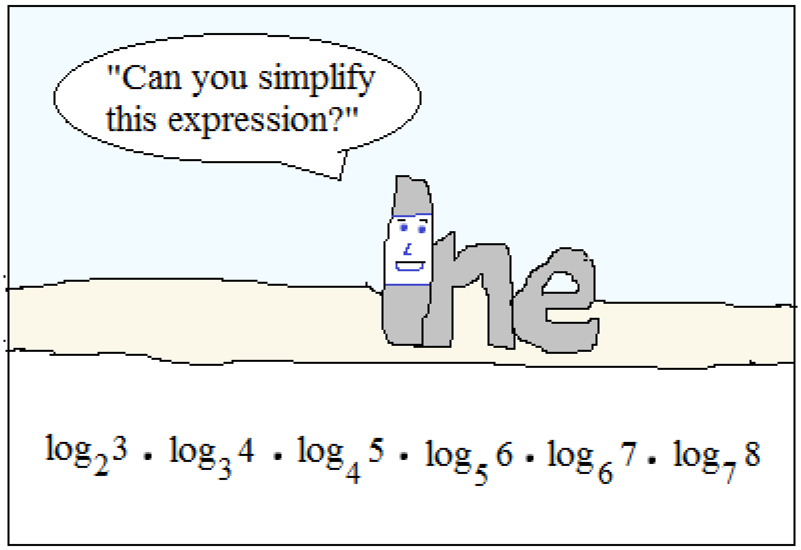
Logarithm and Exponents 2:

Solving equations



(Answer in the back)

Topics include change of base, inverses, inequalities, factoring, intercepts, graphing, and more

Example: $\log_4(x-1) = -1 + \log_4(x)$

$\log_4(x-1) - \log_4(x) = -1$ (Logarithm Quotient Rule)

$\log_4\frac{(x-1)}{x} = -1$ (Change to Exponential Form)

$4^{-1} = \frac{(x-1)}{x}$

$\frac{1}{4} = \frac{(x-1)}{x}$ (Cross Multiply)

$4x - 4 = x$

$\boxed{x = \frac{4}{3}}$

Example: $\log_{\sqrt{a}}(5) = \log_a x$ find x:

$\frac{\log 5}{\log(a^{\frac{1}{2}})} = \frac{\log x}{\log a}$

OR

$\frac{\log 5}{\frac{1}{2}\log a} = \frac{\log x}{\log a}$

$\log 5 = \frac{1}{2}\log x$

$\log 5 = \log\sqrt{x}$

$\boxed{x = 25}$

$\log 5 \cdot \log a = \log x \cdot \log\sqrt{a}$

$\frac{\log 5}{\log x} = \frac{\log\sqrt{a}}{\log a}$

$\log_x 5 = \log_a\sqrt{a}$

$\log_x 5 = \frac{1}{2}$

$\boxed{x = 25}$

Example: $\log(\sqrt[3]{x}) = \sqrt{\log(x)}$

$\log x^{\frac{1}{3}} = \sqrt{\log(x)}$ (Logarithm Power Rule)

$\frac{1}{3}\log(x) = \sqrt{\log(x)}$ (Square both sides)

$\frac{1}{3}\log(x) \cdot \frac{1}{3}\log(x) = \log(x)$

$\frac{1}{9}(\log(x))^2 - \log(x) = 0$

$\log(x) \cdot \left(\frac{1}{9}\log(x) - 1\right) = 0$

$\log(x) = 0$

$x = 10^0$

$\boxed{x = 1}$

$\frac{1}{9}\log(x) - 1 = 0$

$\frac{1}{9}\log(x) = 1$

$\log(x) = 9$

$\boxed{x = 10^9}$

Example: $e^{3x} = \left(\frac{7}{e}\right)^{x+1}$

$e^{3x} = (7 \cdot e^{-1})^{x+1}$

$e^{3x} = 7^{(x+1)} \cdot e^{(-x-1)}$

$\frac{e^{3x}}{e^{(-x-1)}} = 7^{(x+1)}$

$e^{4x+1} = 7^{(x+1)}$

$\ln e^{4x+1} = \ln 7^{(x+1)}$

$(4x+1)\ln e = (x+1)\ln 7$

$4x + 1 = 1.946x + 1.946$

$2.054x = .946$

$\boxed{x = .461}$ approximately

Check:

$e^{(3 \cdot .461)} = \left(\frac{7}{e}\right)^{(.461+1)}$

$e^{1.383} = \left(\frac{7}{e}\right)^{1.461}$

$3.987 = 3.983$ (approx.) ✓

Example: $\log_7(x+5) = \log_7(x-1) - \log_7(x+1)$

$\log_7(x+5) = \log_7\left(\frac{x-1}{x+1}\right)$ (Logarithm Quotient Rule)

$\frac{(x+5)}{1} = \left(\frac{x-1}{x+1}\right)$ (Drop the logarithms)

$(x+5)(x+1) = x - 1$

$x^2 + 5x + 6 = 0$

$(x+2)(x+3) = 0$ $x = -2$ or -3

However, logarithms cannot be negative... Therefore there is

NO SOLUTION!

