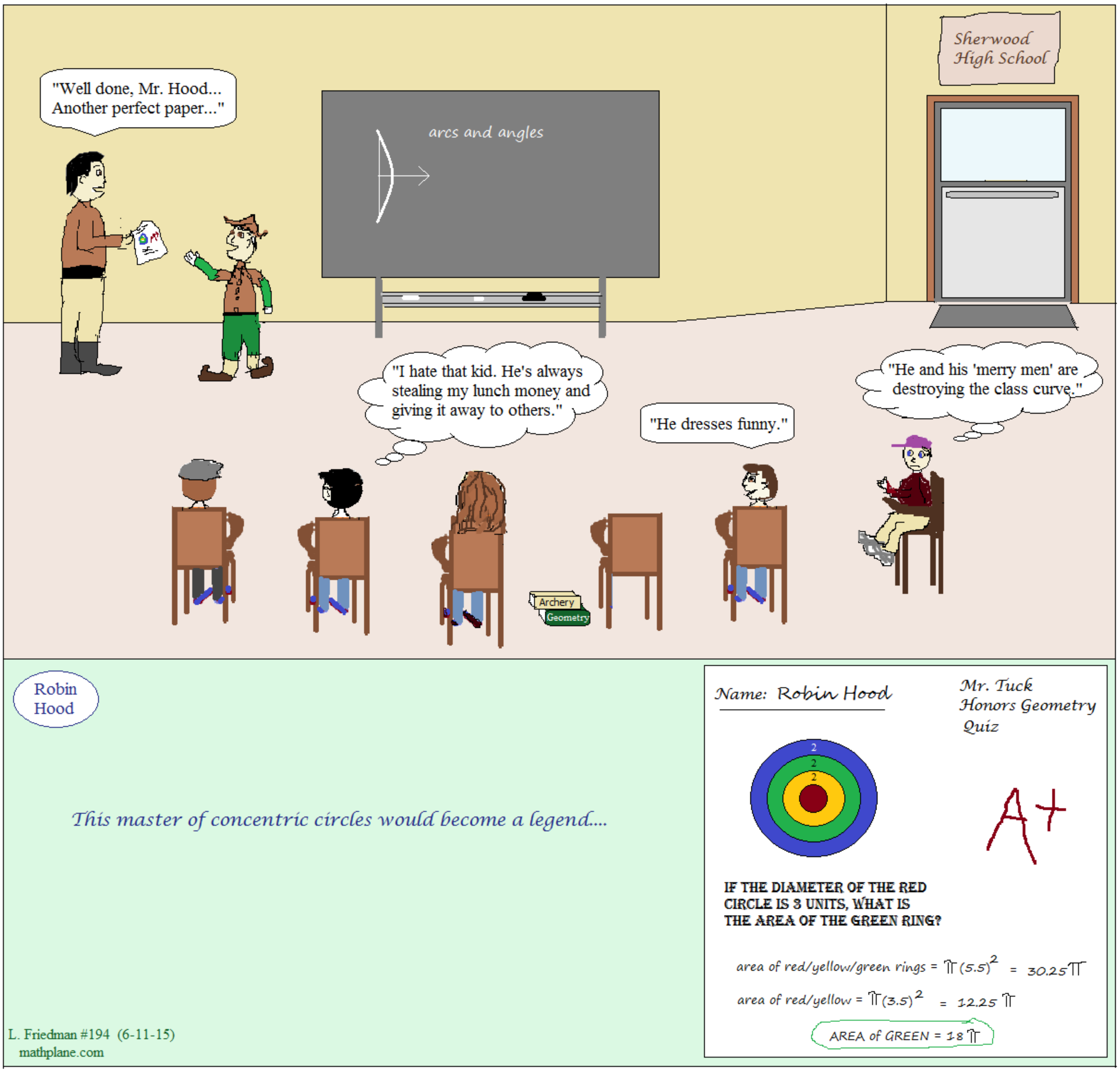


8) Parallelogram



What is the shaded area?





