Algebra: Introduction to Polynomials

Definition, Notes, Examples, and Quizzes (w/Solutions)

Defining Polynomial Expressions

What is a 'polynomial'?

A *sum/difference of terms* that have *variables raised to positive integers* and *coeffients that are real or complex*.

Examples:

$x^3 + 3x^2 - 2x + 7$ yes

$3xy^2 + 6$ yes

$.445z^3 - 3i$ yes

$\frac{3}{x^2}$ no $\frac{3}{x^2} = 3x^{-2}$ (Variables must have exponents with positive integers.)

5^x no

Classifying Polynomial Expressions

There are 2 ways to describe polynomials:

1) The number of monomials ('number of non-zero terms')

# of terms	Classification	Examples		
1	Monomial	xy^2	7	$-2z^3$
2	Binomial	$3x + 4$	$-9y^7 - y^2$	$xyz + x^2z^3$
3	Trinomial	$7 + xyz - 3i$	$x^2 - xy + 3y^4$	
Any	Polynomial	$4x$	$x + 3y + 7z - 23xy^2$	

Comment: When counting terms, consider "like terms"

$3x + 2y^2 + 4y^2$ is a binomial (because there is $3x$ and $6y^2$)

2) The largest degree of any one term

common polynomials ⇒

Degree	Classification	Examples		
0	Constant	3	-5	$4/5$
1	Linear	$y + 6$	$3x$	$.23z - 12$
2	Quadratic	$x^2 + 2x - 7$	$6 + y^2$	$x^2 + y^2 - 6y$
3	Cubic	$x^3 + 1$	$2 + 3y - 4y^2 + y^3$	$xy^2 + x + 3$
4	Quartic	$x^4 + x^2y + 4y + 1$	$yz - 6x^4$	
5	Quintic	$x^2y^3 + 7x - 3$		

Comments:

— xy^2 has degree 3 and x^2y^3 has degree 5

— The order of the terms does not matter.

— There are polynomials with larger degrees that are not listed above.
EX: 'octic' 'nonic' 'decic'

Defining/Classifying Polynomials

Examples:

	Number	Degree	Expression
$x^2 + 6x + 5$	3	2	Quadratic Trinomial
$x^3y^5 - 3y + 7x$	3	8	Octic Trinomial
$y - 2$	2	1	Linear Binomial
$z + 2z^2 - 7z^3$	3	3	Cubic Trinomial
$x^2 + 3x + 8 - x$ (note: you must combine "like terms")	3	2	Quadratic Trinomial

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Quick Quiz:

Define any polynomials in the following expressions:

1) $x + 3y - 2$

2) $2 + 3x - 4x^2$

3) $\frac{1}{y^3}$

4) $12y + 3/4$

5) $z^3 + 1$

6) $3x^2y^5 + 2xy + 8$

7) $x^2y + \sqrt{7}$

8) $4^2r^3s^2$

9) $x^2 + xy - 10x + 3$

10) $x^2\sqrt{x + 2}$

**Challenge: (Using Examples)
Show that multiplying 2 linear binomials can produce a quadratic trinomial.
Then, show that multiplying 2 linear binomials can produce a quadratic binomial.

Quick Quiz:

Define the following Expressions:

	Expression	Number of monomials	Highest Degree	Definition
1)	$x + 3y - 2$	3	1	Linear Trinomial
2)	$2 + 3x - 4x^2$	3	2	Quadratic Trinomial
3)	$\frac{1}{y^3}$	0	0	----------------
4)	$12y + 3/4$	2	1	Linear Binomial
5)	$z^3 + 1$	2	3	Cubic Binomial
6)	$3x^2y^5 + 2xy + 8$	3	7	Trinomial of degree 7
7)	$x^2y + \sqrt{7}$	2	3	cubic binomial
8)	$4^2r^3s^2$	1	5	5th degree monomial
9)	$x^2 + xy - 10x + 3$	4	2	quadratic polynomial
10)	$x^2\sqrt{x + 2}$	0	0	----------------------

**Challenge: (Using Examples)
Show that multiplying 2 linear binomials can produce a quadratic trinomial.
Then, show that multiplying 2 linear binomials can produce a quadratic binomial.

