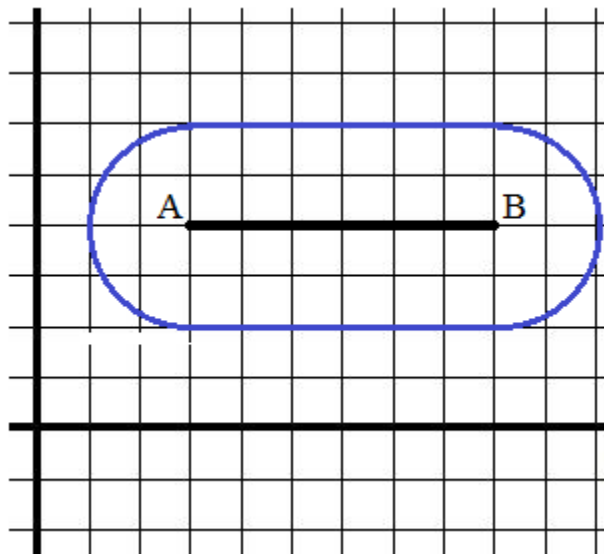


Geometry: Locus of Points

Notes, Examples, and Practice Quiz (with Solutions)



Topics include circles, equidistance theorem, compound locus of points, intersections, and more.

Locus of Points: Describing and Graphing

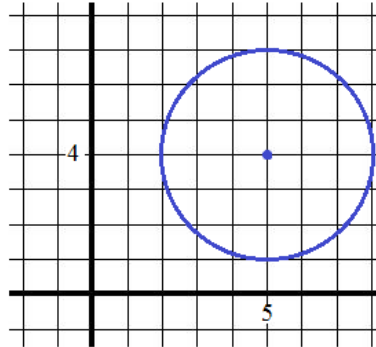
What is a locus of points? "The set of points that satisfies a given condition(s)"

A popular example is a circle. Every point on a circle is equidistant from the center.

The locus of points is "3 units from (5, 4)".

$$(x - 5)^2 + (y - 4)^2 = 9$$

note: radius is 3 units

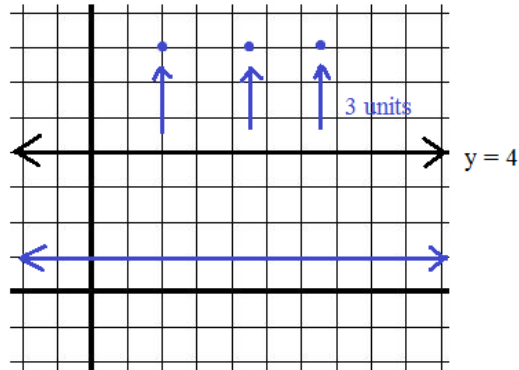
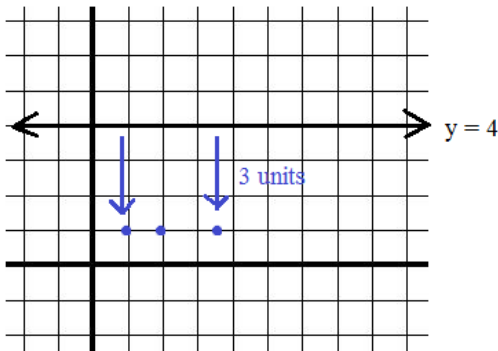


An approach: Keeping in mind, the distance between 2 points is a straight line, sketch points and find the pattern.

Example: Describe all points that are 3 units from $y = 4$

After plotting a few points that are 3 units from points on $y = 3$, a pattern emerges..

Then, plot a few points above $y = 3$...

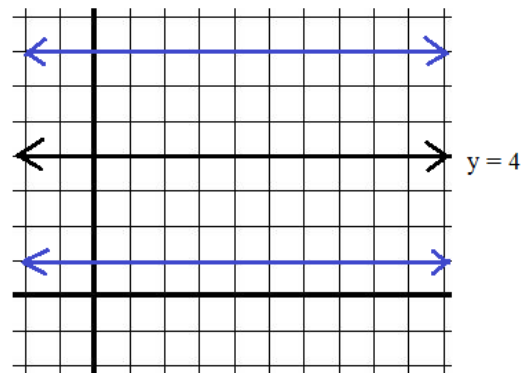


Finally, describe the graph.

In this case,

$y = 7$ and $y = 1$

is the locus of points 3 units from $y = 4$



Locus of Points: Describing and Graphing

An approach: Use geometry properties

Example: Find the locus of points equidistant from (6, 1) and (2, -3)

We know the midpoint of the (6, 1) and (2, -3) is equidistant.

So, how do we find the other points?

Consider the Equidistance Theorem:

If a point is on the *perpendicular bisector* of a segment, then it is equidistant from the endpoints of the segment.

Find the perpendicular bisector:

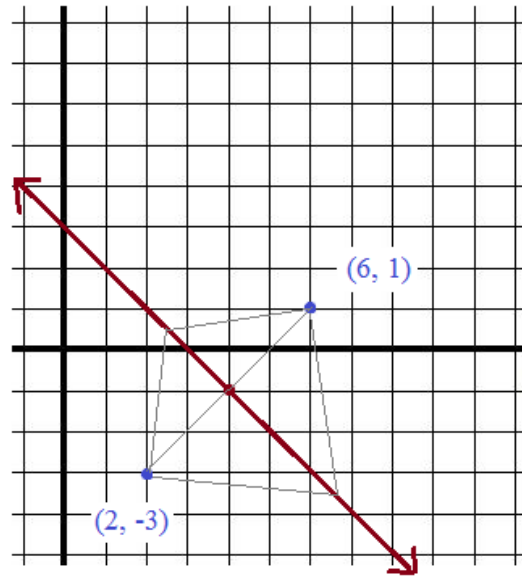
Midpoint: (4, -1)

Slope: Since slope of given points is $\frac{-3 - 1}{2 - 6} = 1$,

the slope of \perp bisector is -1

Therefore, the equation of the line is $y - (-1) = -1(x - 4)$

$$y = -x + 3$$



$x + y = 3$ is the locus of points equidistant from (6, 1) and (2, -3)

Example: Draw the locus of points 3 units from (4, 4)

Definition of a circle: A locus of points that are a fixed distance *from a given point*.

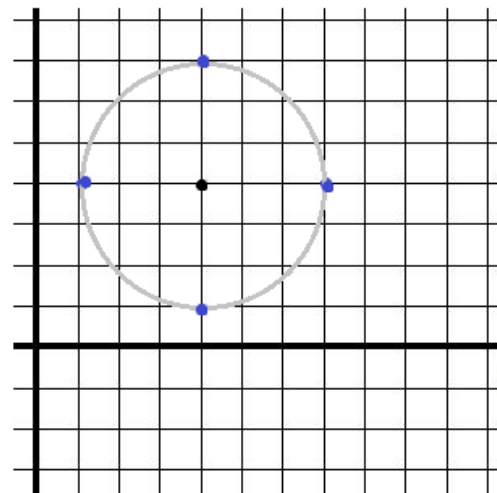
Since we have a given point (4, 4), the locus of points will be a circle.

Plotting (1, 4) (7, 4) (4, 1) and (4, 7) will outline the circle...