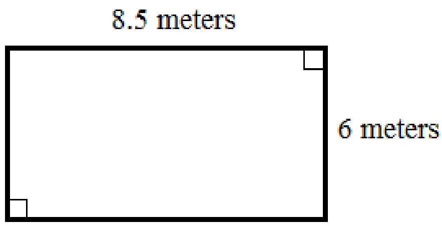
Sherwood High School
"Well done, Mr. Hood... Another perfect paper..."
arcs and angles
"I hate that kid. He's always stealing my lunch money and giving it away to others."
"He dresses funny."
"He and his 'merry men' are destroying the class curve."
Archery
Geometry
Robin Hood
This master of concentric circles would become a legend....
Name: Robin Hood
Mr. Tuck
Honors Geometry
Quiz
2
2
2
A+
IF THE DIAMETER OF THE RED CIRCLE IS 3 UNITS, WHAT IS THE AREA OF THE GREEN RING?
area of red/yellow/green rings = $\pi(5.5)^2 = 30.25\pi$
area of red/yellow = $\pi(3.5)^2 = 12.25\pi$
AREA of GREEN = 18π
L. Friedman #194 (6-11-15)
mathplane.com

SOLUTIONS-→

Determine the area and perimeter of each polygon: SOLUTIONS

Area and Perimeter of Polygons Quiz

1)



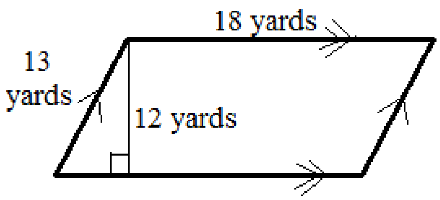
area of rectangle = (length)(width)

= (8.5m)(6m) = $\boxed{\text{51 square meters}}$

perimeter of rectangle = 2(width) + 2(length)

= 2(6m) + 2(8.5m) = $\boxed{\text{29 meters}}$

2)



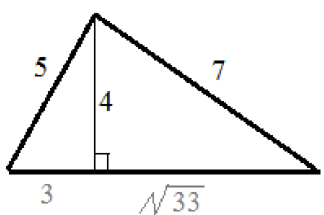
area of parallelogram = (base)(height)

= (18 yds)(12 yds) = $\boxed{216 \text{ yds}^2}$

perimeter of parallelogram = 2(base) + 2(slant height/side)
(add all the sides)

= 2(18 yds) + 2(13 yds) = $\boxed{\text{62 yds}}$

3)



Using quadratic formula (and remembering special right triangles), we can determine the length of the base....

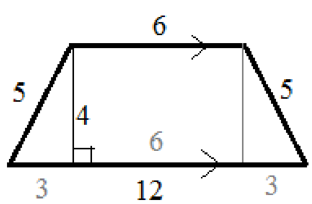
Area of triangle = $\frac{1}{2}$(base)(height)

$= \frac{1}{2}(3 + \sqrt{33})(4) =$ $\boxed{6 + 2\sqrt{33} \text{ or approx. 17.5 square units}}$

Perimeter = $5 + 7 + (3 + \sqrt{33})$

$\boxed{= 15 + \sqrt{33} \text{ or approx. 20.75 units}}$

4)



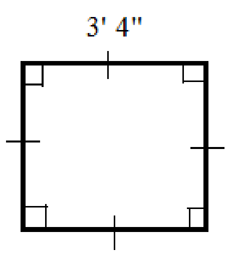
Area of trapezoid = $\frac{1}{2}$(base1 + base2)(height)

$= \frac{1}{2}(12 + 6)(4) =$ $\boxed{\text{36 sq. units}}$

Perimeter = sum of the sides = 5 + 6 + 5 + 12 = $\boxed{\text{28 units}}$

Note: the area of the left triangle is 6 units, the area of the middle rectangle is 24 units, and the area of the right triangle is 6 units... total: 36 units!!

5)



3 feet, 4 inches
= 40 inches

Perimeter of square = 4(side) = 4(40 inches) = $\boxed{\text{160 inches or 13'4"}}$

Area of square = $(\text{side})^2$ = (40inches)(40inches) =
$\boxed{\text{1600 square inches}}$

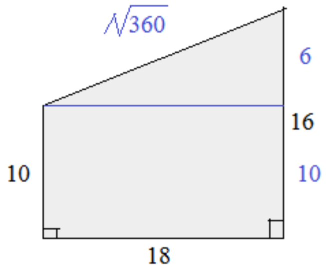
$$\left(\frac{10}{3} \text{ ft}\right)^2 = \frac{100}{9} \text{ ft}^2 = \boxed{\text{11.11 sq. ft}}$$

Determine the area and perimeter of each (shaded) figure:

SOLUTIONS

Area and Perimeter of Polygons Quiz

6)



Area of upper triangle:

$\frac{1}{2}(6)(18) = 54$

Area of lower rectangle:

$(10)(18) = 180$

$\boxed{\text{total area: } 234 \text{ sq. units}}$

Perimeter is sum of all the sides..