$(X + 5)(X + 3) = X^2 + 8X + 15$ Quadratic Trinomial

$(X + 7)(X - 7) = X^2 - 49$ Quadratic Binomial

also,

$(X + 5)(Y + 3) = XY + 3X + 5Y + 15$ Quadratic with 4 terms

Working with Polynomials

Adding/Subtracting Polynomials: Collect/Combine "Like Terms" (adding or subtracting the coefficients)

Example: $(3x^4 + 5x^2 + 5) + (4x^4 + 9x^2 + 11x - 3)$

$(3x^4 + 5x^2 + 5) + (4x^4 + 9x^2 + 11x - 3)$

$7x^4$

There is no x^3 term

$(\quad 5x^2 + 5) + (\quad 9x^2 + 11x - 3)$

$14x^2$

$(\quad + 5) + (\quad + 11x - 3)$

$11x$

$7x^4 + 14x^2 + 11x + 2$

$(\quad + 5) + (\quad - 3)$

2

Example: $(2t^3 - 4t^2 + 12t - 5) - (1 - 3t + 2t^2 + t^3)$

***Remember to combine "Like" terms
and, distribute the negative throughout the entire polynomial

$(2t^3 - 4t^2 + 12t - 5) - (1 - 3t + 2t^2 + t^3)$

$2t^3 - t^3 = t^3$

$(\quad - 4t^2 + 12t - 5) - (1 - 3t + 2t^2 + \quad)$

$-4t^2 - 2t^2 = -6t^2$

$(\quad + 12t - 5) - (1 - 3t + \quad)$

$12t - (-3t) = 15t$

$(\quad - 5) - (1 \quad)$

$-5 - 1 = -6$

$t^3 - 6t^2 + 15t - 6$

Working with Polynomials

"Distribute" -- Multiply *each term* in the polynomial

Example: $5x(3x + 2x^2 - 1)$

$5x\ (3x + 2x^2 - 1)$

$5x \cdot 3x = 15x^2$

$5x \cdot 2x^2 = 10x^3$

$5x \cdot (-1) = -5x$

(then, write in descending order)

$10x^3 + 15x^2 - 5x$

Example: $-2xy(x^3 + 2y^2 + 4y - 1)$

$-2xy \cdot x^3 = -2x^4 y$

$-2xy \cdot 2y^2 = -4xy^3$

$-2xy \cdot 4y = -8xy^2$

$-2xy \cdot (-1) = 2xy$

(multiply carefully, then combine all the terms....)

$-2x^4 y - 4xy^3 - 8xy^2 + 2xy$

Taking out the Greatest Common Factor (GCF)

A useful way to simplify a polynomial is to take out the GCF:

Example: $9x^5 + 12x^3 + 6x$

The greatest common factor of 9, 12, and 6 is 3
and
the greatest common factor of x^5 , x^3 , and x is x

