Factoring and Simplifying Rational Polynomials

Notes, Examples, and Worksheets (with solutions)

Topics included quadratic formula, factor by grouping, difference of squares, and more.

Simplifying Rational Polynomials

What is a "rational polynomial"? A polynomial that has fractional coefficient(s). (i.e. rational coefficients)

Examples:

$.5x^2 - 3x + 10$ — The lead coefficient is .5 or 1/2

$\frac{x^3}{4} + 8x + 1$ — The lead coefficient is 1/4

$x^2 + \frac{3}{5}x + 1/4$ — The coefficient of the *linear term* is 3/5

$3x^2 + 2x + 11$ — The coefficients of this trinomial are 3, 2, and 11 --- all rational numbers!

If we want to "simplify" a rational polynomial, we must collect "like terms".

Examples:

$x^2 + 5 + 3x + .5x^2 + 4$

$\longrightarrow 1.5x^2 + 3x + 9$

$3x + 3y + 10xy + 6 + 4y - 1/3$

$\longrightarrow 3x + 7y + 10xy + \frac{17}{3}$

And, sort the terms in descending order.

Examples:

$3 - 4x + 2x^2$

$\longrightarrow 2x^2 - 4x + 3$ (ignoring the coefficients), we sort from highest *degree* to lowest degree...

$4x + 11x^3 + 3 + 5x^5$

$\longrightarrow 5x^5 + 11x^3 + 3 + 4x$

$x^3 + 3y^4x^2 + 2yx^5 + 7b$

$\longrightarrow 2yx^5 + x^3 + 3y^4x^2 + 7b$

note: we rank according to the degree of the main variable x

Simplifying Rational Polynomials

Also, we may decide to remove any fractions.
If so, (similar to ordinary fractions), we identify the least common denominator (or, least common multiple)

Examples: simplify the following equations:

$$y = \frac{2}{5}x^2 + \frac{1}{3}x + 2$$

The least common multiple of 5, 3, and 1 is 15...
So, multiply the equation by 15....

$$15y = 6x^2 + 5x + 30$$

$$y = .6x^3 + .3x^2 + 1.5$$

there are 2 ways to look at this equation:
a) decimals -- simply multiply by 10 to remove the decimals...

$$10y = 6x^3 + 3x^2 + 15$$

b) fractions -- convert to fraction and find least common multiple

$$y = \frac{3}{5}x^3 + \frac{3}{10}x^2 + \frac{3}{2}$$

least common multiple of 2, 5, and 10 is 10....
so, multiply both sides of equation by 10

$$10y = 6x^3 + 3x^2 + 15$$

$$y = 4 + \frac{x}{7} - \frac{x^2}{3} + \frac{x^3}{2}$$

The coefficients of the polynomial are 1, 1/7, -1/3, and 1/2..
The least common multiply of 1, 7, 3, and 2 is 42....
So, multiply the polynomial by 42...
(and, write the terms in descending order)

$$42y = 21x^3 - 14x^2 + 6x + 168$$

Example: Simplify the following:

$$y = 3x^2 + 2xy + 4 - \frac{x^2}{3} + 3 + .2x^3$$

Combine "like" terms: $y = \frac{8x^2}{3} + 2xy + 7 + .2x^3$

Order terms: $y = .2x^3 + \frac{8x^2}{3} + 2xy + 7$

(Optional) Remove fractions: $15y = 3x^3 + 40x^2 + 30xy + 105$

Factoring (4 term) Polynomials: Grouping

> *Factor by 'Grouping'*
>
> 1) Separate polynomial into groups
> 2) Factor each group (using Greatest Common Factor)
> 3) Merge and re-group

Example 1: $y^3 + 2y^2 - 81y - 162$

Solution A: $y^3 + 2y^2 \quad - 81y - 162$ — *Separate the polynomial*

$y^2(y + 2) \quad - 81(y + 2)$ — *Factor each group* (using GCF)

$(y^2 - 81)(y + 2)$ — *Merge and re-group*

$(y - 9)(y + 9)(y + 2)$

Solution B: $y^3 - 81y \quad + 2y^2 - 162$

$y(y^2 - 81) \quad + 2(y^2 - 81)$

$(y + 2)(y^2 - 81)$

$(y + 2)(y + 9)(y - 9)$

Note: Although Solutions A and B approach the polynomial differently, the outcome is the same!