$10 + 18 + 16 + \ ?$

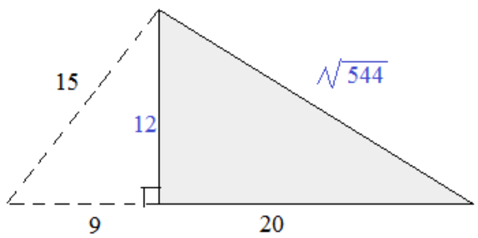
use pythagorean theorem

$18^2 + 6^2 = c^2$

$c = \sqrt{360} = 6\sqrt{10}$

$\boxed{\text{perimeter} = 44 + 6\sqrt{10}}$

7)



Using the small right triangle, we find the height is 12
(9-12-15 Pythagorean Triple)

then, A = 1/2 (base)(height)

$\boxed{A = 1/2\ (20)(12) = 120}$

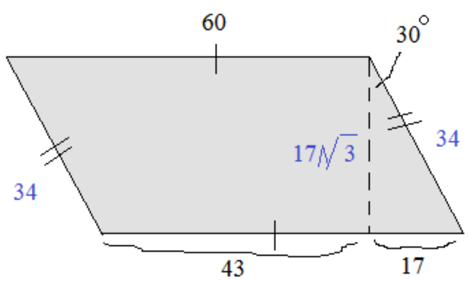
Using Pythagorean Theorem:

$12^2 + 20^2 = c^2$

$c = \sqrt{544} = 4\sqrt{34}$

$\boxed{\text{Perimeter} = 32 + 4\sqrt{34}}$

8)



Recognizing the 30-60-90 right triangle, we can find the length of the parallelogram's, small sides.

$2 \times 17 = 34$

$\boxed{\text{perimeter} = 2(34) + 2(60) = 188}$

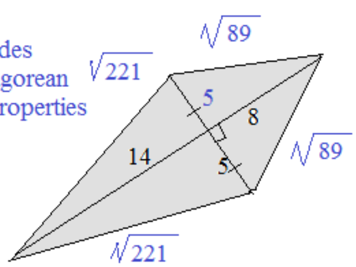
Then, we know the height of the parallelogram is $17\sqrt{3}$

(30-60-90 right triangle)

area = base x height

$= 60 \times 17\sqrt{3} = \boxed{1020\sqrt{3}}$

9) Label all the sides (utilizing pythagorean theorem and properties of a kite)



Perimeter is sum of 4 sides:

$\boxed{2\sqrt{89} + 2\sqrt{221} \text{ units}}$

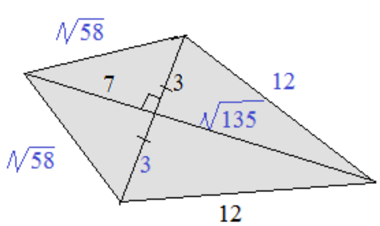
$\text{Area} = \frac{1}{2} d_1 d_2$

$= \frac{1}{2}(10)(22) = \boxed{110 \text{ sq. units}}$

Also, area of kite is sum of 4 right triangles' areas:

35 + 35 + 20 + 20 = 110 sq. units

10)



$\boxed{\text{Perimeter} = 24 + 2\sqrt{58}}$

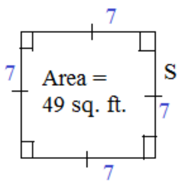
$\text{Area} = \frac{21}{2} + \frac{21}{2} + \frac{3}{2}\sqrt{135} + \frac{3}{2}\sqrt{135}$

$\boxed{= 21 + 9\sqrt{15}}$

or, $\frac{1}{2}(6)(7 + \sqrt{135}) = 21 + 3\sqrt{135}$

Area and Perimeter: finding parts of geometry figures

SOLUTIONS



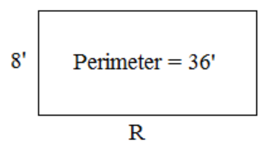
Area of square = $(\text{side})^2$

(S)(S) = 49 sq. ft

S = 7 feet

Perimeter = 28 feet

Rectangle



Perimeter of rectangle = 2(length) + 2(width)

36' = 2R + 2(8')

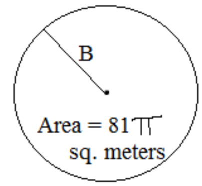
20' = 2R

R = 10 feet

Area = length • width

8' x 10' = 80 sq. ft.