The GCF of the polynomial is 3x

**So divide *each term* by 3x

$9x^5 + 12x^3 + 6x$ /3x

$3x(3x^4 + 4x^2 + 2)$

Example: $4a^3 bc - 10ab^4 + 20ac^2$

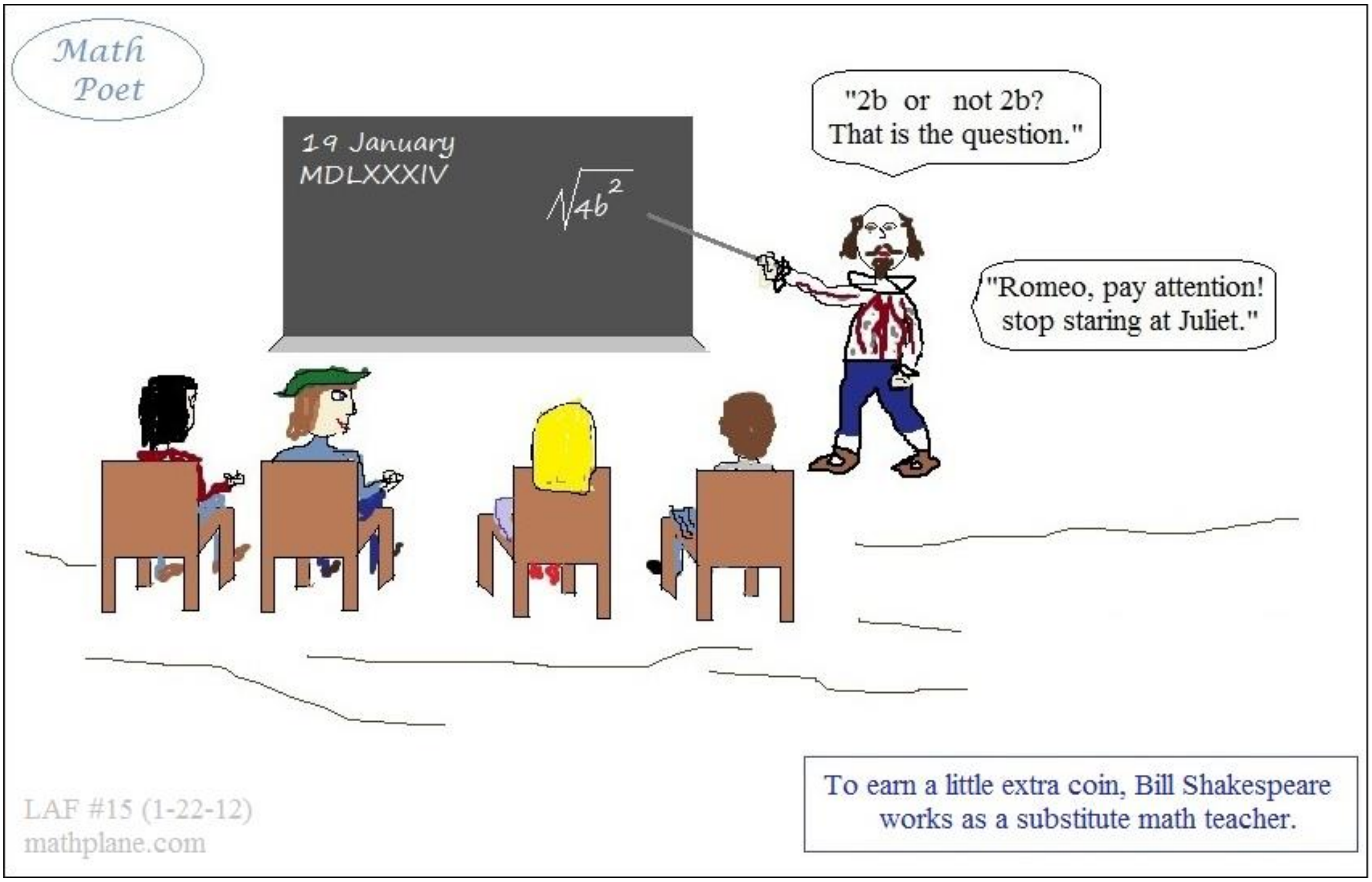
The greatest common factor of 4, 10, and 20 is 2
and
the greatest common factor of a^3 , a, and a is a
then..
the GCF of b, b^4 and no b is 1
the GCF of c, no c, and c^2 is 1

The GCF of the polynomial is 2a

(divide each term by 2a)