Example: $\log(4x) - \log(24 + \sqrt{x}) = 2$

$\log_{10}\frac{4x}{(24+\sqrt{x})} = 2$

$\frac{4x}{(24+\sqrt{x})} = 100$

$4x = 2400 + 100\sqrt{x}$

$4x - 100\sqrt{x} - 2400 = 0$

$x - 25\sqrt{x} - 600 = 0$

(let $A = \sqrt{x}$)

$A^2 - 25A - 600 = 0$

$(A - 40)(A + 15) = 0$

$A = 40$ or -15

therefore, $\sqrt{x} = 40$ or -15

$\boxed{x = 1600}$ or ~~225~~

Check: $\log(4(1600)) - \log(24 + \sqrt{1600}) = 2$

$\log(6400) - \log(64) = 2$ ✓

$\log(4(225)) - \log(24 + \sqrt{225}) = 2$

$\log(900) - \log(39) = 2$ ✗

Exponents and Logarithms

Example: Find $3^x = 21$

Method 1: Convert to logarithmic form...

$$3^x = 21$$

$\log_3 21 = x$ Then, input into a calculator... $x = 2.771244$

Method 2: Use the common log (base 10)

$$3^x = 21$$

$\log(3^x) = \log(21)$ "raise" both sides to the common log

$x\log(3) = \log(21)$ logarithm power rule...

$$3^x = 21$$

$\log_3 21 = x$ "Change of Base Formula"

$$x = \frac{\log 21}{\log 3} = \frac{1.3222}{.4771}$$

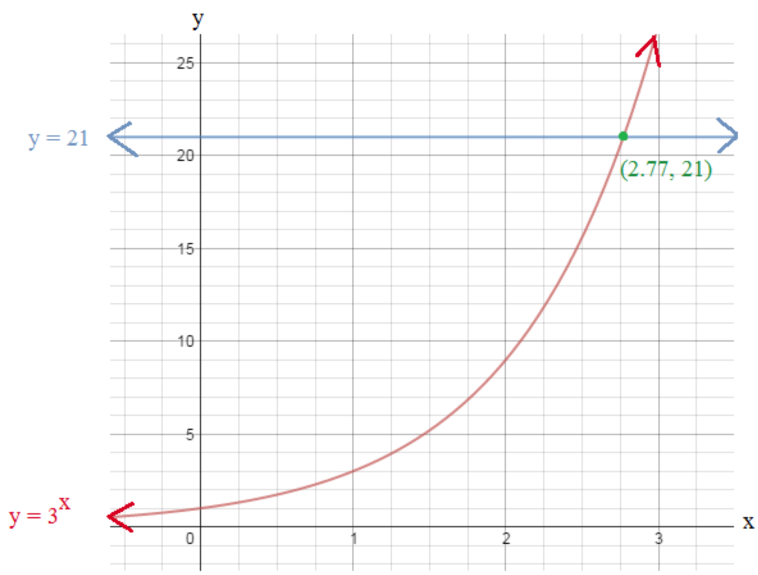
Method 3: Graphing each side

The intersection of

$y = 3^x$

and

$y = 21$ is the solution...



Method 4: Guess and check

$3^x = 21$ If $x = 2$, then $3^2 = 9$ Greater...

If $x = 3$, then $3^3 = 27$ Less...

If $x = 2.8$, then $3^{2.8} = 21.67$ Less...

If $x = 2.7$, then $3^{2.7} = 19.4$ Greater ...

If $x = 2.75$, then $3^{2.75} = 20.52$ Greater...

We've determined the answer is between 2.75 and 2.8

If $x = 2.77$, then $3^{2.77} = 20.97$

Example: $36 = 10\left(1 + \frac{.08}{4}\right)^{4x}$

NOTE: this is a model of a compounding interest function!
"how long will it take 10 to grow to 36 if compounded at 8% quarterly?"

Rule of 72: $72/8 = 9$
it will take approx. 9 years to double...
10... 20 (9 years)... 40 (18 years)
so, the answer should be a bit under 18 years!

Let's see....

$\log 3.6 = (4x)\log(1.02)$

$.5563025 = (4x)(.00860017)$

$x = 16.17$ (approximately)

Example: Find the inverse of $g(x) = 2^{(x-4)} + 6$

Using $\log_2$

$y = 2^{(x-4)} + 6$ change $g(x)$ to y

$x = 2^{(y-4)} + 6$ switch x and y

$x - 6 = 2^{(y-4)}$ solve for y

$\log_2(x-6) = \log_2\left(2^{(y-4)}\right)$

$\log_2(x-6) = (y-4)\log_2 2$

$\log_2(x-6) = (y-4)(1)$

$\log_2(x-6) + 4 = y$

$g^{-1}(x) = \log_2(x-6) + 4$

Using log ($\log_{10}$)

$y = 2^{(x-4)} + 6$ change $g(x)$ to y

$x = 2^{(y-4)} + 6$ switch x and y

$x - 6 = 2^{(y-4)}$ solve for y

$\log(x-6) = \log 2^{(y-4)}$

$\log(x-6) = (y-4)\log 2$

$\frac{\log(x-6)}{\log 2} = (y-4)$

$\log_2(x-6) = (y-4)$

Example: Solve algebraically... Then, support your answer graphically.

$\log_3 x + 7 = 4 - \log_5 x$

$\log_3 x + \log_5 x = 4 - 7$

$\frac{\log x}{\log 3} + \frac{\log x}{\log 5} = -3$

$\frac{1}{.477}\log x + \frac{1}{.699}\log x = -3$

$2.10\log x + 1.43\log x = -3$

$\log x = -.85$

$x = .141$

x	y
(.141,	5.22)

To solve on TI-Nspire CX CAS

"*solve*($\log_3 x + \log_5 x + 3 = 0$, x)"

"enter"