Area = 80 sq. feet



Area = $\pi(\text{radius})^2$

$81\pi = \pi B^2$

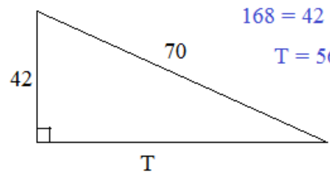
B = 9

B = 9 meters

Circumference = $2\pi(\text{radius})$ OR $\pi(\text{diameter})$

Circumference = 18π meters

Perimeter = 168 inches



168 = 42 + 70 + T

T = 56

Area = 1/2(base)(height)

T = 56 inches

1/2(T)(42")

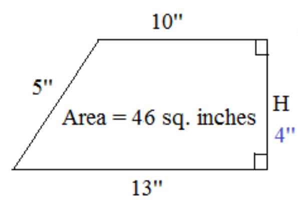
1/2(56")(42")

1176 sq. inches

Area = 1176 inches^2

Trapezoid

Area = $1/2(b_1 + b_2)h$



46 = 1/2(10 + 13)h

92sq. in. = 23"(h)

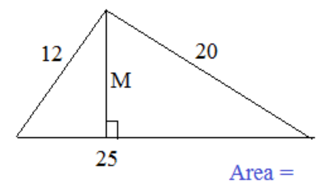
h = 4"

P = 5 + 10 + 4 + 13

H = 4 inches

Perimeter = 32 inches

Area = 100 sq. units



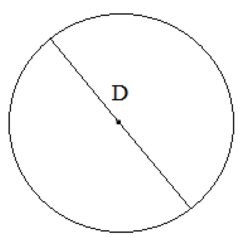
Area = 1/2(base)(height)

M = 8 units

100 = 1/2(25)(M)

M = 8 units

Circumference = 12 feet



(diameter) D = 3.82 feet (approx.)

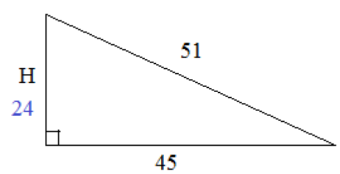
Area = $3.65\,\pi \text{ feet}^2$ or 11.5 sq. feet (approx.)

12 feet = πD

D = 3.82 (approx.)

Circumference = $2\pi(\text{radius})$ OR $\pi(\text{diameter})$

Area = 540 sq. meters



H = 24 meters

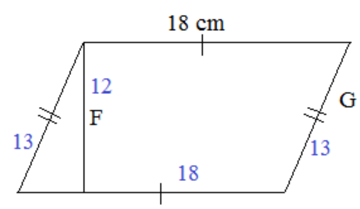
Perimeter = 120 meters

Triangle

Area = 1/2(base)(height)

540 = 1/2(45)(H)

1080 = 45H



Area = 216 cm^2

Perimeter = 62 cm

F = 12cm

G = 13cm

Area of parallelogram = (base)(height)

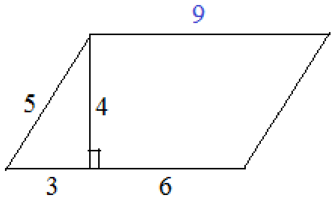
216 cm^2 = 18cm (F)

12cm = F

Answer the following questions: SOLUTIONS

Area and Perimeter of Polygons Quiz

1) What is the area of the parallelogram? The perimeter?



Area = (base)(height) = (9)(4) = $\boxed{36 \text{ sq. units}}$

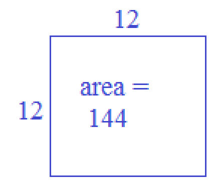
Perimeter = sum of the sides = 5 + 9 + 5 + 9 = $\boxed{28 \text{ units}}$

2) If the area of a square is 144 square feet, what is the perimeter?

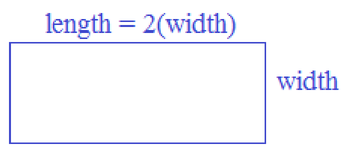
Area = (side)(side)

144 = $(\text{side})^2$ square root both sides 12 = side

If each side = 12, then the perimeter is 4 x 12 = $\boxed{48 \text{ units}}$



3) The length of a rectangle is twice its width. If the perimeter is 66cm, what is the area?