The equation is $(x - 4)^2 + (y - 4)^2 = 9$

$$(x + h)^2 + (y + k)^2 = r^2$$

circle with center (h, k) and radius length r

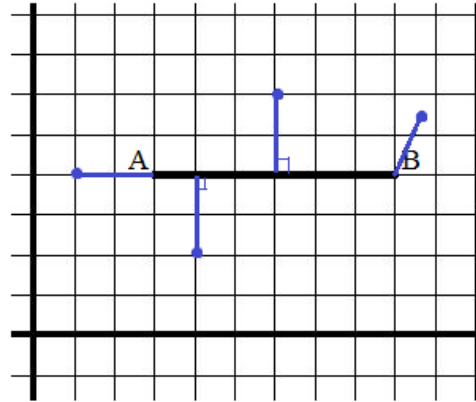


Sketching Locus of Points

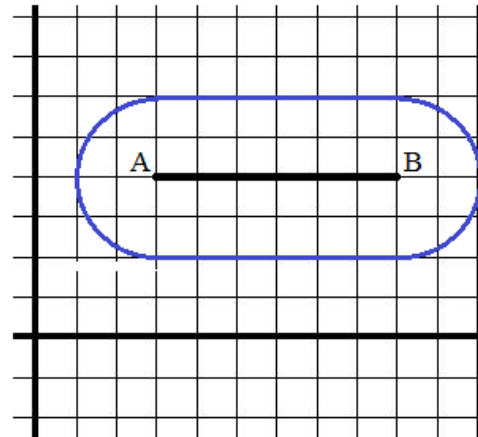
Example: Sketch all points that are
2 units from line segment \overline{AB}

The (shortest) distance from a point to a line (segment) is a perpendicular straight line (segment).

And, the shortest distance between 2 points is a straight line.



These 4 segments are each 2 units in length



Example: Describe and graph the locus of points
2 units from $x^2 + y^2 = 16$

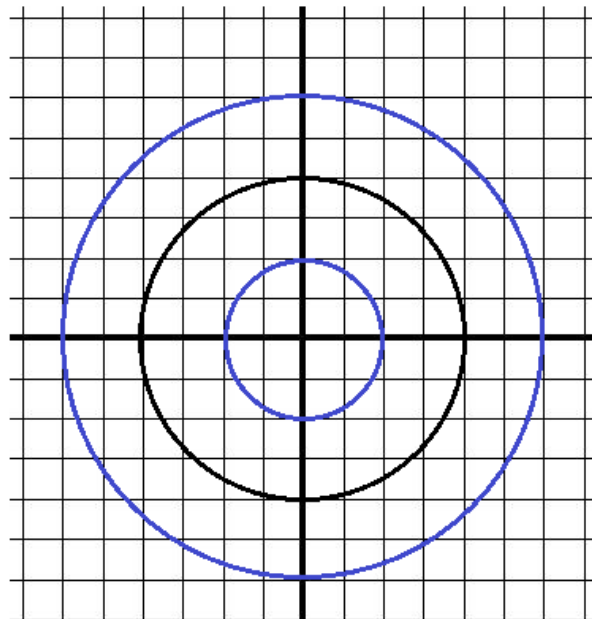
Since the radius of the given circle is 4,

we could describe and graph a circle with
a radius of 6....

$$x^2 + y^2 = 36$$

Then, we also must describe and graph a
circle with a radius of 2....

$$x^2 + y^2 = 4$$



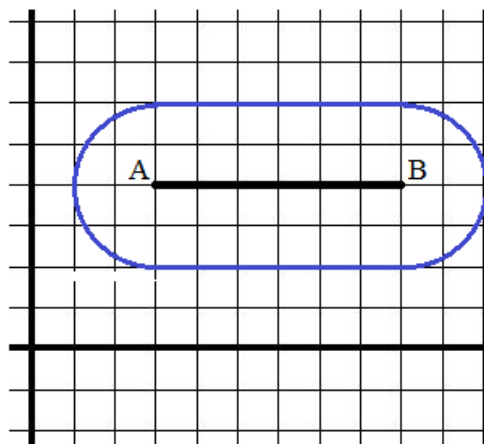
A topic for discussion: Different interpretations

Example: Sketch all points that are 2 units from line segment \overline{AB}

Answer 1: Any point where the (minimum) distance to the segment is exactly 2 units.

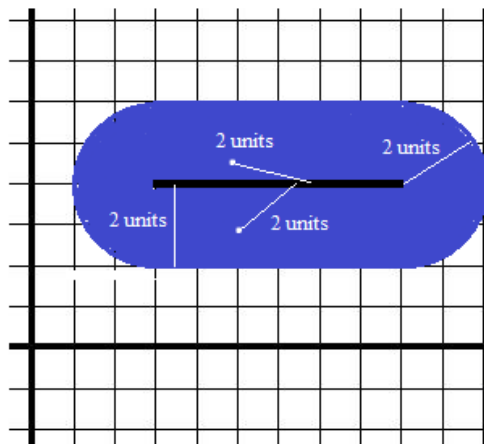
Every point on the track is 2 units from the *nearest* point on the segment.

(note: in Euclidean Geometry, the distance from a point to a line is the shortest distance possible.)



Answer 2: Any point that is exactly 2 units from anywhere on the line segment.

Every point in the shaded area is 2 units from *some* point on the segment.

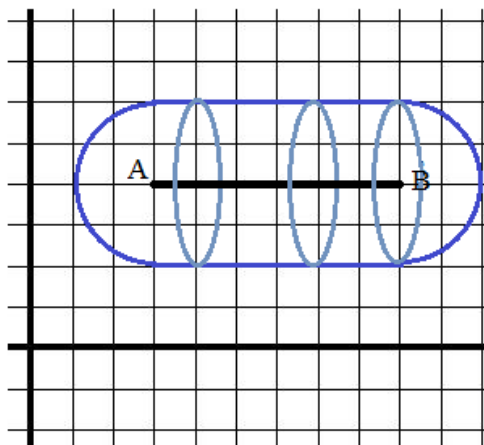


Answers 3 and 4: \overline{AB} is in 3-dimensional *space* (instead of a 2-d *plane*)

In space, the locus of points 2 units from \overline{AB} could be

a "hollow pill"

or, a "solid pill"...



Intersection of loci

Example: Find the locus of points that are 3 units from $(4, 5)$ and 2 units from $x = 8$.

Step 1: Find the locus of points that are 3 units from $(4, 5)$...

$$\text{The circle } (x - 4)^2 + (y - 5)^2 = 9$$

Step 2: Find the locus of points that are 2 units from $x = 8$...

$$\text{The lines } x = 6 \text{ and } x = 10$$

Step 3: Identify any intersecting points...