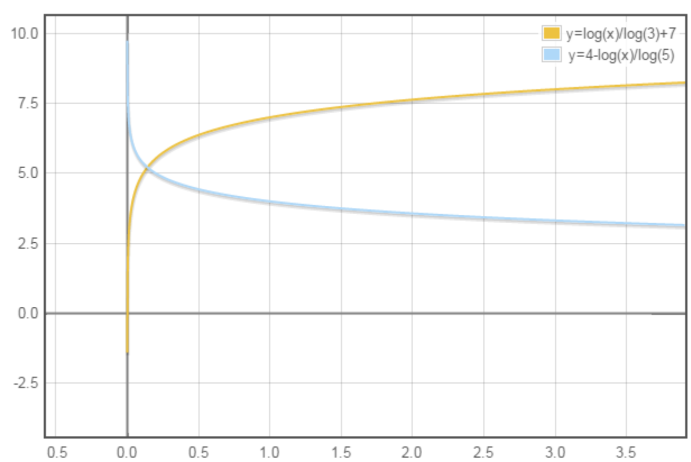
solve graphically on calculator

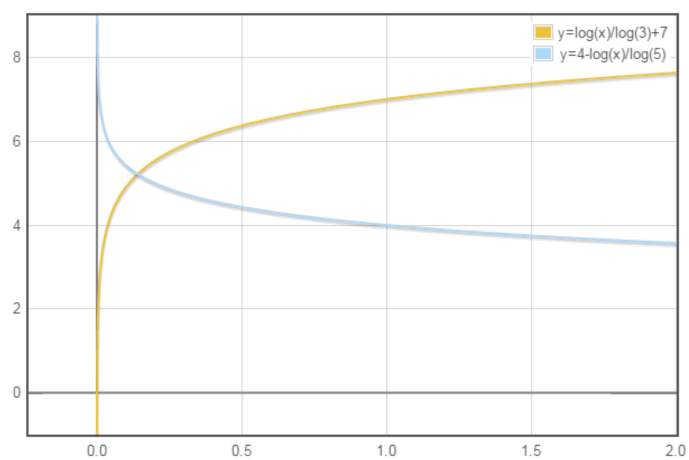
graph $\log_3 x + 7$: $\frac{\log x}{\log 3} + 7$

then,

graph $4 - \log_5 x$ $4 - \frac{\log x}{\log 5}$

The intersection is the solution!





Describing a Logarithmic Curve

Example: Find the equation of the curve using $\log_2$

Step 1: Find 2 points on the curve...

(1, 3) and (2, 4)

Since the inverse of the log function is an exponential function, we can apply...

Step 2: Use the reflected points to find the inverse...

(3, 1) and (4, 2)

$y = ab^x$ exponential model

$1 = ab^3$ $\quad 2 = ab^4$

solve the system: $a = \frac{1}{b^3}$ substitute into second equation

$$2 = \left(\frac{1}{b^3}\right) b^4$$

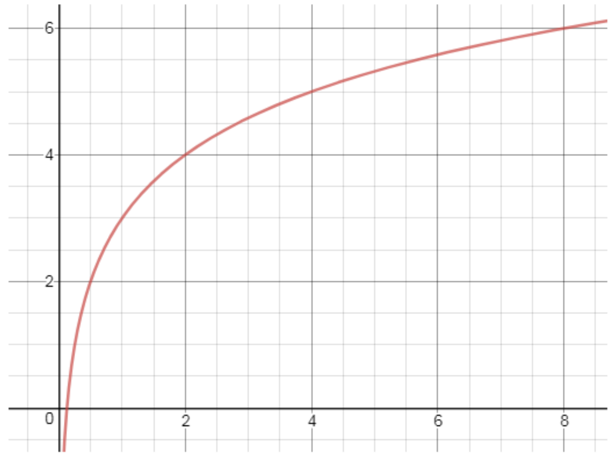
$b = 2$

then, $1 = a(2)^3$

$a = 1/8$

$y = ab^x$

$y = (1/8)(2)^x$



Step 3: Find the inverse (logarithm model)

$x = (1/8)(2)^y$ "switch x and y"

$8x = 2^y$ "solve for y"

$\log_2(8x) = y$

Example: Find the logarithmic equation for the given graph:

Step 1: Recognize the vertical asymptote

$y = \log_a(x - 2)$

Step 2: use the point (3, -3) to find the vertical shift

$y = \log_a(x - 2) + k$

$-3 = \log_a(3 - 2) + k$

$-3 = 0 + k$

vertical shift $k = -3$

$y = \log_a(x - 2) - 3$

Step 3: use the point (4, -4) to find the base

$-4 = -\log_a(4 - 2) - 3$ ⟸ since the log graph was reflected over the x-axis, include a negative

$-1 = -\log_a(2)$ base $a = 2$

$y = -\log_2(x - 2) - 3$

Logarithm 2 Practice Test

I. Logarithm rules and properties

Simplify 1) $\ln e^3 + (\ln e)^2 - \ln(4e^2) =$

2) $2\log_4 8 + (\log_3 162 - \log_3 2) =$

Solve for x 3) $\log(x + 3) = \log x + \log 3$

4) $6 + \log(x^2 - 80) = 6$

5) $2\log_2 x + \log_2\left(\frac{1}{x-1}\right) = 5$

6) $\log_2(x + 7) - \log_2(x - 7) = 3$

7) $3\log_2 x = -\log_2 27$

8) $\log_3(-81) = x$

II. Exponentials and Bases

Logarithm 2 Practice Test

Solve for x:

1) $8^{5x} = 16^{3x+4}$

2) $4^{3-x} \cdot \left(\frac{1}{8}\right)^{2x+5} = 16^{x+3}$

3) $2^{x+1} = 3^{x-1}$

4) $2^{x+3} = 3^{2x-1}$

5) $4^{3x+1} = 5^{x-2}$

6) $2^{2/\log_5 x} = \frac{1}{16}$

Solve for x and y:

7) $4^{x+y} = 64$

$2^{2x-y} = 128$

8) $5^{2x+y} = 21$

$7^{4x-y} = 25$

III. Using Change of Base

Logarithm 2 Practice Test

Simplify:

1) $\log_{10}11 \cdot \log_{11}12 \cdot \log_{12}13 \cdot \ldots \cdot \log_{999}1000 =$

2) $\dfrac{\log_{25}(3)}{\log_{5}(81)}$

Solve for x:

3) $\log_{4}x + \log_{16}x = 1$

4) $3^{x-9} = \dfrac{\log_{5}8}{\log_{5}2}$

Find y:

5) $(\log_{3}x)(\log_{x}4x)(\log_{4x}y) = \log_{x}x^2$

6) $\log_{9}\left(\dfrac{1}{27}\right) = \dfrac{y}{2}$

Logarithm 2 Practice Test

IV. Factoring exponentials

Solve for x:

1) $2^{2x} - 2^{x} - 6 = 0$

2) $3^{2x+1} - 7 \cdot 3^{x} + 2 = 0$

3) $4^{x} - 2^{x+1} = 3$

4) $e^{x} - 6e^{-x} = 1$

5) $(\log_3 x)^2 - \log_3 (x^2) = 3$

Logarithm 2 Practice Test

V. Exponential and Logarithm inequalities

1) $\ln(x + 2)^2 > 3$

2) $6^{n - 1} < 11^n$

3) $\ln(x^2) \geq \ln(x + 2)$

4) $2\ln 3 - \ln(x + 3) > \ln 6$

5) When is $\log_2(x - 2) > \log_4(x)$?