Perimeter: 2(length) + 2(width) = 66cm

2(2w) + 2w = 66cm

6w = 66cm

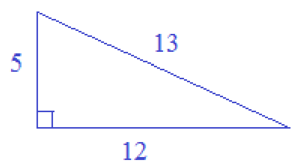
width = 11

length = 22

Area = (11cm)(12cm)

$\boxed{= 132 \text{ cm}^2}$

4) What is the area of a 5-12-13 special right triangle?



$$\text{Area} = \frac{1}{2}(\text{base})(\text{height})$$

$$= \frac{1}{2}(12)(5) = \boxed{30 \text{ square units}}$$

**Challenge: Given: Rectangle ABCD

$\overline{AC} = 2x + 5$

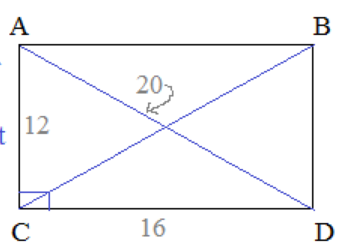
$\overline{AD} = 6x - 1$

$\overline{BD} = 3y - 6$

$\overline{BC} = 2y + 8$

What is the perimeter of $\square$ABDC?

diagonals of a rectangle are congruent



1) $\overline{AD} = \overline{BC}$

$6x - 1 = 2y + 8$

$6x - 2y = 9$

2) $\overline{AC} = \overline{BD}$

$2x + 5 = 3y - 6$

$2x - 3y = -11$

3) 2 equations with 2 unknowns

$6x - 2y = 9$

$2x - 3y = -11$

$6x - 9y = -33$

(elimination method)

4) $7y = 42$

$y = 6$

then, $x = \frac{7}{2}$

5) (Pythagorean theorem)

$AC^2 + CD^2 = BC^2$

$(12)^2 + CD^2 = (20)^2$

$CD = 16$

Perimeter =

AB + BD + CD + AC

16 + 12 + 16 + 12

$\boxed{56 \text{ units}}$

Can you answer the following?

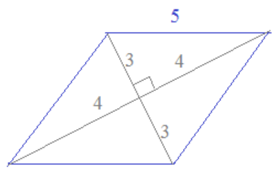
SOLUTIONS

Area and Perimeter of Polygons Quiz

A) The diagonals of a rhombus are 6 and 8. What is the height?

Step 1: Use the formula for a rhombus to find the area...

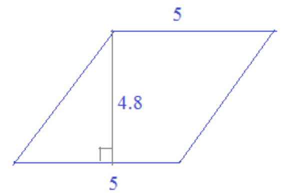
$$\text{Area} = \frac{1}{2}(\text{diagonal 1})(\text{diagonal 2}) = \frac{1}{2}(6 \times 8) = 24$$



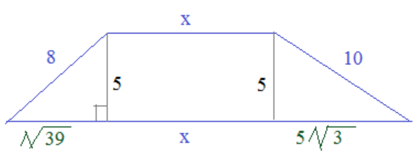
Diagonals are perpendicular bisectors...

Step 2: Use the formula for a parallelogram to identify the height...

Area = (base)(height) ----> 24 = (5)(height) $\boxed{\text{height} = 4.8}$



B) A trapezoid with perimeter 44 has non-parallel sides of length 8 and 10.
If the height is 5, what is the area of the trapezoid?
What is the length of each base?



(Use Pythagorean Theorem to get missing lengths)

$$8 + 10 + 2x + \sqrt{39} + 5\sqrt{3} = 44$$

$$2x + 6.24 + 8.66 = 26$$

$$2x = 11.1$$

$$x = 5.55$$

Bases: 5.55
20.45

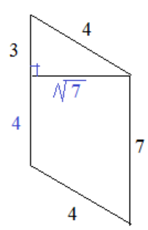
Area of Trapezoid =

$$\frac{1}{2}(\text{base 1} + \text{base 2})(\text{height})$$

$$\frac{1}{2}(26)(5) = \boxed{65 \text{ sq units}}$$

C) What is the area and perimeter of each parallelogram?

1)

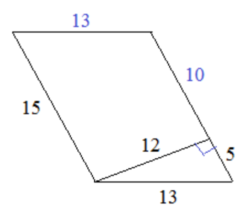


Area = (base)(height)

$= 7\sqrt{7}$

Perimeter = 22

2)

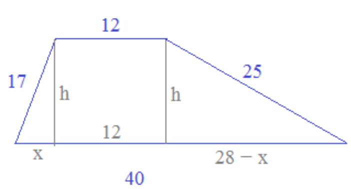


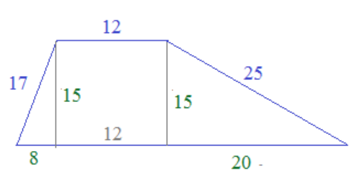
Perimeter = 56 units

Area = (1/2)(15)(12)

= 90 sq. units

D) Find the area of a trapezoid with sides 12, 17, 40, and 25
where 12 and 40 are the bases.