$4a^3 bc - 10ab^4 + 20ac^2$ /2a

$2a(2a^2 bc - 5b^4 + 10c^2)$



Practice Quiz (and Solutions)-→

Introduction to Polynomials Quiz

I. Addition/Subtraction: Simplify the following

a) $(2x^2 - 4x + 6) + (x^2 + 13x - 5)$

b) $(2x + 5 - 7x^2) + (x^2 - 11x + 3)$

c) $(x^2 - 3x + 6) - (2x^2 + 6x - 5)$

d) $(3ab^2 + 2a^2b - b^3) + (5a^2 + 6ab^2 + 2b^3)$

e) $3(x + y) + 2c(x + y) + d(x + y)$

II. Multiplication: Expand the following

a) $(x + 3)(x + 7)$

b) $(x + 4)^2$

c) $(x - 11)(x + 4)$

d) $(x + 2y)(3x - y)$

e) $-(x - 3)(x + 8)$

f) $(x + 5)(x^2 + 7x - 1)$

g) $(x + 9)(x - 9)$

h) $(x + 1)(x + 2)(x + 3)$

i) $(x^2 - 2)(x^3 + 6x + 14)$

Introduction to Polynomials Quiz

III. Classifying Polynomials: Determine which are polynomials; then, identify the type, degree, and lead coefficients

a) $3x + 3$

b) $5 + 4x - 5x^3$

c) $4^Y + 1$

d) $z^3 + z^2 + z + \frac{1}{Z}$

e) $4c^3 d + 3cd^2 + d^3$

f) $9x^3y^3$

IV. Simplify and Classify: Solve and identify each polynomial (type and degree)

a) $(2x + 3)^2 + (3x^2 + 6)$

b) $(2a^2 + b)(2b + a)$

c) $-3(6 - 2s^2 + 5s^3)$

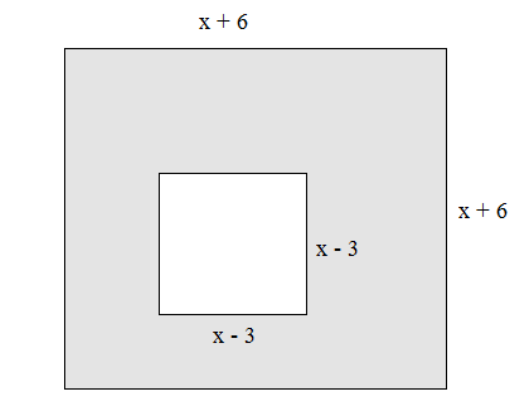
V. More Questions

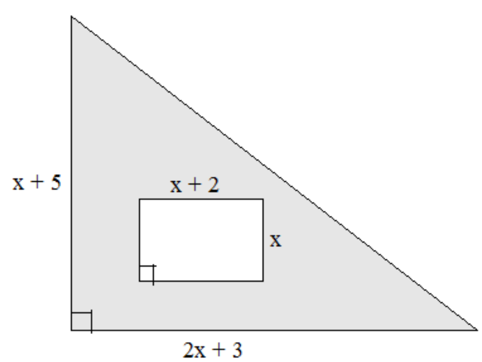
Introduction to Polynomials Quiz

a) Classify the following $3x^2y^A + 42x$

b) If $f(x)$ is a polynomial, describe the domains of A and B $f(x) = 5x^{A-7} - 2x^{(B/2)}$

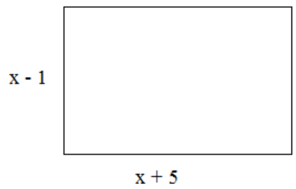
c) What is the area of each shaded region? (Write an expression in simplified form)



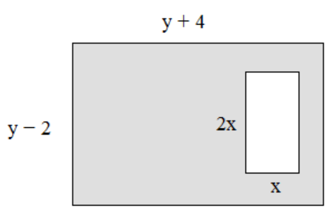


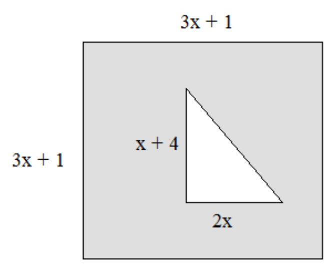
Polynomials Applications: Area

1) If the area of the rectangle is 72, then what are the dimensions?

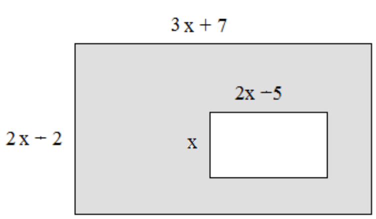


2) Write equations to describe the shaded areas..