VI. Miscellaneous Questions

1) What are the intercepts? (x-intercept and y-intercept)

$$y = \log_3 (x + 9) - 3$$

2) The vertical asymptote is at $x = 2$
containing point (18, -5)

What is the function in the log form

$$f(x) = \log_4 (x + A) + B\ ?$$

3) $\log_{10} 2 = .30$

$\log_{10} 3 = .48$

What is $\log_3 4$?

(no calculator)

4) Rewrite using base 5:

a) $y = 2(25)^{0.4x}$

b) $y = (4)^{-0.2x}$

5) Find the inverses:

$f(x) = 4e^{(x+2)} + 16$

$h(x) = 3 - \log(2 + x)$

6) Word Problems

A) You deposit $10,000 into an investment account that earns 7% interest.
How many years will it take to increase to $30,000?

a) Use the "rule of 72" to get an estimate...

b) Use logarithms to get an actual value....

B) A six year old savings account has $21,000...
It has been compounding interest continuously at 4%.

What was the original savings deposit?

C) If 300 mg of a sample decays to 200 mg
in 48 hours, find the half-life of the sample...

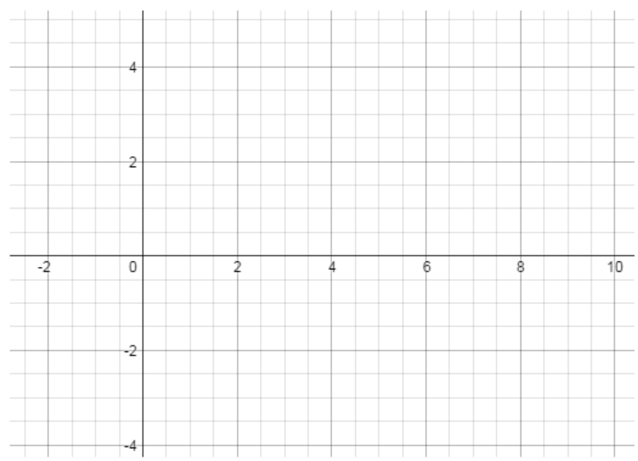
***VII. Challenge Questions

1) $3^{x} \cdot \frac{-4}{3^{x+1}} = 8$

2) $\log_5(x+3) = \log_5(x-1) + \log_3 9 + 6^{\log_6 2}$

3) $2\log_4(x) = \log_4(11x+4) - .5\log_4 9$

4) Graph $\log_3(9x)$ (hint: 9x is "9 times x")



5) $x + 7x^{(2/3)} + 10x^{(1/3)} = 0$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SOLUTIONS-→

I. Logarithm rules and properties

SOLUTIONS

Logarithm 2 Practice Test

Simplify 1) $\ln e^3 + (\ln e)^2 - \ln(4e^2) =$

$3\ln e + (1)^2 - (\ln 4 + \ln e^2)$

$4 - \ln 4 - 2\ln e$

$\boxed{2 - \ln 4}$

2) $2\log_4 8 + (\log_3 162 - \log_3 2) =$

$\log_4 8^2 + (\log_3 \frac{162}{2})$

$3 + 4 = \boxed{7}$

Solve for x 3) $\log(x + 3) = \log x + \log 3$

$\log(x + 3) = \log(x \cdot 3)$

$x + 3 = 3x$

$3 = 2x$

$\boxed{x = 3/2}$

4) $6 + \log(x^2 - 80) = 6$

$\log(x^2 - 80) = 0$

$10^0 = x^2 - 80$

$x^2 - 81 = 0$

$\boxed{x = 9 \text{ and } -9}$

5) $2\log_2 x + \log_2\left(\frac{1}{x-1}\right) = 5$

logarithm power rule $\log_2 x^2 + \log_2\left(\frac{1}{x-1}\right) = 5$

logarithm product rule $\log_2\left(\frac{x^2}{x-1}\right) = 5$

change to exponential form $\frac{x^2}{x-1} = 32$

cross multiply $x^2 = 32(x - 1)$

quadratic formula $x^2 - 32x + 32 = 0$

$\boxed{x = 1.033 \text{ or } 30.967}$

6) $\log_2(x + 7) - \log_2(x - 7) = 3$

$\log_2 \frac{(x+7)}{(x-7)} = 3$

$2^3 = \frac{(x+7)}{(x-7)}$

$8x - 56 = x + 7$

$7x = 63$

$\boxed{x = 9}$

7) $3\log_2 x = -\log_2 27$

$\log_2 x^3 = \log_2 27^{-1}$

$x^3 = \frac{1}{27}$

$\boxed{x = \frac{1}{3}}$

8) $\log_3(-81) = x$

no solution!

3^x cannot equal -81

II. Exponentials and Bases

SOLUTIONS

Logarithm 2 Practice Test

Solve for x:

1) $8^{5x} = 16^{3x+4}$

$(2^3)^{5x} = (2^4)^{3x+4}$

$2^{15x} = 2^{12x+16}$

$15x = 12x + 16$

$3x = 16$

$x = 16/3$

2) $4^{3-x} \cdot \left(\frac{1}{8}\right)^{2x+5} = 16^{x+3}$

$\left(2^2\right)^{3-x} \cdot \left(2^{-3}\right)^{2x+5} = \left(2^4\right)^{x+3}$

$2^{6-2x} \cdot 2^{-6x-15} = 2^{4x+12}$

$2^{-8x-9} = 2^{4x+12}$

$-8x - 9 = 4x + 12$

$-21 = 12x$

$x = -21/12 = -7/4$

3) $2^{x+1} = 3^{x-1}$

$\log 2^{x+1} = \log 3^{x-1}$

$(x+1)\log 2 = (x-1)\log 3$

$(x+1)(.301) = (x-1)(.477)$

$.301x + .301 = .477x - .477$

$.778 = .176x$

$x = 4.42$ (approx.)