To find the values, solve the system:

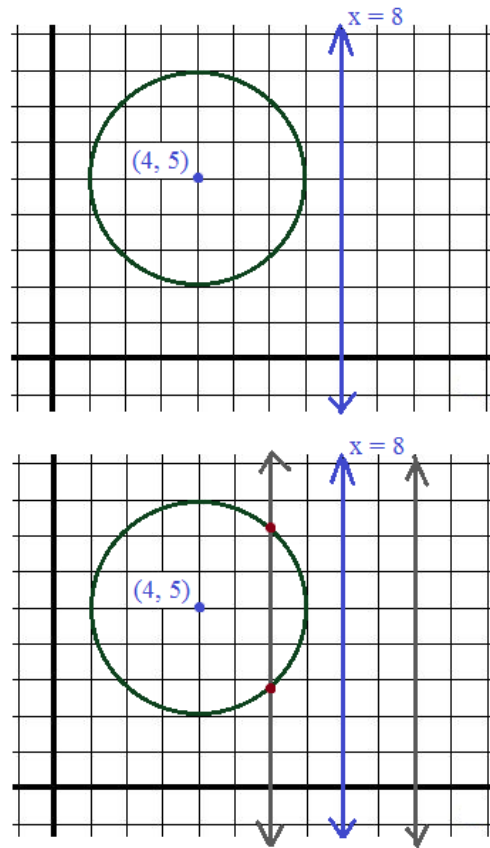
$$\begin{aligned} x &= 6 \\ (x - 4)^2 + (y - 5)^2 &= 9 \end{aligned}$$

Direct substitution:

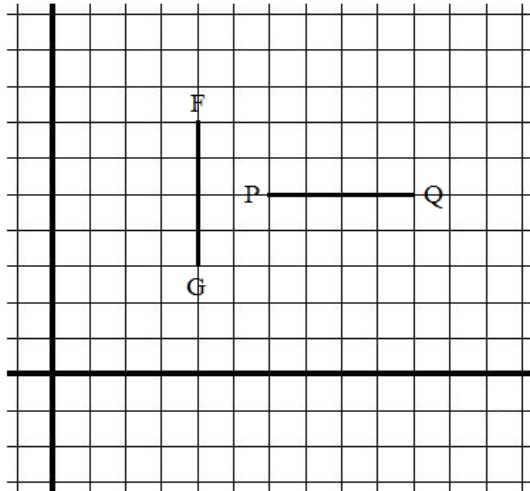
$$\begin{aligned} (6 - 4)^2 + (y - 5)^2 &= 9 \\ (y - 5)^2 &= 5 \end{aligned}$$

$$x = 6 \quad y = 5 \pm \sqrt{5}$$

the locus of points are $(6, 5 + \sqrt{5})$ and $(6, 5 - \sqrt{5})$



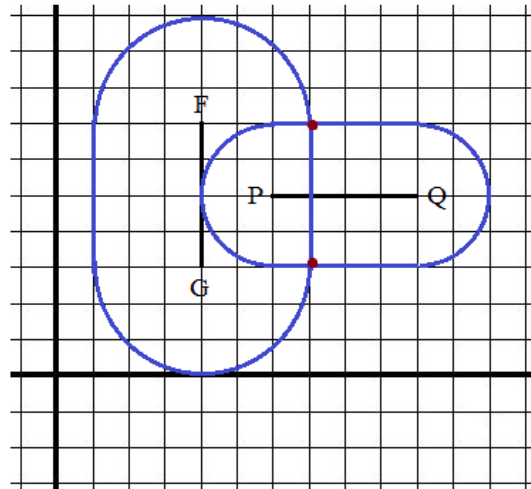
Example: Identify the locus of point 3 units from \overline{FG} and 2 units from \overline{PQ}



Step 1: Draw a *race track oval* around \overline{FG}

Step 2: Draw a *race track oval* around \overline{PQ}

Step 3: Identify the points of intersection



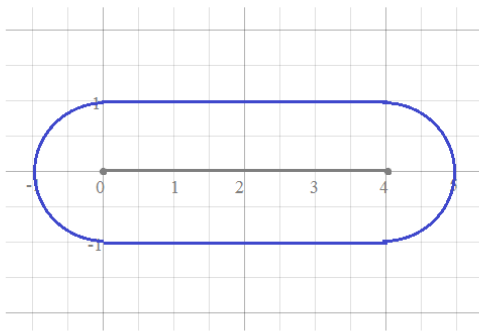
The locus of points both 3 units from \overline{FG} and 2 units from \overline{PQ} are

$(7, 3)$ and $(7, 7)$

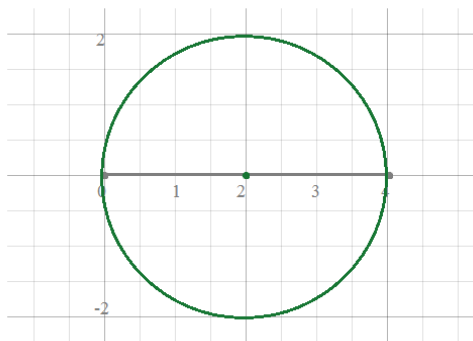
Example: Can you find the locus of points that are 1 unit from a 4-unit line segment AND 2 units from the midpoint of that line segment?
How many points are there?

"Compound Locus of Points"

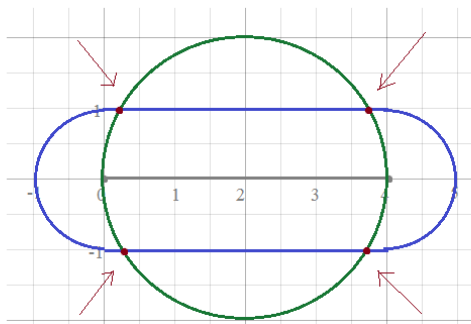
First locus of points



Second locus of points



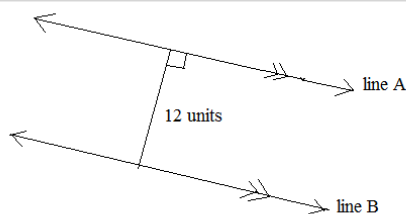
The intersection is the compound locus of points...



These 4 points are 2 units from the segment AND 4 units from the midpoint of the segment..

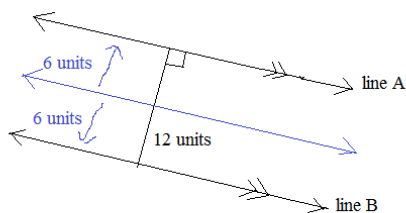
Example: Describe the locus of points that are

- a) 6 units from A and B
- b) 4 units from A and B
- c) 10 units from A and B



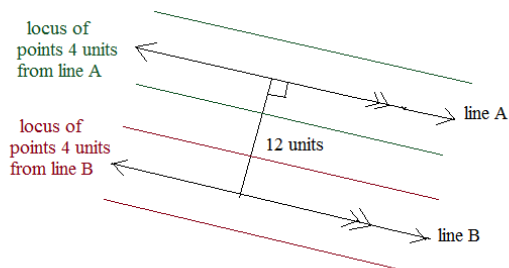
Since the lines are parallel, the distance between them is constant.

a) 6 units from both A and B: a line



b) 4 units from both A and B: Empty set

Since there is no intersection of loci, there is NO SOLUTION (an empty set)

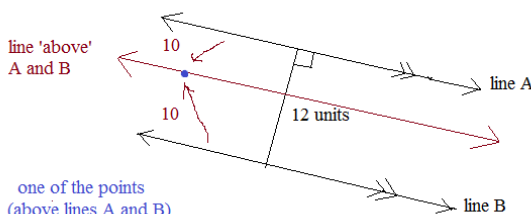


c) 10 units from A and B

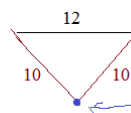
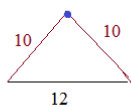
In a 2-dimensional plane, there is no intersections. Empty set (i.e. no solutions)