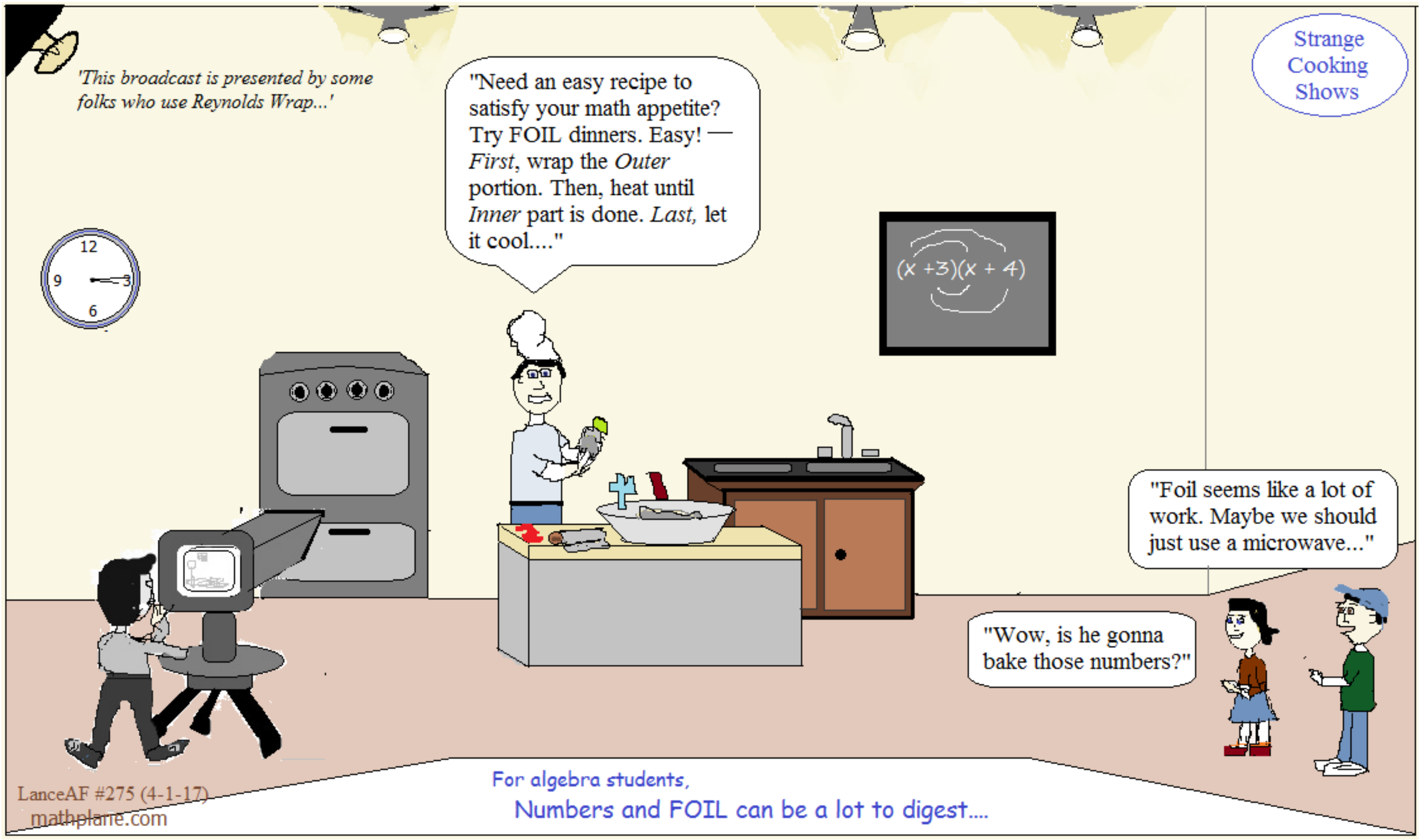




3) Find x



Shaded area is 273


'This broadcast is presented by some folks who use Reynolds Wrap...'
"Need an easy recipe to satisfy your math appetite? Try FOIL dinners. Easy! — *First*, wrap the *Outer* portion. Then, heat until *Inner* part is done. *Last,* let it cool...."
Strange Cooking Shows
12
9
3
6
(x +3)(x + 4)
"Foil seems like a lot of work. Maybe we should just use a microwave..."
"Wow, is he gonna bake those numbers?"
LanceAF #275 (4-1-17)
mathplane.com
For algebra students,
Numbers and FOIL can be a lot to digest....