$$x^2 + h^2 = 17^2$$

$$(28 - x)^2 + h^2 = 25^2$$

$$h^2 = 289 - x^2$$

$$h^2 = 625 - (784 - 56x + x^2)$$

$$289 - x^2 = 625 - 784 + 56x - x^2$$

$$448 = 56x$$

$$x = 8 \qquad \text{then, } h = 15$$

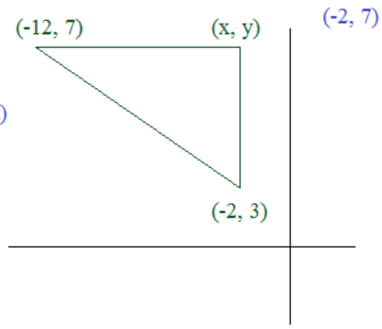
$$\text{area} = (1/2)(40 + 12)(15) = \boxed{390 \text{ sq units}}$$

Identify the missing coordinates. Then, find the area of each figure.

SOLUTIONS

Quadrilaterals, Triangles, and Coordinates

1) Right Triangle

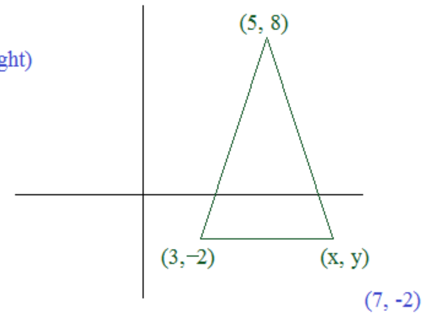


Area = $\frac{1}{2}$(base)(height)

= $\frac{1}{2}$ (4)(10)

= 20 sq. units

2) Isosceles Triangle



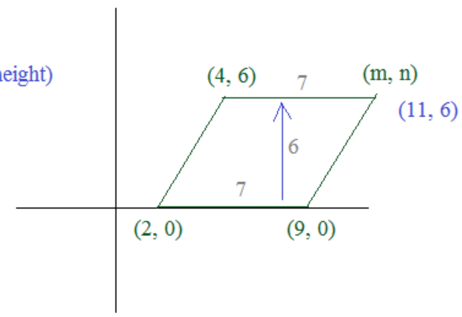
Area = $\frac{1}{2}$(base)(height)

= $\frac{1}{2}$ (4)(10)

= 20

3) Parallelogram

(opposite sides congruent)

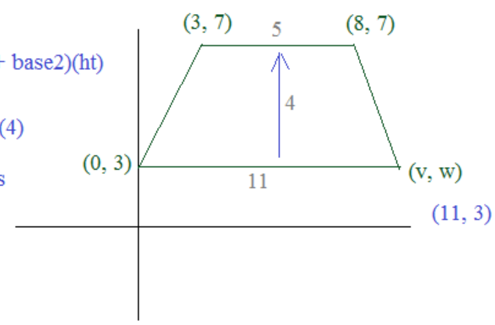


Area = (base)(height)

= (7)(6)

= 42

4) Isosceles Trapezoid

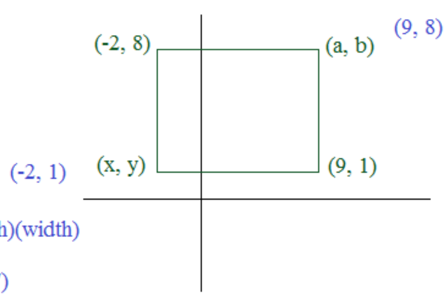


Area = $\frac{1}{2}$(base1 + base2)(ht)

= $\frac{1}{2}$(11 + 5)(4)

= 32 sq. units

5) Rectangle

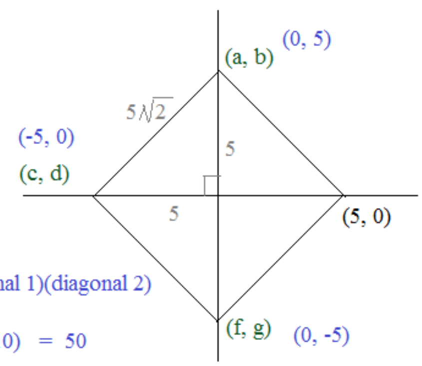


Area = (length)(width)

= (11)(7)