BUT, in 3-dimensional space, the locus of points is in a line 'above' A and B and a line 'below' A and B

(note: line A, line B, and the line above (or below) would form the edges of an isosceles triangular prism with base 12, sides 10, and height 8)



one of the points (above lines A and B)

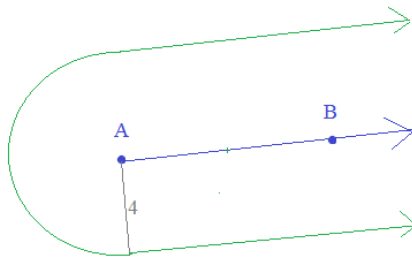


one of the points (below lines A and B)

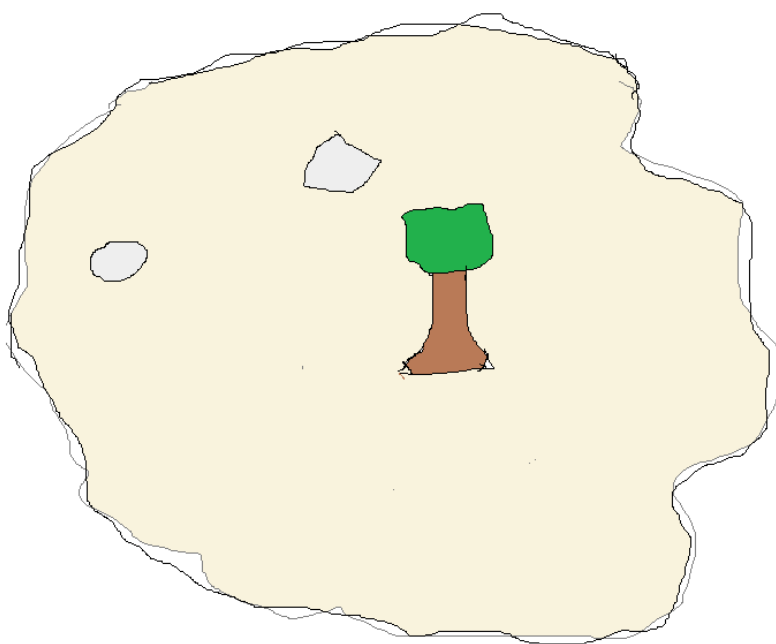
Example: Describe the locus of points that are 4 units from the ray \overrightarrow{AB}

Two parallel rays that are 4 units from \overrightarrow{AB} ...

And, a semicircle whose center is A and radius is 4



Example: Buried treasure.



There is a buried treasure that awaits you....

It is 10 feet from the big tree on the island...

and, it is the same distance from each of the big rocks...

Can you find the treasure?!?!

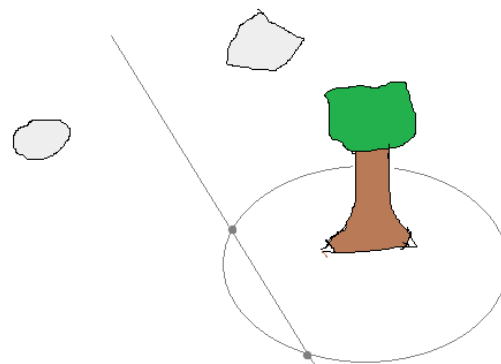
Definition of Locus:

- A place;
- A place where something happens;
- A center of great activity...

The latin word 'locus' means "place"

The plural 'loci' means "places"

Answer: The treasure may be buried in 2 possible places...



(compound locus of points)

Gallery Exhibit...

"I don't get it..."

"Me, neither"

Title: "Lines in Space"

Abstract Art and Math

(Sometimes, it takes multiple views to understand and appreciate...)

Geometry Lesson...

Lines in space

$y = 4$ is parallel to the xz -plane

"I don't get it..."

"Me, neither."

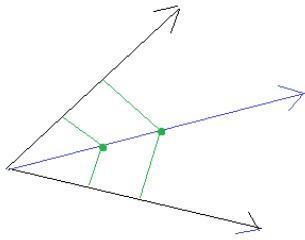
LanceAF #244 (7-4-16)
Mathplane.com

Topic for Discussion-→

A topic of discussion: Locus of points equidistant from 2 rays (with same endpoint)

Here are 2 potential answers:

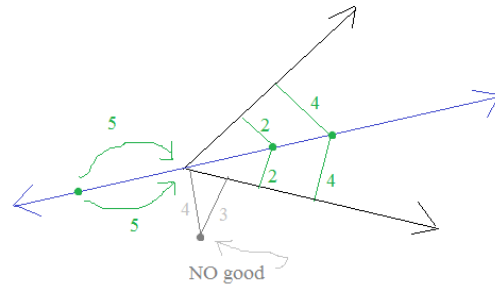
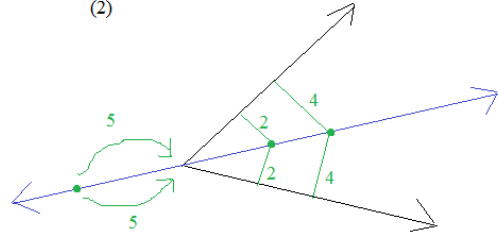
(1)



2 Rays in Black
Locus of points in Blue
Equidistant lengths in Green

← OR →

(2)



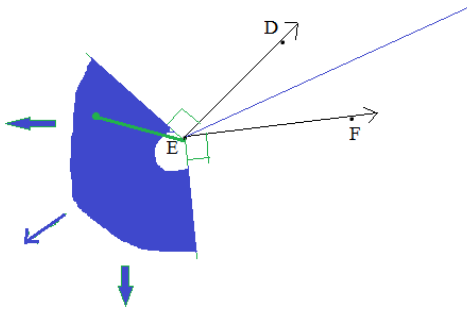
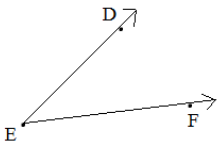
Note:

Locus of points equidistant "between" 2 rays that share a common endpoint ----> ANGLE BISECTOR (i.e. a ray)

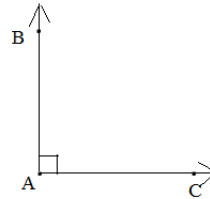
Locus of points "where" equidistant from 2 rays that share a common endpoint.. LINE that bisects angle

OR, is there another possible answer?!?!?

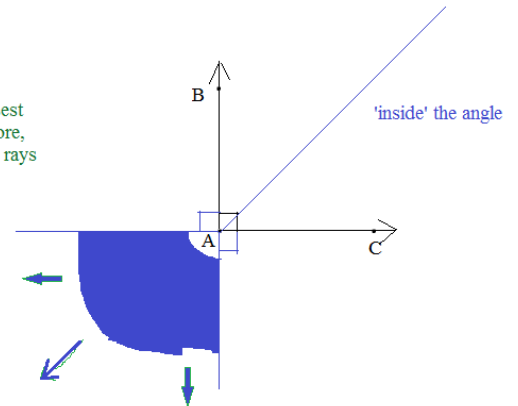
(3)



Describe and graph the locus of points equidistant from 2 rays that form a 90 degree angle:



'outside' the angle
***Every point is closest to vertex A; therefore, equidistant to both rays

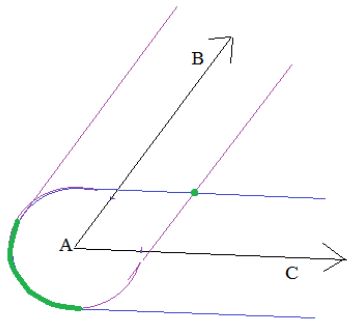


A topic of discussion: Locus of points equidistant from 2 rays (with same endpoint)

Drawing a conclusion by using a compound locus of points....