ANSWERS-→

Introduction to Polynomials Quiz

Solutions

I. Addition/Subtraction: Simplify the following

a) $(2x^2 - 4x + 6) + (x^2 + 13x - 5)$

Collect "like terms"

$3x^2 + 9x + 1$

b) $(2x + 5 - 7x^2) + (x^2 - 11x + 3)$

$-6x^2 - 9x + 8$

c) $(x^2 - 3x + 6) - (2x^2 + 6x - 5)$

distribute -1

$x^2 - 3x + 6 + (-2x^2 - 6x + 5)$

$-x^2 - 9x + 11$

d) $(3ab^2 + 2a^2b - b^3) + (5a^2 + 6ab^2 + 2b^3)$

$9ab^2 + 2a^2b + 5a^2 + b^3$

e) $3(x + y) + 2c(x + y) + d(x + y)$

same term

$(3 + 2c + d)(x + y)$

II. Multiplication: Expand the following

a) $(x + 3)(x + 7)$

FOIL (First, Outer, Inner, Last)

$x^2 + 7x + 3x + 21$

$x^2 + 10x + 21$

b) $(x + 4)^2$

$(x + 4)(x + 4)$

$x^2 + 8x + 16$

c) $(x - 11)(x + 4)$

$x^2 + 4x - 11x - 44$

$x^2 - 7x - 44$

d) $(x + 2y)(3x - y)$

$3x^2 - xy + 6xy - 2y^2$

$3x^2 + 5xy - 2y^2$

e) $-(x - 3)(x + 8)$

$-1 \cdot (x^2 + 5x - 24)$

$-x^2 - 5x + 24$

f) $(x + 5)(x^2 + 7x - 1)$

$x^3 + 7x^2 - x$

$+ \quad 5x^2 + 35x - 5$

$x^3 + 12x^2 + 34x - 5$

g) $(x + 9)(x - 9)$

$x^2 - 9x + 9x - 81$

$x^2 - 81$

('difference of squares')

h) $(x + 1)(x + 2)(x + 3)$

$(x^2 + 3x + 2) \cdot (x + 3)$

$x^3 + 3x^2 + 2x$

$+ \quad 3x^2 + 9x + 6$

$x^3 + 6x^2 + 11x + 6$

i) $(x^2 - 2)(x^3 + 6x + 14)$

$x^5 + 6x^3 + 14x^2$

$+ \quad -2x^3 - 12x - 28$

$x^5 + 4x^3 + 14x^2 - 12x - 28$

Introduction to Polynomials Quiz

Solutions

III. Classifying Polynomials: Determine which are polynomials; then, identify the type, degree, and lead coefficients

a) $3x + 3$ — binomial (2 terms); degree 1 (linear); lead coefficient: 3

b) $5 + 4x - 5x^3$ — trinomial (3 terms); degree 3 (cubic); lead coefficient: -5

$-5x^3 + 4x + 5$

c) $4^Y + 1$ — NOT a polynomial -- Exponent cannot be a variable

d) $z^3 + z^2 + z + \frac{1}{z}$ — NOT a polynomial -- All exponents must be whole numbers

z^{-1}

e) $4c^3 d + 3cd^2 + d^3$ — trinomial (3 terms); degree 4; lead coefficient: 4

f) $9x^3y^3$ — monomial (1 term); degree 6 ; lead coefficient: 9

IV. Simplify and Classify: Solve and identify each polynomial (type and degree)

a) $(2x + 3)^2 + (3x^2 + 6)$

$4x^2 + 6x + 6x + 9 + 3x^2 + 6$

$7x^2 + 12x + 15$

Quadratic Trinomial

b) $(2a^2 + b)(2b + a)$

$4a^2 b + 2a^3 + 2b^2 + ba$

Four-term polynomial of degree 3

c) $-3(6 - 2s^2 + 5s^3)$

$-18 + 6s^2 - 15s^3$

$-15s^3 + 6s^2 - 18$

Cubic Trinomial

V. More Questions

Introduction to Polynomials Quiz

Solutions

a) Classify the following $3x^2 y^A + 42x$

If A is a whole number, then this is a binomial with degree $A + 2$

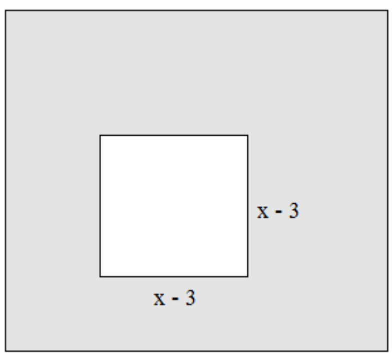
If A is not a whole number, then this is not a polynomial!