Check: $2^{4.42+1} = 3^{4.42-1}$

$2^{5.42} = 3^{3.42}$ (approximately)

$42.81 = 42.82$ ✓

4) $2^{x+3} = 3^{2x-1}$

take the log of both sides:

$\log 2^{x+3} = \log 3^{2x-1}$

$(x+3)\log 2 = (2x-1)\log 3$

$.301x + .903 = .954x - .477$

$1.380 = .653x$

$x = 2.11$

5) $4^{3x+1} = 5^{x-2}$

one method:

take log (base4) of both sides...

$\log_4 4^{3x+1} = \log_4 5^{x-2}$

$3x + 1 = (x-2)(\log_4 5)$

$3x + 1 = (x-2)(1.16)$

$1.84x = -3.32$

$x = -1.80$ approximately

check: $4^{3(-1.80)+1} = 5^{-1.80-2}$

$.00224 \approx .00221$ ✓

6) $2^{2/\log_5 x} = \frac{1}{16}$

$2^{2/\log_5 x} = 2^{-4}$

$\frac{2}{\log_5 x} = -4$

$2 = (-4)\log_5 x$

$\frac{-1}{2} = \log_5 x$

$x = 5^{-1/2}$ or $\frac{1}{\sqrt{5}}$

Solve for x and y:

7) $4^{x+y} = 64$

$2^{2x-y} = 128$

$4^{x+y} = 4^3$

$2^{2x-y} = 2^7$

$x + y = 3$

$2x - y = 7$

$x = 10/3$

$y = -1/3$

8) $5^{2x+y} = 21$

$7^{4x-y} = 25$

$\log_5(21) = 2x + y$

$\log_7(25) = 4x - y$

$1.8917 = 2x + y$

$+\ 1.6542 = 4x - y$

$3.5459 = 6x$

solve system: combination/ elimation method

$x = .591 \quad y = .710$

check: $5^{2(.591)+(.710)} = 21.01$

$7^{4(.591)-(.710)} = 24.99$

III. Using Change of Base — SOLUTIONS — Logarithm 2 Practice Test

Simplify:

1) $\log_{10}11 \cdot \log_{11}12 \cdot \log_{12}13 \cdot \ldots \cdot \log_{999}1000 =$

Using change of base formula:

$$\frac{\log 11}{\log 10} \cdot \frac{\log 12}{\log 11} \cdot \frac{\log 13}{\log 12} \cdots \frac{\log 999}{\log 998} \cdot \frac{\log 1000}{\log 999}$$

$$\frac{\log 1000}{\log 10} = \frac{3}{1} = \boxed{3}$$

2) $\dfrac{\log_{25}(3)}{\log_5(81)}$

change of base $\dfrac{\frac{\log 3}{\log 25}}{\frac{\log 81}{\log 5}}$

$$\frac{\log 3}{\log 25} \cdot \frac{\log 5}{\log 81}$$

$$\frac{\log 3}{\log 81} \cdot \frac{\log 5}{\log 25}$$

$$\log_{81}(3) \cdot \log_{25}(5)$$

$$\frac{1}{4} \cdot \frac{1}{2} = \boxed{\frac{1}{8}}$$

Solve for x:

3) $\log_4 x + \log_{16} x = 1$

$\log_4 x + \dfrac{\log_4 x}{\log_4 16} = 1$ — use change of base (to base 4)

$\log_4 x + \dfrac{\log_4 x}{2} = 1$

$2\log_4 x + \log_4 x = 2$ — log power rule

$\log_4 x^2 + \log_4 x = 2$ — log product rule

$\log_4 x^3 = 2$ — convert to exponential form

$x^3 = 16$

$\boxed{x = 2\sqrt[3]{2}}$

4) $3^{x-9} = \dfrac{\log_5 8}{\log_5 2}$

$$\frac{\frac{\log 8}{\log 5}}{\frac{\log 2}{\log 5}} = \frac{\log 8}{\log 2}$$

Instead of using base 10, let's use base 2...

$$3^{x-9} = \frac{\log_2 8}{\log_2 2}$$

$$3^{x-9} = \frac{3}{1}$$

$$3^{x-9} = 3^1$$

$\boxed{x = 10}$

Find y:

5) $(\log_3 x)(\log_x 4x)(\log_{4x} y) = \log_x x^2$

Using change of base formula: $\dfrac{\log x}{\log 3} \cdot \dfrac{\log 4x}{\log x} \cdot \dfrac{\log y}{\log 4x} = \log_x x^2$

Simplify:

$$\frac{\log y}{\log 3} = \log_x x^2$$

$$\frac{\log y}{\log 3} = 2$$

$$\log_3 y = 2$$

$\boxed{y = 3^2 = 9}$

6) $\log_9\left(\dfrac{1}{27}\right) = \dfrac{y}{2}$

change of base (to base 3)

$$\frac{\log_3\left(\frac{1}{27}\right)}{\log_3 9} = \frac{y}{2}$$

$$\frac{-3}{2} = \frac{y}{2}$$

$\boxed{y = -3}$

OR

change to exponential form

$$9^{\frac{y}{2}} = \frac{1}{27}$$

$$(3^2)^{\frac{y}{2}} = \frac{1}{27}$$

$$3^{\frac{2y}{2}} = 3^{-3}$$

$$\frac{2y}{2} = -3$$

$\boxed{y = -3}$

IV. Factoring exponentials

Solve for x:

SOLUTIONS

Logarithm 2 Practice Test

1) $2^{2x} - 2^x - 6 = 0$

Hint: $2^{2x} = (2^x)^2$

$(2^x)^2 - 2^x - 6 = 0$ $\qquad A^2 - A - 6 = 0$

$(2^x - 3)(2^x + 2) = 0$ $\qquad (A-3)(A+2) = 0$

$A = 3, -2$

$2^x = 3$

$x = \frac{\log 3}{\log 2}$ approx. 1.585

$2^x = -2$ No solution

2) $3^{2x+1} - 7 \cdot 3^x + 2 = 0$

Hint: recognize 3^x as a term and use exponent rules

$3^{2x+1} = 3^{2x} \cdot 3^1$

Let $A = 3^x$

then, $3^{2x} = A^2$

$3A^2 - 7A + 2 = 0$

$(3A - 1)(A - 2) = 0$

$A = 1/3$ or 2

$3^x = 1/3$ $\qquad x = -1$

$3^x = 2$ $\qquad x = \frac{\log 2}{\log 3}$ $\qquad x = .63$ (approx.)