Describe and graph the locus of points equidistant from 2 rays that form an angle

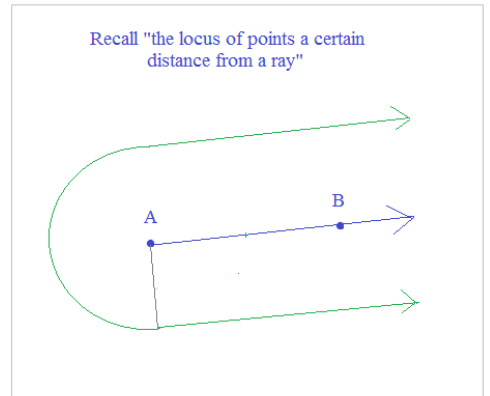
First, draw 2 locus of points -- each 2 units from each ray



Locus of points that are 2 units from AB

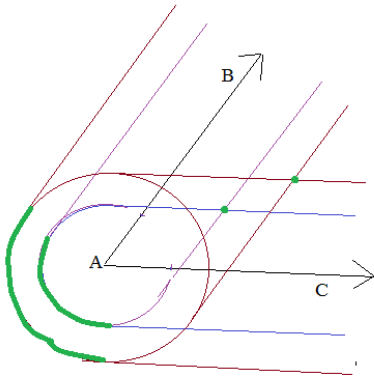
Locus of points that are 2 units from AC

The green represents the intersection, showing the compound locus of points (i.e. points that are 2 units from BOTH rays!)



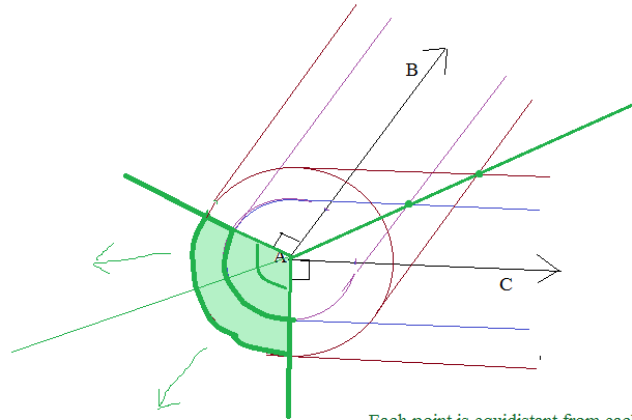
Recall "the locus of points a certain distance from a ray"

Then, add locus points that is 3 units from each ray...

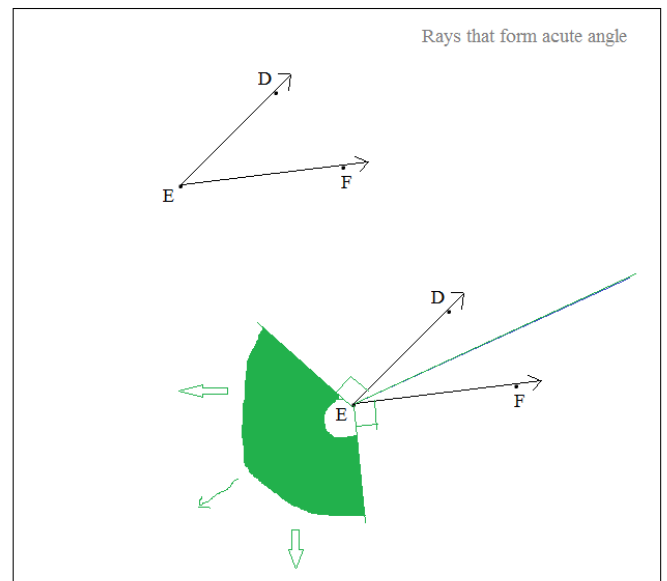
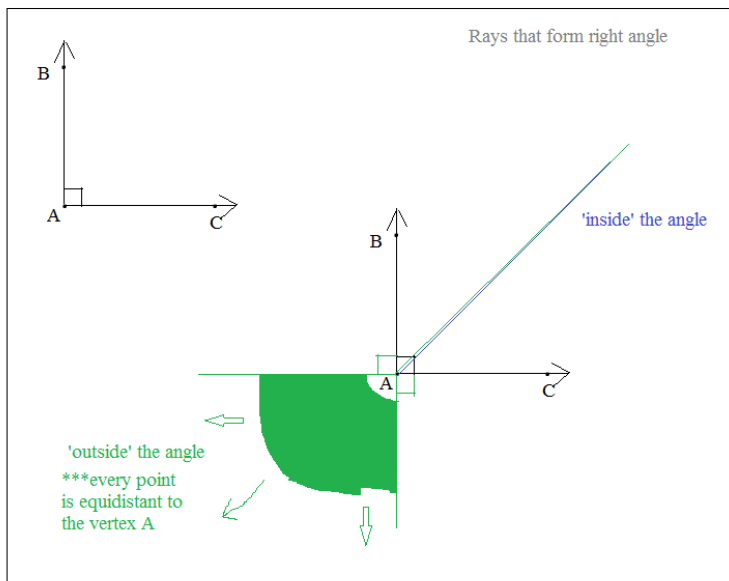