Example 2: $b^3 + b^2 = 64b + 64$

$b^3 + b^2 - 64b - 64 = 0$ — *Write equation (setting polynomial equal to zero)*

$b^2(b + 1) \quad - 64(b + 1) = 0$ — *Separate into groups and find GCF's*

$(b^2 - 64)(b + 1) = 0$ — *Merge and regroup*

$(b + 8)(b - 8)(b + 1) = 0$ — *Factor further*

$b = -8, 8, -1$ — *Solve*

Then, check your solutions:

Substitute into the original equation

$b = -8$: $(-8)^3 + (-8)^2 = 64(-8) + 64$

$-512 + 64 = -512 + 64$ ✓

$b = 8$: $(8)^3 + (8)^2 = 64(8) + 64$

$512 + 64 = 512 + 64$ ✓

$b = +1$: $(-1)^3 + (-1)^2 = 64(-1) + 64$

$-1 + 1 = -64 + 64$ ✓

Factoring (4 term) polynomials: Grouping (continued)

Example 3: $-4m^4 - 10m^3 + 16m^2 + 40m = 0$

$-m(4m^3 + 10m^2 - 16m - 40) = 0$ — *Greatest common factor*

$-m(4m^3 + 10m^2 \quad - 16m - 40) = 0$ — *Factor (by grouping)*

$-m(2m^2(2m+5) - 8(2m+5)) = 0$

$-m(2m^2 - 8)(2m+5) = 0$

Simplify further...

$-2m(m^2 - 4)(2m+5) = 0$

$-2m(m+2)(m-2)(2m+5) = 0$

Solve

$m = 0, -2, 2, -5/2$

Comments:

1) Instead of factoring out m, I factored out -m, because I prefer working with a leading coefficient that is positive.

2) Do not divide the equation by m, because you will "lose one of the solutions". Instead, factor out the variable.

Check solutions:

$m = 0: \quad -4(0)^4 - 10(0)^3 + 16(0)^2 + 40(0) = 0$

$0 = 0$ ✓

$m = -2: \quad -4(-2)^4 - 10(-2)^3 + 16(-2)^2 + 40(-2) = 0$

$-4(16) - 10(-8) + 16(4) - 80 = 0$

$-64 + 80 + 64 - 80 = 0$ ✓

$m = 2: \quad -4(2)^4 - 10(2)^3 + 16(2)^2 + 40(2) = 0$

$-64 - 80 + 64 + 80 = 0$ ✓

$m = -5/2: \quad -4(5/2)^4 - 10(-5/2)^3 + 16(-5/2)^2 + 40(-5/2) = 0$

$-4\left(\frac{625}{16}\right) - 10\left(\frac{-125}{8}\right) + 16\left(\frac{25}{4}\right) - 100 = 0$

$-\frac{625}{4} - \frac{-625}{4} + 100 - 100 = 0$ ✓

Example 4: The volume of the sketched box is 60 cubic feet. What are the measurements of the length, width, and height?

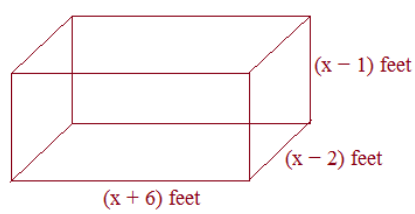
Volume = (length)(width)(height)

$60 = (x+6)(x-2)(x-1)$

$60 = (x^2 + 4x - 12)(x-1)$

$60 = x^3 + 4x^2 - 12x - x^2 - 4x + 12$

$60 = x^3 + 3x^2 - 16x + 12$



Find x:

$x^3 + 3x^2 - 16x - 48 = 0$

$x^2(x+3) - 16(x+3) = 0$

$(x^2 - 16)(x+3) = 0$

$(x-4)(x+4)(x+3) = 0$

$x = 4, -4, -3$

Answer question/check solutions:

~~If x = -3, then length = 3, width = -5, height = -4~~

~~If x = -4, then length = 2, width = -6, height = -5~~

Cannot have negative measurements!!

If x = 4, then length is 10 feet
width is 2 feet
height is 3 feet

Volume is 60 cubic feet...


Googol Search
"Siri, how many zeros are in a googol?"
"There are 2 O's in google."
"Alexa, what is the number 'googol'?"
"The phone number of google headquarters is 6-5-0-...."
"Maybe you should try a basic internet search engine
... or, a dictionary..."
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search results....
Unfortunately, 0 were useful....