b) If $f(x)$ is a polynomial, describe the domains of A and B $f(x) = 5x^{A-7} - 2x^{(B/2)}$

$A \geq 7$ where A is a whole number

$B = 0$ or any positive even number

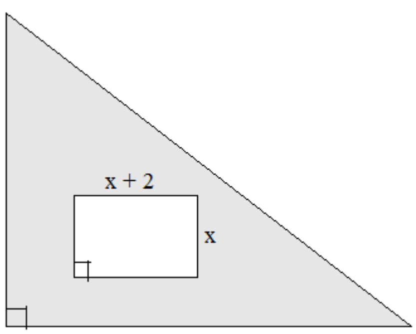
c) What is the area of each shaded region? (Write an expression in simplified form)



$(x + 6)(x + 6) - (x - 3)(x - 3)$

$x^2 + 12x + 36 - (x^2 - 6x + 9)$

$\boxed{18x + 27}$



$\frac{1}{2}(x + 5)(2x + 3) - (x + 2)(x)$

$\frac{1}{2}(2x^2 + 3x + 10x + 15) - (x^2 + 2x)$

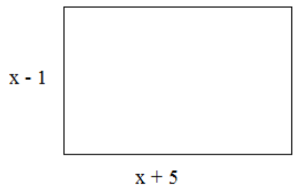
$x^2 + \frac{13}{2}x + \frac{15}{2} - x^2 - 2x$

$\boxed{\frac{9}{2}x + \frac{15}{2}}$

Polynomials Applications: Area

SOLUTIONS

1) If the area of the rectangle is 72, then what are the dimensions?



x = 7,

so the dimensions are

6 and 12

First, multiply/expand the binomals...

$(x - 1)(x + 5) = 72$

$x^2 + 4x - 5 = 72$

collect like terms on the left...

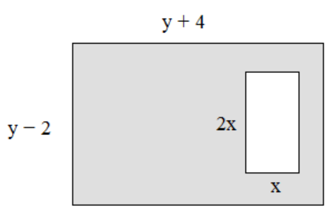
$x^2 + 4x - 77 = 0$

Then, factor and solve... $(x + 11)(x - 7) = 0$

x can be -11 or 7...

since sides cannot be negative, we'll exclude -11..

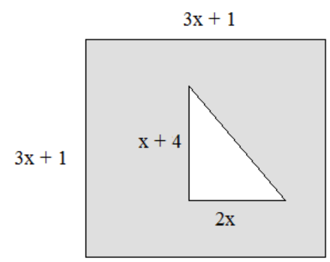
2) Write equations to describe the shaded areas..



$(y - 2)(y + 4) = y^2 + 2y - 8$

$2x(x) = 2x^2$

$y^2 + 2y - 8 - 2x^2$



area of square:

$(3x + 1)^2 = 9x^2 + 6x + 1$

area of triangle:

$\frac{1}{2}(x + 4)(2x) = \frac{1}{2}(2x^2 + 8x)$

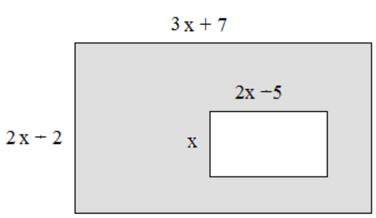
$= x^2 + 4x$

shaded area:

$9x^2 + 6x + 1 - (x^2 + 4x)$

$8x^2 + 2x + 1$

3) Find x



Shaded area is 273

Area of big rectangle: $(2x - 2)(3x + 7) = 6x^2 + 8x - 14$

Area of small rectangle: $(x)(2x - 5) = 2x^2 - 5x$

Shaded area: $4x^2 + 13x - 14 = 273$

$4x^2 + 13x - 287 = 0$

$(4x + 41)(x - 7) = 0$

$x = 7$ or ~~-41/4~~

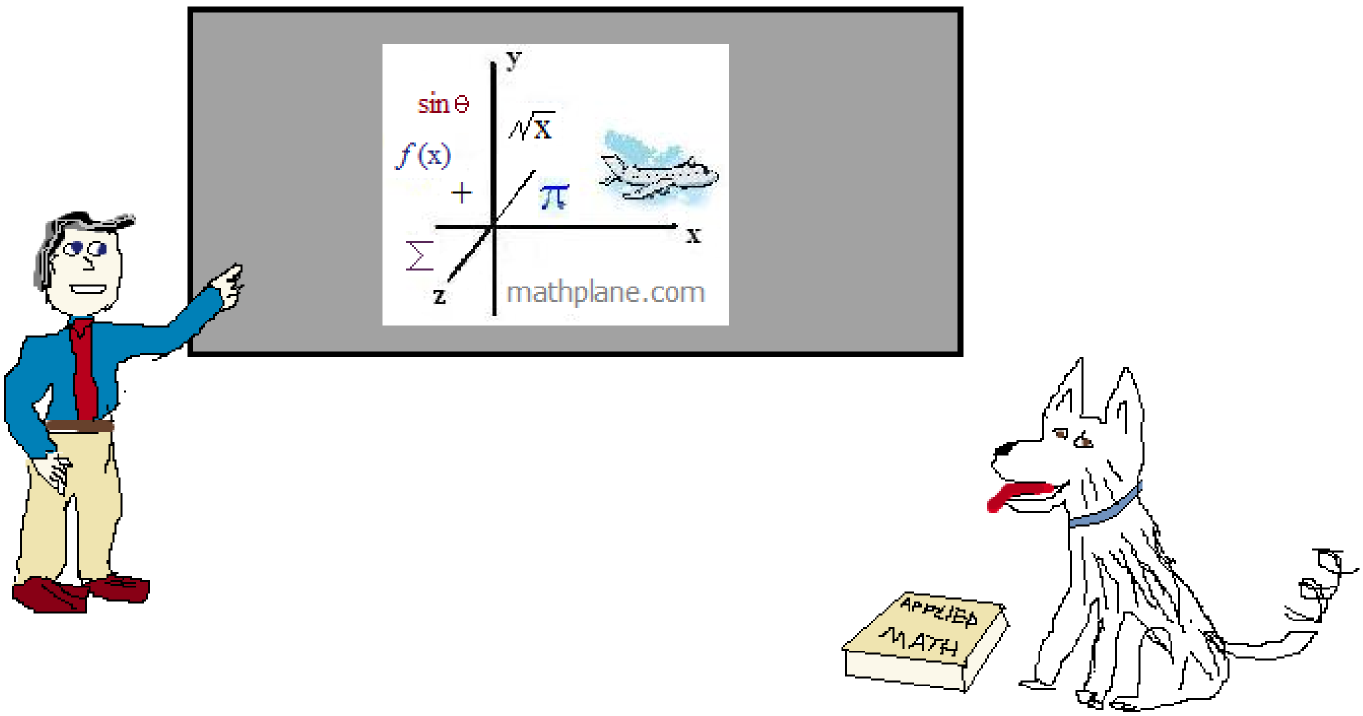
large rectangle: 12 x 28 = 336
small rectangle: 7 x 9 = 63

336 - 63 = 273 ✓

Thanks for visiting. (Hope it helped!)

If you have questions, suggestions, or requests, let us know.

Good luck!



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And, Mathplane *Express* for mobile at Mathplane.ORG

Challenge Question:

$$\text{If } (x+y)^2 = 80 \quad \text{and} \quad -xy = 2,$$

$$\text{what is } x^2 + y^2 \ ?$$

The answer is…..

$$\text{If } (x+y)^2 = 80 \quad \text{and} \quad -xy = 2$$

$$\text{then what is } x^2 + y^2 \ ?$$

SOLUTION:

$$x^2 + 2xy + y^2 = 80$$

$$xy = -2$$

$$x^2 + 2(-2) + y^2 = 80$$

$$x^2 + y^2 = 84$$