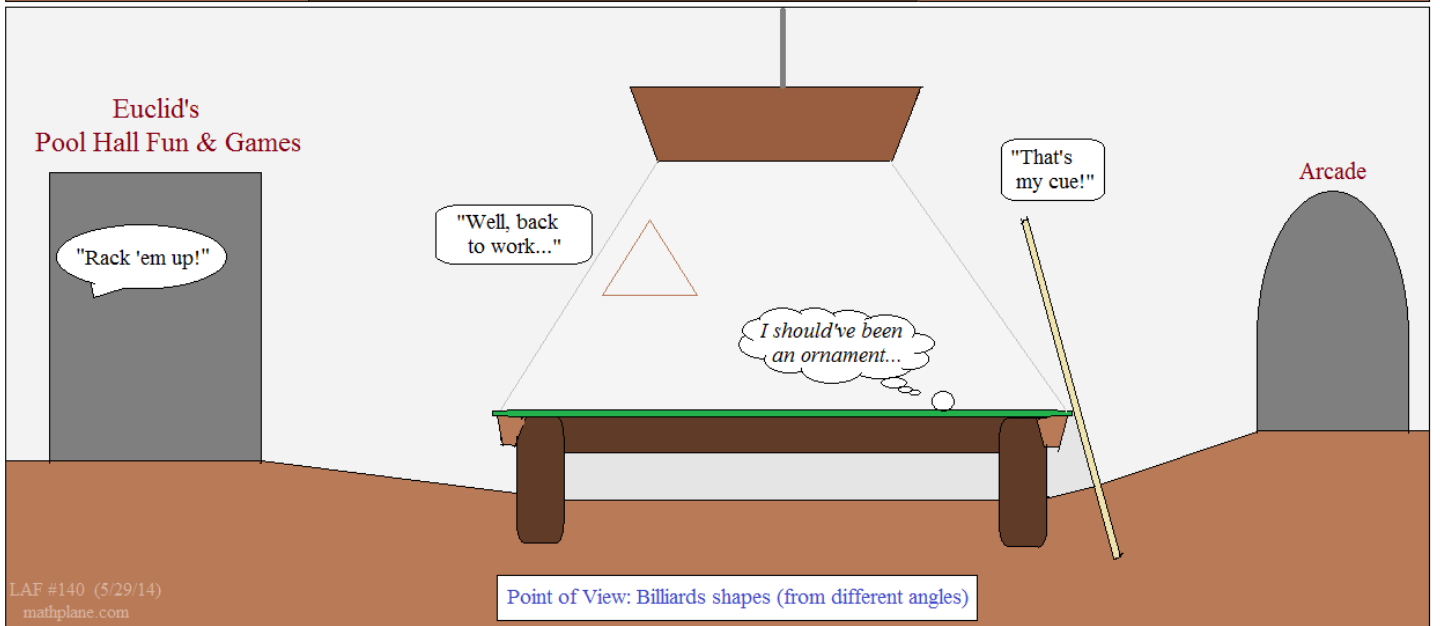
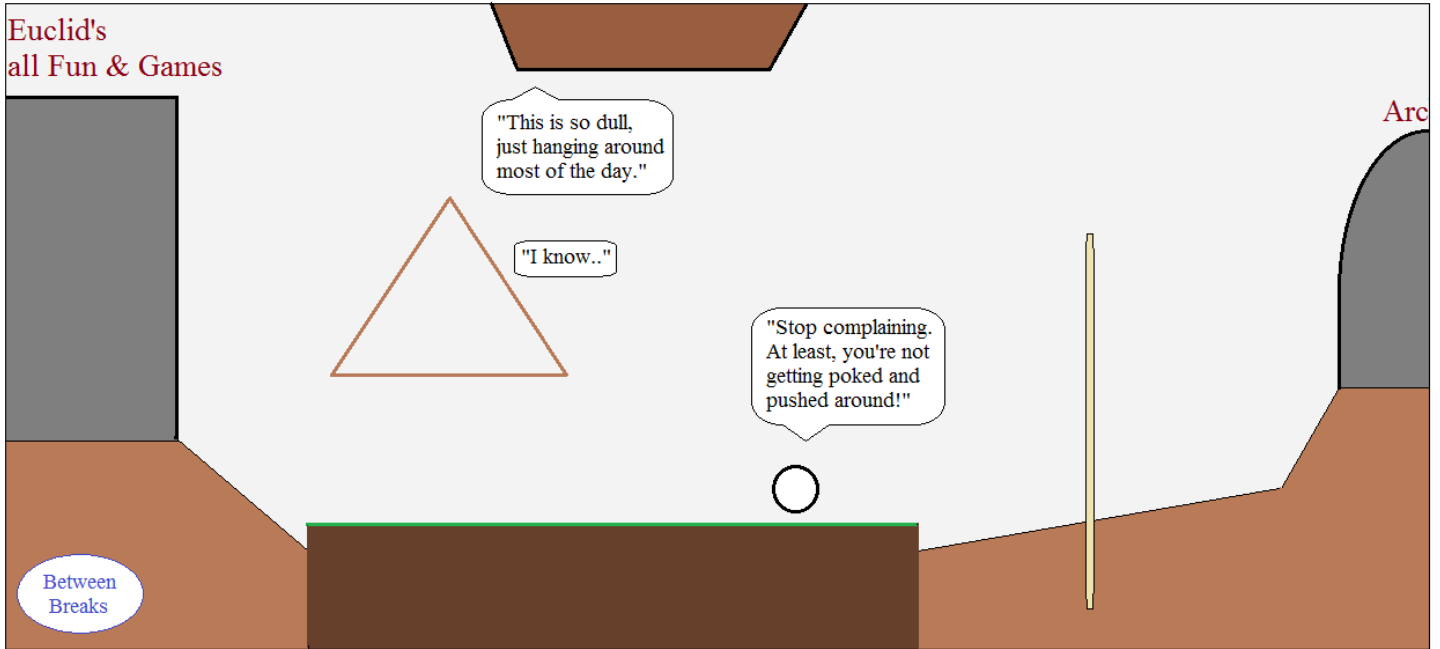
The green points and arcs represent any point that is 2 or 3 units from both of the rays...

Finally, add locus of points that are x units from each ray....



Each point is equidistant from each ray...



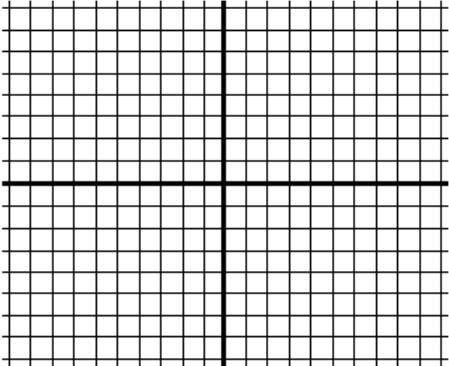


Practice Quizzes (and Solutions) ->

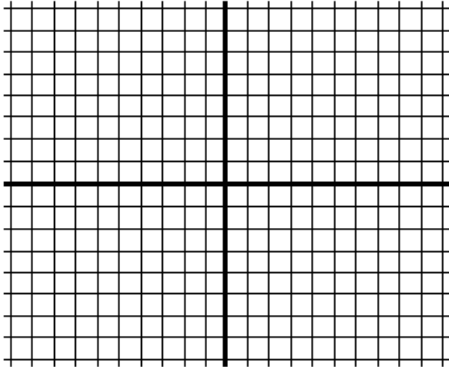
Locus of Points Quiz

Describe the locus of points (in the xy-plane) and graph.

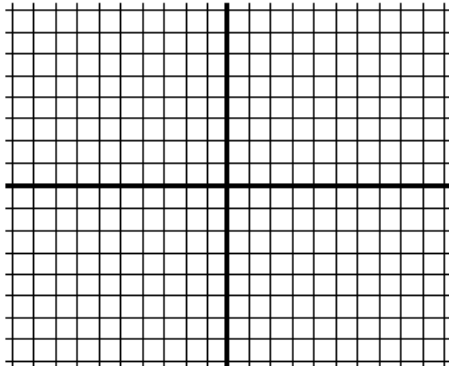
1) All points 6 units from the origin



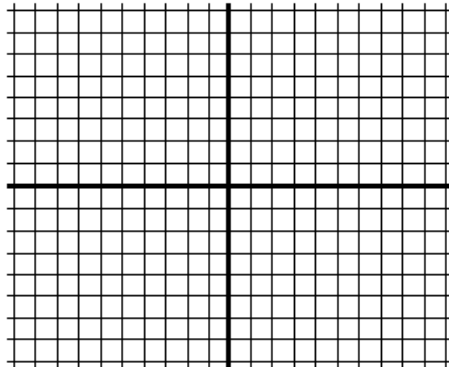
2) All points 3 units from $y = -2$



3) All points equidistant from the (x and y) axes



4) All points equidistant from (2, 4) and (0, 2)



6) Describe the locus of points that are 4 units from a circle with radius 4.