(substitute into original equation to check!)

3) $4^x - 2^{x+1} = 3$

$4^x - 2^{x+1} - 3 = 0$

$(2^2)^x - (2^x)(2^1) - 3 = 0$

$(2^x)^2 - (2^x)(2^1) - 3 = 0$

Let $y = 2^x$

$y^2 - 2y - 3 = 0$

Check:

$(y-3)(y+1) = 0$

$y = -1, 3$

therefore, $2^x = -1$ and 3

-1 is extraneous!

$2^x = 3$ → approx. 1.585

4) $e^x - 6e^{-x} = 1$

$e^x \cdot (e^x - 6e^{-x} - 1) = 0 \cdot e^x$

$e^{2x} - 6e^0 - e^x = 0$

$e^{2x} - e^x - 6 = 0$

let $A = e^x$

$A^2 - A - 6 = 0$

$(A-3)(A+2) = 0$

$A = -2, 3$

$e^x = -2$ or 3

-2 is extraneous, because e^x will never be negative.

$e^x = 3$

take natural log of each side

$\ln e^x = \ln 3$

$x \ln e = 1.0986$ (approximately)

$x = 1.0986$ (approximately)

5) $(\log_3 x)^2 - \log_3 (x^2) = 3$

$(\log_3 x)^2 - 2(\log_3 x) = 3$

$(\log_3 x)^2 - 2(\log_3 x) - 3 = 0$

$A^2 - 2A - 3 = 0$

$(\log_3 x - 3)(\log_3 x + 1) = 0$

$(A - 3)(A + 1) = 0$

$(\log_3 x - 3) = 0$ $\qquad \log_3 x = 3$ $\qquad x = 27$

$(\log_3 x + 1) = 0$ $\qquad \log_3 x = -1$ $\qquad x = 1/3$

V. Exponential and Logarithm inequalities

SOLUTIONS

Logarithm 2 Practice Test

1) $\ln(x+2)^2 > 3$

$\log_e (x+2)^2 = 3$

$e^3 = (x+2)^2$

$\pm\sqrt{20.08} = x + 2$

$x = -2 \pm\sqrt{20.08}$

2) $6^{n-1} < 11^n$

$(n - 1)\log 6 = n\log 11$

$n\log 6 - \log 6 = n\log 11$

$n\log 6 - n\log 11 = \log 6$

$n(\log 6 - \log 11) = \log 6$

$n\log(6/11) = \log 6$

$n = \log 6/\log(6/11)$

$n > -2.9560$

If n = 0,

then $6^{0-1} < 11^0$

$\frac{1}{6} < 1$ ✓

3) $\ln(x^2) \geq \ln(x+2)$

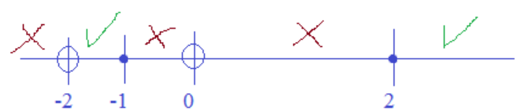
$x^2 > (x+2)$ assume terms are equal to determine the 'critical values'

$x - x - 2 = 0$

Then, test values in each region...

$(x-2)(x+1) = 0 \quad x = -1, 2$

(-2, -1] ∪ [2, ∞)



ln cannot = 0 or be negative...

4) $2\ln 3 - \ln(x+3) > \ln 6$

$\ln 3^2 - \ln(x+3) = \ln 6$

$\ln \frac{9}{(x+3)} = \ln 6$

$\frac{9}{(x+3)} = 6$

$6x + 18 = 9$

$x = -3/2$

Test x = -2

$2\ln 3 - \ln(-2+3) > \ln 6$?

$\ln 9 - \ln(1) > \ln 6$

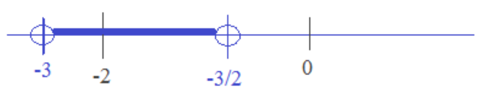
$\ln 9 > \ln 6$ YES

Test x = 0

$2\ln 3 + \ln(0+3) > \ln 6$?

$\ln 9 + \ln 3 > \ln 6$

$\ln \frac{9}{3} > \ln 6$ NO



So, x < -3/2... BUT, it must be greater than -3
(otherwise, ln(x + 3) is undefined)

$-3 < x < -3/2$

5) When is $\log_2(x-2) > \log_4(x)$?

(first, find where sides are equal...)

$\frac{\log_2(x-2)}{\log_2 2} = \frac{\log_2 x}{\log_2 4}$ use change of base formula

$\frac{\log_2(x-2)}{1} = \frac{\log_2 x}{2}$

$2\log_2(x-2) = \log_2 x$

$\log_2(x-2)^2 = \log_2 x$

$(x-2)^2 = x$

$x^2 - 5x + 4 = 0$

$(x-4)(x-1) = 0$

$x = 1, 4$

But, we eliminate 1, because $\log_2(x - 2)$ does not exist when x = 1

test x = 3....
and, the inequality does not work..

$x \geq 4$

VI. Miscellaneous Questions

SOLUTIONS

Logarithm 2 Practice Test

1) What are the intercepts? (x-intercept and y-intercept)

$y = \log_3 (x + 9) - 3$

y-intercept occurs when $x = 0$ (0, ?)