= 77

6) Square



Area = (side)^2

= $(5\sqrt{2})^2$

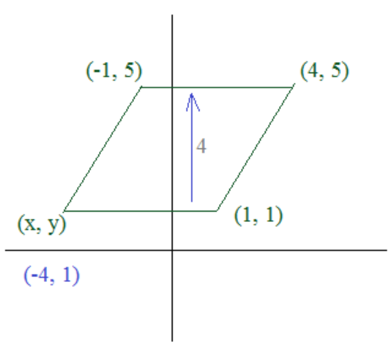
= 50

Or,

Area = $\frac{1}{2}$ (diagonal 1)(diagonal 2)

= $\frac{1}{2}$ (10)(10) = 50

7) Rhombus



Area = (base)(height)

= (4)(5)

= 20

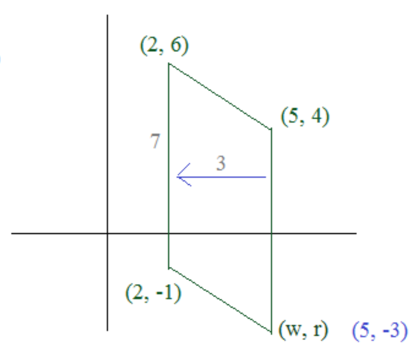
Or,

Area = $\frac{1}{2}$(d1)(d2)

= $\frac{1}{2}$ $(\sqrt{80})(\sqrt{20})$

= 20

8) Parallelogram

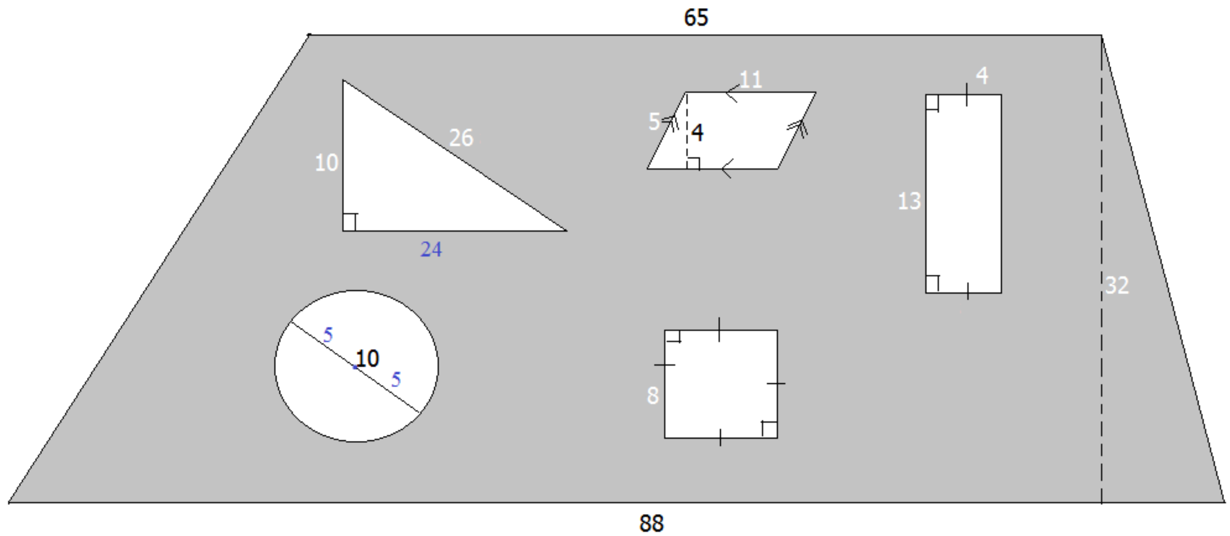


Area = (base)(height)

= (7)(3)

= 21 sq. units

What is the shaded area?



square: $S^2 = (8)^2 = 64$

circle: $\pi\,(\text{radius})^2 = \pi\,(5)^2 = 25\pi$

triangle: $\frac{1}{2}(\text{base})(\text{height}) = \frac{1}{2}(10)(24) = 120$

parallelogram: $(\text{base})(\text{height}) = (11)(4) = 44$

rectangle: $(\text{length})(\text{width}) = (13)(4) = 52$

total ("cut out") area $= 280 + 25\pi$ or approximately 358.5

trapezoid: $\frac{1}{2}(\text{base}_1 + \text{base}_2)(\text{height}) = \frac{1}{2}(65 + 88)(32) = 2448$