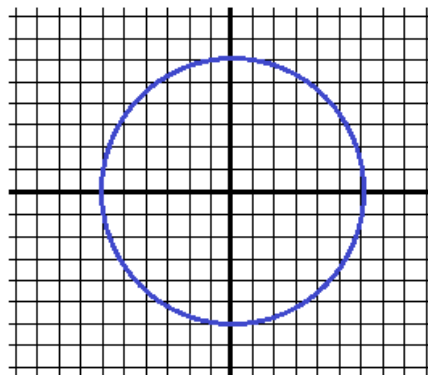
7) Describe the locus of points 7 units from ray \overrightarrow{AB} and 7 units from point B

8) Describe the locus of points 7 units from ray \overrightarrow{AB} and 7 units from point A

- 1) All points 6 units from the origin

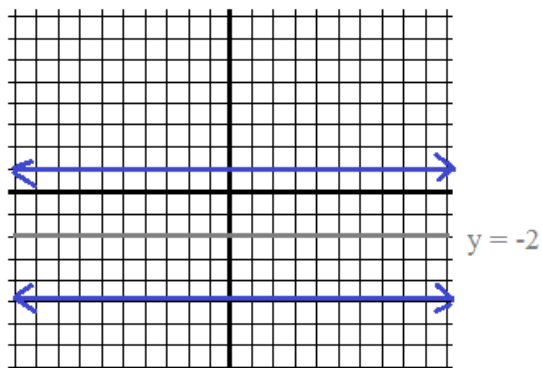
Circle with center
at $(0, 0)$
and radius 6

$$x^2 + y^2 = 36$$



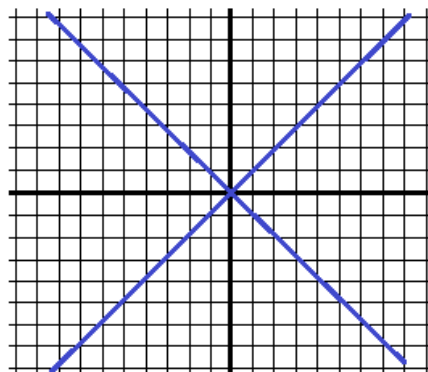
- 2) All points 3 units from
- $y = -2$

the lines $y = 1$ and
 $y = -5$



- 3) All points equidistant from the
- $(x$
- and
- $y)$
- axes

All points on
 $y = x$
or
 $y = -x$

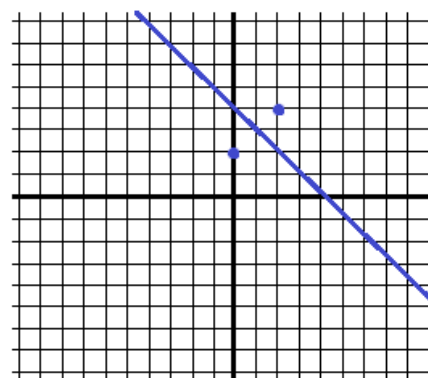


- 4) All points equidistant from
- $(2, 4)$
- and
- $(0, 2)$

find midpoint: $(1, 3)$

then, construct line
that is *perpendicular* to the
segment that joins $(2, 4)$ and $(0, 2)$
and goes through the midpoint $(1, 3)$

$$y = -x + 4$$

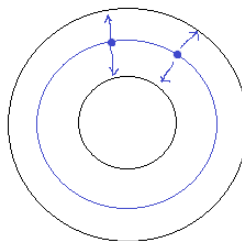


Describe the compound locus of points. Also, draw a quick sketch...

1) Locus of points equidistant from 2 concentric circles

A (concentric) circle in between

ex: if the radii of concentric circles is 6 and 10, then radius of circle from locus of points is 8...

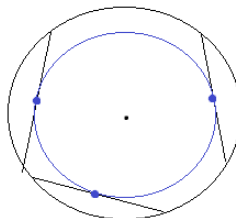


SOLUTIONS

2) Midpoint of all chords that are congruent to a given chord in a circle

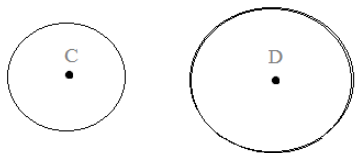
Circle

ex: diagram shows 3 congruent chords and their midpoints...

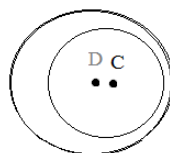


3) (In a plane), the locus of points 3 units from point C and 5 units from point D

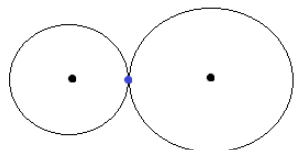
Case 1: No points exist...



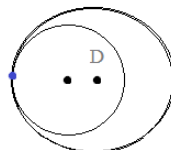
Distance between points C and D is greater than 8 units or less than 2 units



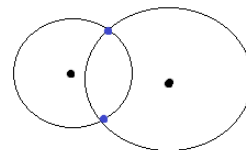
Case 2: 1 point



Distance between C and D is exactly 8 units or exactly 2 units



Case 3: 2 points...

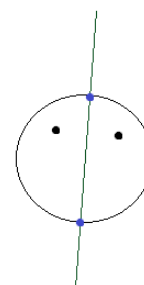
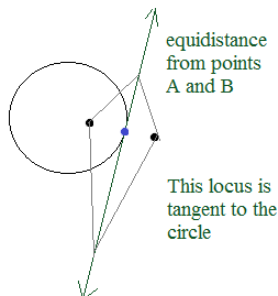


Distance between C and D is less than 8 units and greater than 2 units

4) Equidistant from 2 points AND lying on the same circle

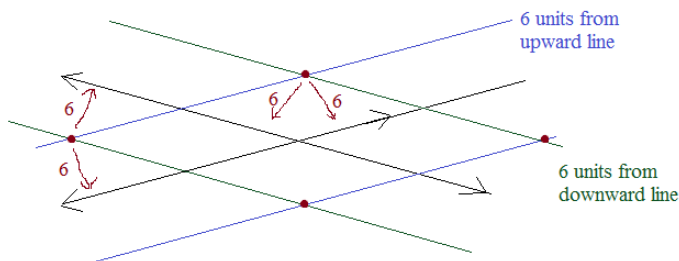
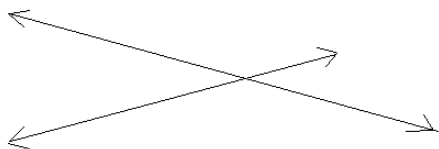
Case 1: 1 point

Case 2: 2 points



5) 6 units from two (non-parallel) lines

4 points

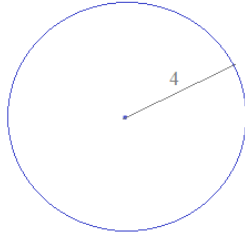


The 4 intersections are the locus of points

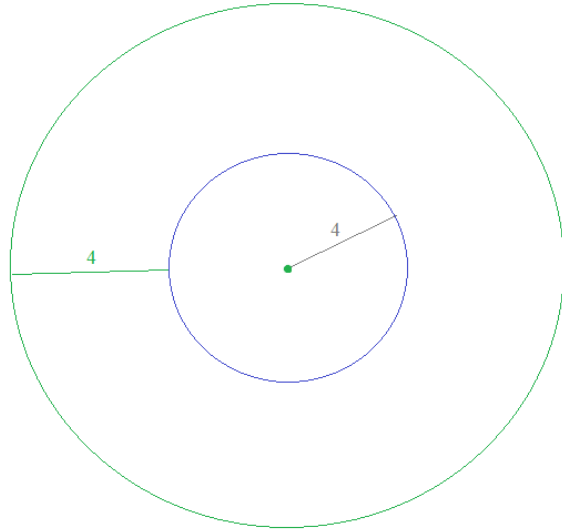
6) Describe the locus of points that are 4 units from a circle with radius 4.