$y = -1$

(0, -1)

x-intercept occurs when $y = 0$ (?, 0)

$3 = \log_3 (x + 9)$

$x + 9 = 27 \qquad x = 18$

(18, 0)

2) The vertical asymptote is at $x = 2$ containing point (18, -5)

What is the function in the log form

$f(x) = \log_4 (x + A) + B$?

since asymptote is $x = 2$,

$f(x) = \log_4 (x - 2) + B$

then, to find B, substitute the point (18, -5)

$-5 = \log_4 (18 - 2) + B$

$-5 - B = \log_4 (16)$

$B = -7$

$f(x) = \log_4 (x - 2) - 7$

3) $\log_{10} 2 = .30$

$\log_{10} 3 = .48$

What is $\log_3 4$?

(no calculator)

$\log_3 (2)^2$

$2 \cdot \log_3 (2)$

$2 \cdot \frac{\log 2}{\log 3} = 2 \cdot \frac{.30}{.48} = \frac{.60}{.48} =$ **1.25**

4) Rewrite using base 5:

a) $y = 2(25)^{0.4x}$

$y = 2(5^2)^{0.4x}$

$y = 2(5)^{0.8x}$

Find $5^x = 2$

$\log 5^x = \log 2$

$x = \frac{\log 2}{\log 5} = .43$

$5^{.43} = 2$

$y = 5^{(0.8x + .43)}$ (approx)

b) $y = (4)^{-0.2x}$

Find $5^x = 4$

$\log 5^x = \log 4$

$x = \frac{\log 4}{\log 5} = .86$

$y = (5^{.86})^{-0.2x}$

$y = (5)^{-.17x}$ (approx)

5) Find the inverses: SOLUTIONS

$f(x) = 4e^{(x+2)} + 16$

$y = 4e^{(x+2)} + 16$ switch x and y

$x = 4e^{(y+2)} + 16$

$x - 16 = 4e^{(y+2)}$

$\frac{x-16}{4} = e^{(y+2)}$

$\ln\frac{x-16}{4} = y + 2$

$f^{-1}(x) = \ln\frac{x-16}{4} - 2$

$h(x) = 3 - \log(2 + x)$

$y = 3 - \log(2 + x)$ switch x and y

$x = 3 - \log(2 + y)$ solve for y

$3 - x = \log(2 + y)$

$10^{3-x} = 2 + y$

$h^{-1}(x) = 10^{3-x} - 2$

6) Word Problems

A) You deposit $10,000 into an investment account that earns 7% interest.
How many years will it take to increase to $30,000?

a) Use the "rule of 72" to get an estimate...

"rule of 72" estimates it'll take 72/7, or approx. 10 years to double..
$10,000 to $20,000 will take 10 years...
$20,000 to $40,000 will take another 10 years...
Since we are looking for an estimate for $30,000, half-way, it takes approx 15 years...

b) Use logarithms to get an actual value....

$A = Pe^{rt}$

$30{,}000 = 10{,}000e^{.07t}$

$3 = e^{.07t}$

$\ln 3 = \ln e^{.07t}$

$t = \frac{\ln 3}{.07} =$ 15.69 years (approx)

B) A six year old savings account has $21,000...
It has been compounding interest continuously at 4%.

What was the original savings deposit?

$A = Pe^{rt}$

$21{,}000 = Pe^{(.04)(6)}$

$21{,}000 = P(1.27)$

$P = 16{,}519$

C) If 300 mg of a sample decays to 200 mg in 48 hours, find the half-life of the sample...

Step 1: Find the rate r

$A = Pe^{rt}$

$200\text{mg} = 300\text{mg}(e)^{r(48)}$

$\frac{2}{3} = e^{48r}$

$\ln\left(\frac{2}{3}\right) = 48r(\ln e)$

$r = -.008447$

Step 2: Find the half-life (t)

$A = Pe^{rt}$

$150\text{mg} = 300\text{mg}(e)^{-.008447t}$

$\ln\left(\frac{1}{2}\right) = -.008447t$

82 hours

***VII. Challenge Questions

SOLUTIONS

1) $3^{x} \cdot \frac{-4}{3^{x+1}} = 8$

$-4 \cdot \frac{3^x}{3^{x+1}} = 8$

$\frac{3^x}{3^{x+1}} = -2$

$3^{-1} = -2$

NO SOLUTION

2) $\log_5(x+3) = \log_5(x-1) + \log_3 9 + 6^{\log_6 2}$

$\log_5(x+3) - \log_5(x-1) = 2 + 2$

$\log_5 \frac{(x+3)}{(x-1)} = 4$

$\frac{(x+3)}{(x-1)} = 625$

$625x - 625 = x + 3$

$624x = 628$

$x = \frac{157}{156}$

3) $2\log_4(x) = \log_4(11x+4) - .5\log_4 9$

$\log_4(x)^2 = \log_4(11x+4) - \log_4 9^{.5}$

$\log_4(x^2) = \log_4 \frac{(11x+4)}{3}$

$3x^2 = 11x + 4$

$3x^2 - 11x - 4 = 0$

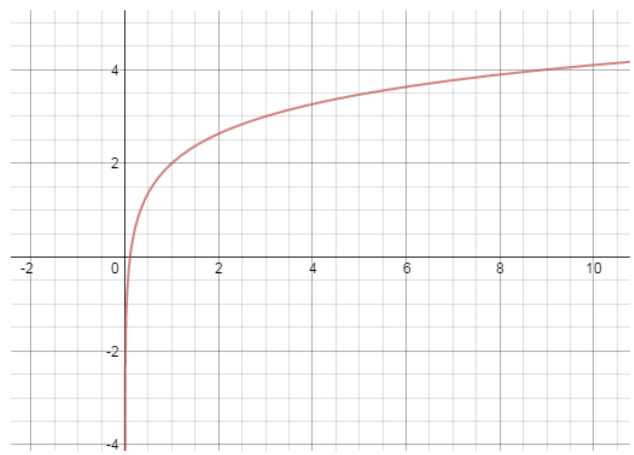
$(3x+1)(x-4) = 0$

$x = 4$ or ~~$-1/3$~~

ONLY $x = 4$

4) Graph $\log_3(9x)$ (hint: 9x is "9 times x")