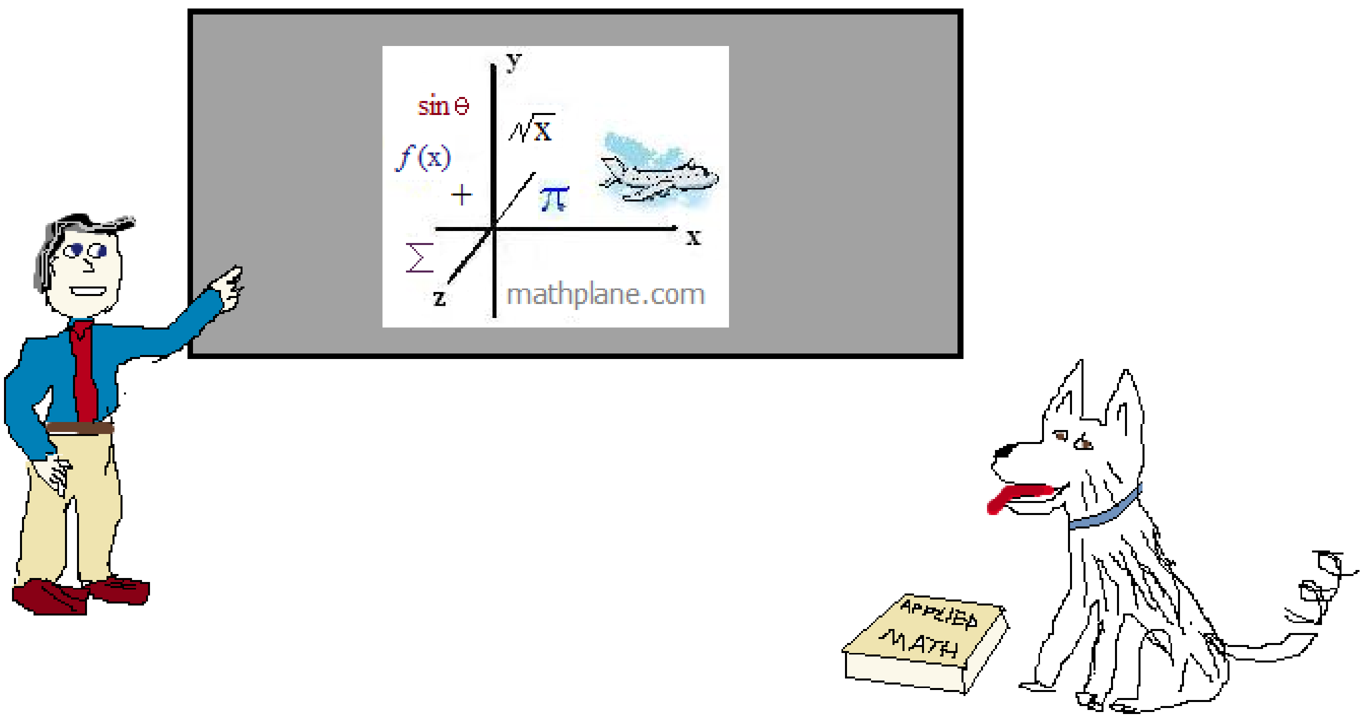
Total shaded area = trapezoid − "cut out" shapes

$= 2448 - 358.5 = 2089.5$

Thanks for visiting the site. (Hope it helped!)

If you have questions, suggestions, or requests, just let us know.

Best of luck!



Also, at mathplane.ORG for mobile

And, our stores at TES and TeachersPayTeachers

One more question!

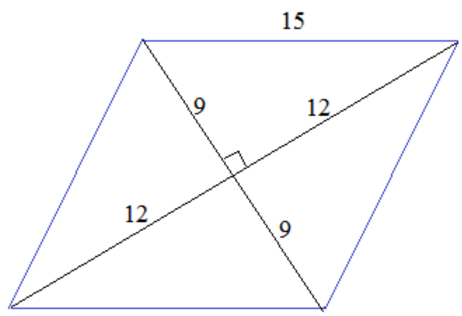
A rhombus has diagonals that measure 18 and 24.

What is the <u>height</u> of the rhombus?

(Answer on next page)

Challenge Question:	A rhombus has diagonals that measure 18 and 24. What is the <u>height</u> of the rhombus?

Step 1: Draw a diagram



Step 2: Recognize useful formulas and quadrilateral properties

the diagonals of a rhombus are perpendicular bisectors

Therefore, the sides of the rhombus are 15

Step 3: Solve

area of rhombus: $\frac{1}{2}$(diagonal 1)(diagonal 2)

$$\frac{1}{2}(18)(24) = 216$$

area of a parallelogram: (base)(height)

216 = (base)(height)

216 = (15)(height)   height = 14.4 units