SOLUTIONS

Step 1: Draw the circle



Step 2: Consider the locus of points

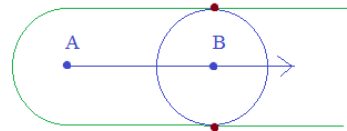


Step 3: Describe

It's a circle with radius 8 AND the center point

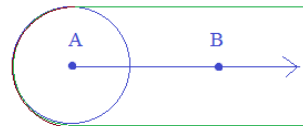
7) Describe the locus of points 7 units from ray \vec{AB} and 7 units from point B

Two points: 7 units above
and 7 units below B



8) Describe the locus of points 7 units from ray \vec{AB} and 7 units from point A

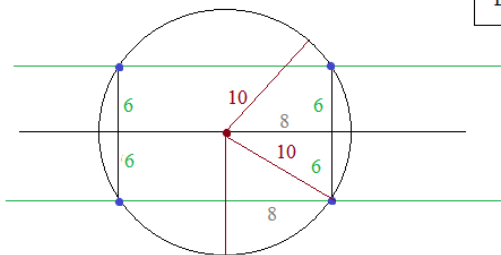
Semicircle with radius of 7 around left of A



Describe the following. Then, draw a sketch.

- 1) Locus of points 6 inches from a line that lie on a circle with a 10-inch radius that has endpoint on the line

vertices of a rectangle that is 12 x 16



SOLUTIONS

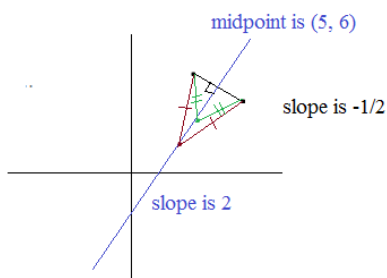
- 2) Locus of points that are equidistant from (3, 7) and (7, 5)

slope between two points: $\frac{7-5}{3-7} = -1/2$

locus of points is the perpendicular bisector...

slope is 2, and it goes through the midpoint (5, 6)

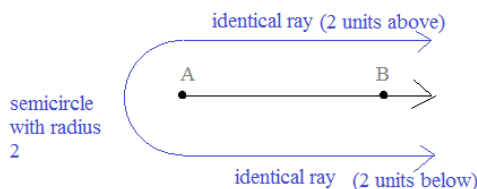
$y - 6 = 2(x - 5)$



Note: This question utilizes several geometry topics
midpoint
slope of perpendicular lines
equidistant theorem
equation of lines

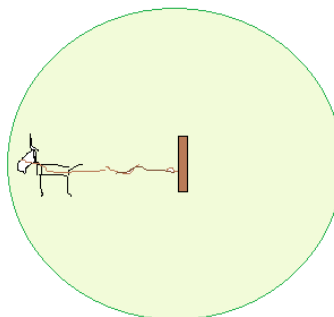
- 3) All points that are 2 units from ray \overrightarrow{AB}

semicircle with center A and radius 2 and 2 parallel rays that are 2 units from \overrightarrow{AB}



- 4) A goat is tied to a fence post in an open pasture. If the rope is 20 feet long, describe the area it is free to graze and roam through.

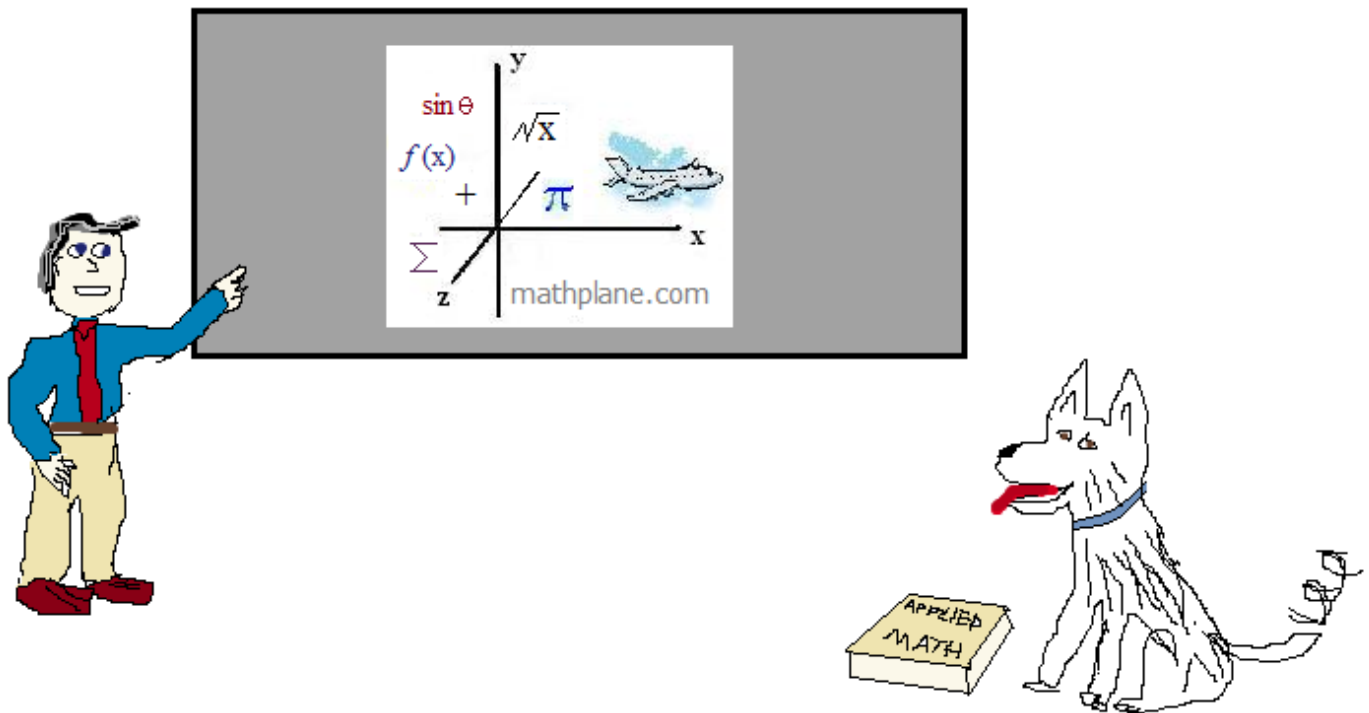
The locus of points is a circle with radius 20, and all the point inside the circle!
Total area = $\pi (\text{radius})^2 = 400\pi$ square ft



Thanks for visiting. (Hope it helped!)

If you have questions, suggestions, or requests, let us know.

Enjoy



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