Worksheets-→

Polynomial Factoring Test

I. Simplify

$(3x^2 + 4x - 17) + (15 + 2x^2) =$

$3(x + 4) + 2(x^2 + 3x - 1) =$

$2(y^2 + 5y + 8) - 3(y^2 - y + 12) =$

II. Factor

$s^2 + 6s + 9$

$x^2 - 10x + 9$

$3y^2 + 18y + 24$

$4x^2 - 49$

$4n^2 + 12n + 9$

$5x^2 - 13x - 6$

III. Find all solutions

$x^2 + 11x + 28 = 0$

$x^2 - 2x - 35 = 0$

$x^3 + 7x^2 - 18x = 0$

$z^2 - 19z + 90 = 0$

$4m^2 - m - 5 = 0$

$3x^2 + 4x + 1 = 0$

Polynomial Factoring Test (continued)

IV: Simplify

$$\frac{5}{(x-3)} + \frac{(x+7)}{(x^2-9)} =$$

$$\frac{3}{x^2} + \frac{4}{x} =$$

$$\frac{5x}{x+3} - \frac{3}{x+8} =$$

$$\frac{(x^2-25)}{(x^2+6x+5)} \cdot \frac{(2x^3+2x^2)}{(x-5)} =$$

$$\frac{x^2+8x+7}{x^2-1} \cdot \frac{3x-3}{x+7} =$$

$$\frac{x^2-10x-11}{x-5} \div (x^2+6x+5) =$$

$$\left(\frac{x^2+5x+4}{x^2+2x-8}\right) \div \left(\frac{3x^2+x-2}{x^2-4}\right) =$$

V: Find solutions using quadratic formula

$x^2+3x-8=0$

$x^2-5x-14=0$

$3x^2+x-10=0$

$x^2+3x+8=0$

$\frac{x^2}{2} + 6x+3=0$

Polynomial Factoring Test

SOLUTIONS

I. Simplify

$(3x^2 + 4x - 17) + (15 + 2x^2) =$

(add "like" terms)

$5x^2 + 4x - 2$

$3(x + 4) + 2(x^2 + 3x - 1) =$

(distribute)

$3x + 12 \quad + \quad 2x^2 + 6x - 2$

(combine terms)

$2x^2 + 9x + 10$

$2(y^2 + 5y + 8) - 3(y^2 - y + 12) =$

$2y^2 + 10y + 16 - \left(3y^2 - 3y + 36\right)$

$-y^2 + 13y - 20$

II. Factor

$s^2 + 6s + 9$

what multiplies to 9 and adds to 6? 3, 3

$(s + 3)(s + 3)$

$x^2 - 10x + 9$

multiplies to 9 and adds to -10 ?
-1 and -9

$(x - 9)(x - 1)$

$3y^2 + 18y + 24$

(Take out Greatest Common Factor)

$3(y^2 + 6y + 8)$

Then, factor quadratic...

$3(y + 2)(y + 4)$

$4x^2 - 49$

Difference of squares!

square root of 1st term: 2x
square root of 2nd term: 7

$(2x + 7)(2x - 7)$

$4n^2 + 12n + 9$

A = 4 B = 12 C = 9

Since $\sqrt{AC} = 2B$,

it's a "squared binomial"
(or, a "perfect square trinomial")

$\sqrt{4n^2} = 2n \qquad \sqrt{9} = 3$

$(2n + 3)(2n + 3) = (2n + 3)^2$

$5x^2 - 13x - 6$

what multiplies to -30 and adds to -13?