$\log_3(9) + \log_3(x) = 2 + \log_3(x)$



Points include: (1, 2) (9, 4) and (1/9, 0)

5) $x + 7x^{(2/3)} + 10x^{(1/3)} = 0$

Use substitution

(choose the "smallest variable exponent")

Let $U = x^{(1/3)}$

$U^3 + 7U^2 + 10U = 0$

$U(U+2)(U+5) = 0$

$U = -2, -5, 0$

$U = -2$: $-2 = x^{(1/3)}$ → $x = -8$

$U = 0$: $0 = x^{(1/3)}$ → $x = 0$

$U = -5$: $-5 = x^{(1/3)}$ → $x = -125$

(plug in solutions to original equation to check)

$-8 + 7(4) + 10(-2) = 0$

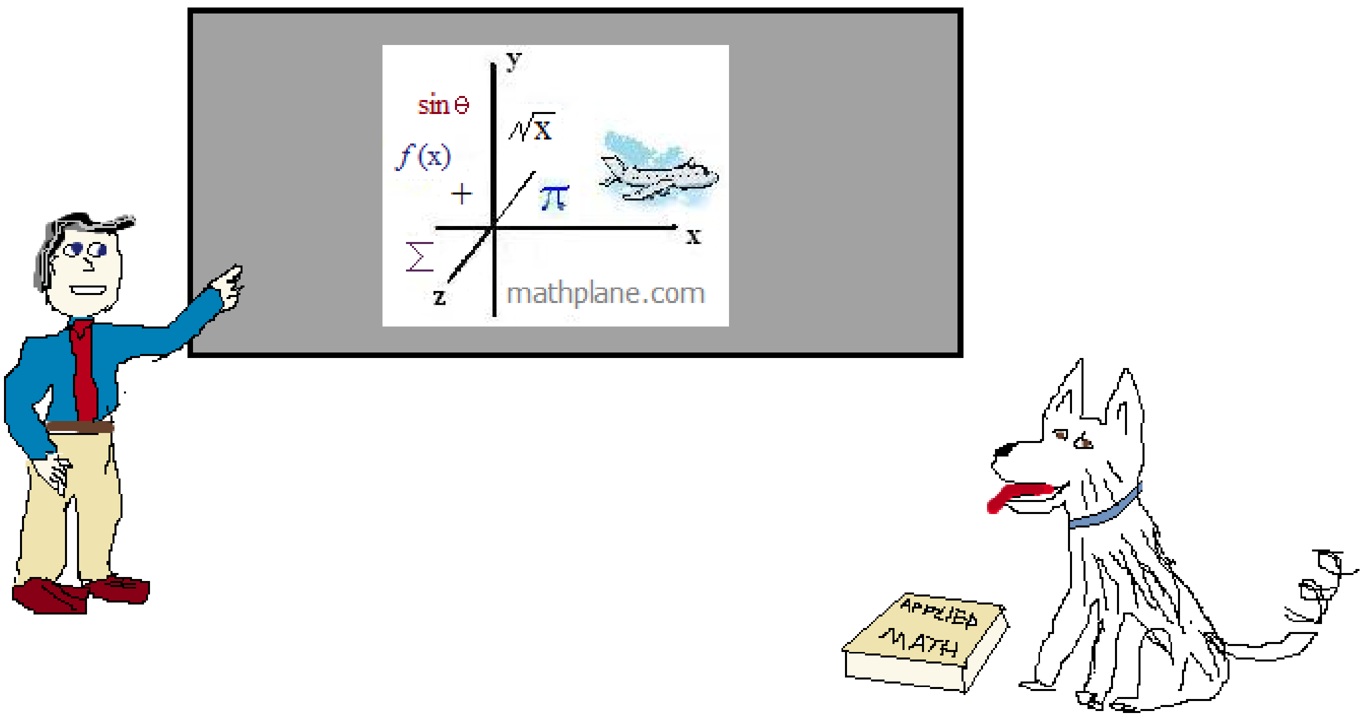
$0 + 7(0) + 10(0) = 0$

$-125 + 7(25) + 10(-5) = 0$

Thanks for visiting. (Hope it helped!)

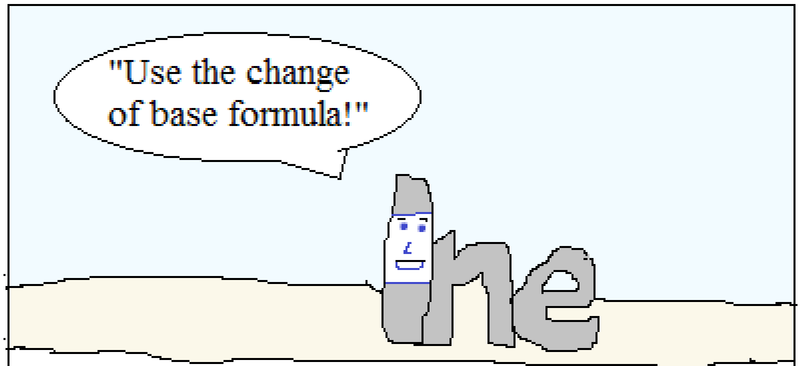
If you have questions, suggestions, or requests, let us know.

Cheers



Also, at Mathplane.ORG.

Or, check out our stores at Teacherspayteachers and TES



$$\log_2 3 \cdot \log_3 4 \cdot \log_4 5 \cdot \log_5 6 \cdot \log_6 7 \cdot \log_7 8$$

Using Change of Base Formula:

$$\frac{\log 3}{\log 2} \cdot \frac{\log 4}{\log 3} \cdot \frac{\log 5}{\log 4} \cdot \frac{\log 6}{\log 5} \cdot \frac{\log 7}{\log 6} \cdot \frac{\log 8}{\log 7}$$

$$\frac{\log 8}{\log 2} = \log_2 8 = \boxed{3}$$