-15 and 2

$(5x^2 + -15x) + (2x - 6)$

factor and re-group

$5x(x - 3) + 2(x - 3)$

$(5x + 2)(x - 3)$

note: to check answer, FOIL

First: $5x^2$
Outer: $-15x$
Inner: $2x$
Last: -6

$5x^2 - 13x - 6$ ✓

III. Find all solutions

$x^2 + 11x + 28 = 0$

$(x + 7)(x + 4) = 0$

$x = -4, -7$

$x^2 - 2x - 35 = 0$

$(x - 7)(x + 5) = 0$

$x = -5, 7$

$x^3 + 7x^2 - 18x = 0$

Factor out GCF: x

$x(x^2 + 7x - 18) = 0$

$x(x + 9)(x - 2) = 0$

$x = 0, -9, 2$

$z^2 - 19z + 90 = 0$

multiplies to 90 and adds to -19 ? -9 and -10

$(z - 9)(z - 10) = 0$

$z = 9, 10$

note: to check solutions, plug answers into original equation

$9^2 - 19(9) + 90 = 81 - 171 + 90 = 0$ ✓

$10^2 - 19(10) + 90 = 100 - 190 + 90 = 0$ ✓

$4m^2 - m - 5 = 0$

$(4m - 5)(m + 1) = 0$

$m = 5/4, -1$

$3x^2 + 4x + 1 = 0$

$(3x + 1)(x + 1) = 0$

$3x + 1 = 0 \quad x = -1/3$

$x + 1 = 0 \quad x = -1$

Polynomial Factoring Test (continued)

SOLUTIONS

IV: Simplify

$$\frac{5}{(x-3)} + \frac{(x+7)}{(x^2-9)} =$$

$$\left(\frac{(x+3)}{(x+3)}\right)\frac{5}{(x-3)} + \frac{(x+7)}{(x+3)(x-3)} =$$

$$\frac{5x+15 \quad + \quad (x+7)}{(x+3)(x-3)} = \boxed{\frac{6x+22}{(x+3)(x-3)}}$$

$$\frac{3}{x^2} + \frac{4}{x} =$$

$$\frac{3}{x^2} + \frac{4x}{x^2} =$$

$$\boxed{\frac{4x+3}{x^2}}$$

$$\frac{5x}{x+3} - \frac{3}{x+8} =$$

$$\frac{(x+8) \quad 5x}{(x+8)(x+3)} - \frac{3 \quad (x+3)}{(x+8)(x+3)} =$$

$$\frac{5x^2+40x - (3x+9)}{(x+8)(x+3)} = \boxed{\frac{5x^2+37x-9}{(x+8)(x+3)}}$$

$$\frac{(x^2-25)}{(x^2+6x+5)} \cdot \frac{(2x^3+2x^2)}{(x-5)} =$$

(factor) $$\frac{(x+5)(x-5)}{(x+1)(x+5)} \cdot \frac{2x^2(x+1)}{(x-5)}$$

(cancel) $$\frac{\cancel{(x+5)}\cancel{(x-5)}}{\cancel{(x+1)}\cancel{(x+5)}} \cdot \frac{2x^2\cancel{(x+1)}}{\cancel{(x-5)}}$$

$$\boxed{2x^2}$$

$$\frac{x^2+8x+7}{x^2-1} \cdot \frac{3x-3}{x+7} =$$

$$\frac{(x+7)(x+1)}{(x+1)(x-1)} \cdot \frac{3(x-1)}{(x+7)}$$ (factor)

$$\frac{\cancel{(x+7)}\cancel{(x+1)}}{\cancel{(x+1)}\cancel{(x-1)}} \cdot \frac{3\cancel{(x-1)}}{\cancel{(x+7)}}$$ (cancel)

$$\boxed{3}$$

$$\frac{x^2-10x-11}{x-5} \div (x^2+6x+5) =$$

(invert and multiply)

$$\frac{x^2-10x-11}{x-5} \cdot \frac{1}{(x^2+6x+5)} =$$

(factor and cancel)

$$\frac{(x-11)\cancel{(x+1)}}{(x-5)} \cdot \frac{1}{\cancel{(x+1)}(x+5)} = \boxed{\frac{x-11}{(x+5)(x-5)}}$$

$$\left(\frac{x^2+5x+4}{x^2+2x-8}\right) \div \left(\frac{3x^2+x-2}{x^2-4}\right) =$$

$$\frac{(x+1)(x+4)}{(x+4)(x-2)} \cdot \frac{(x+2)(x-2)}{(3x-2)(x+1)} =$$

$$\frac{\cancel{(x+1)}\cancel{(x+4)}}{\cancel{(x+4)}\cancel{(x-2)}} \cdot \frac{(x+2)\cancel{(x-2)}}{(3x-2)\cancel{(x+1)}}$$

$$\boxed{\frac{(x+2)}{(3x-2)}}$$

V: Find solutions using quadratic formula

$$x^2+3x-8=0$$

A = 1
B = 3
C = − 8

$$\frac{-3 \pm \sqrt{3^2-4(1)(-8)}}{2(1)}$$

$$\boxed{x = \frac{-3 \pm \sqrt{41}}{2}}$$

approximately 1.70 and −4.70

$$x^2-5x-14=0$$

$$\frac{5 \pm \sqrt{25+56}}{2} = x$$

$$\frac{5+9}{2} = 7$$

$$\frac{5-9}{2} = -2$$

$$3x^2+x-10=0$$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$\frac{-1 \pm \sqrt{(1)^2-4(3)(-10)}}{2(3)}$$

$$\frac{-1 \pm \sqrt{121}}{6} = \boxed{\frac{5}{3}, \ -2}$$

$$x^2+3x+8=0$$

The discriminant is b^2-4ac

$$(3)^2-4(1)(8)<0$$

There are no real solutions

$$\frac{x^2}{2} + 6x + 3 = 0$$

$$\frac{-6 \pm \sqrt{(6)^2-4(1/2)(3)}}{2(1/2)}$$

$$\frac{-6 \pm \sqrt{30}}{1}$$

approximately: -11.4 and -0.52

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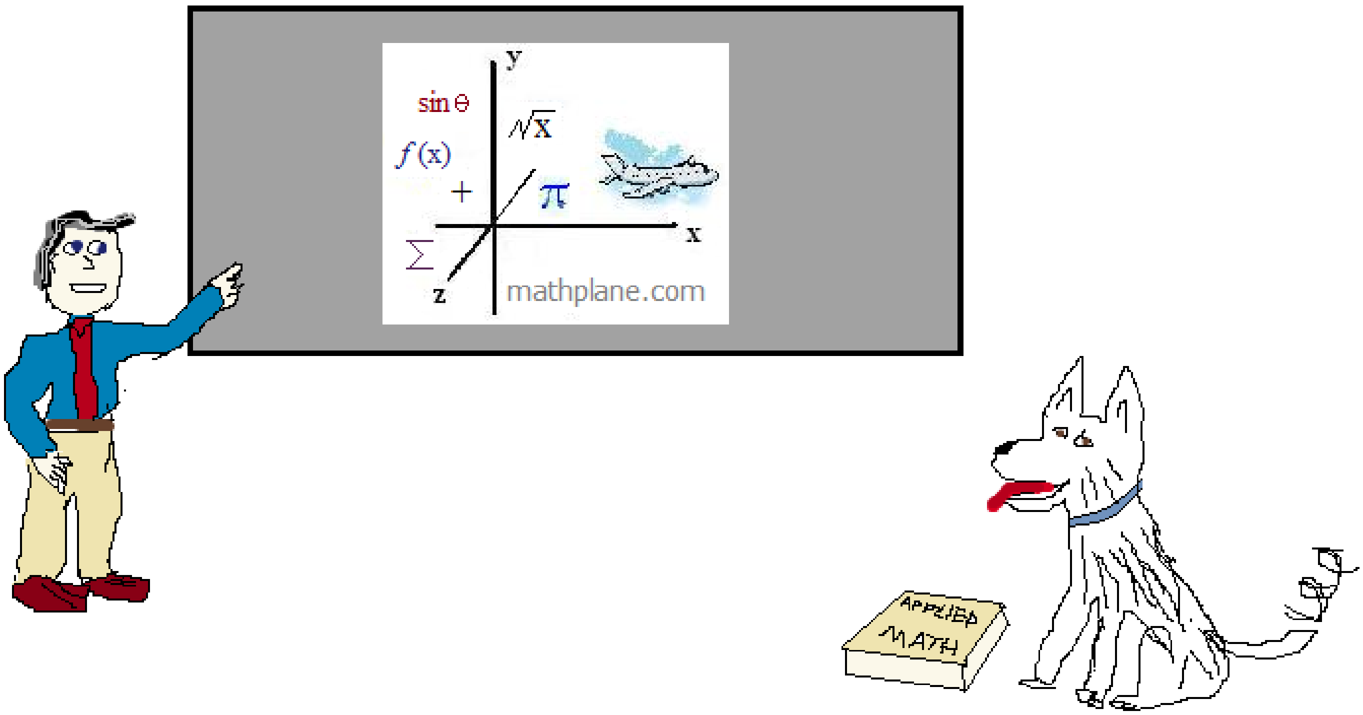
Thank you for visiting. (Hope it helped!)

If you have questions, suggestions, or requests, let us know.

Cheers.



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