

Logarithms Practice Test

(with detailed Solutions)

Topics include logarithm laws, graphing, exponential equations, growth and decay models, $\frac{1}{2}$ life, and more...

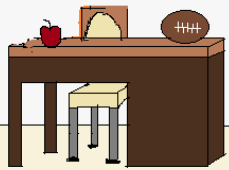


"Show me the one e ...
Say it with me one time, Jerry.
SHOW ME THE ONE e !!!"



euler Natural Logarithm
 $e \rightarrow \text{approx. } 2.718$
 $\ln(x) = 1$

Announcements
7/30/16 Seminar: "Learning about the Kwan"
Reminder: YOU must COMPLETE ch.2 for ME



"Dorothy, do you understand what he's saying?"

"I don't know. But, he had me at hello."

"Why is Mr. Tidwell wearing cleats?"

"At least, he kept his shirt on today.."



Pre-Calc

Jerry Maguire

1) Between what 2 consecutive integers are the following:

a) $\log 500$

b) $\log_5 (.5)$

2) Solve: $2^{5x+3} = 3^{2x+1}$

3) $\log_2 x + \log_4 x + \log_8 x = 11$ Find x

$$4) |\log_4 x| = 3$$

$$5) (\log_5 x)^3 - (\log_5 x)^2 - \log_5 x^9 + 9 = 0$$

$$6) \log_2(\log_3(\log_4 x)) = 0$$

$$7) x^{\sqrt{\log x}} = 10^8$$

$$8) f(x) = \log_6(3x)$$

$$g(x) = 2 \cdot 6^{5x}$$

find $f(g(x))$

$$9) \ln(x) = 1 + \ln(3x - 4)$$

Logarithms Practice Test

$$10) \log(x+1)^4 = 20$$

$$11) \log_3 27^{x-1}$$

$$12) \log_x \frac{81}{x^3} = -1$$

$$13) (\log_3 x)(\log_x 2x)(\log_{2x} y) = \log_x x^2$$

$$14) (\log_3 x)^3 + (\log_3 x)^2 = \log_3 x^{17} - 15$$

15) Solve for t using logarithms with base a

a) $2a^{t/3} = 11$

b) $4a^{2t} = B + 5$

c) $M = Sa^{ct} + D$

16) Use Natural Logarithms to solve for x in terms of y :

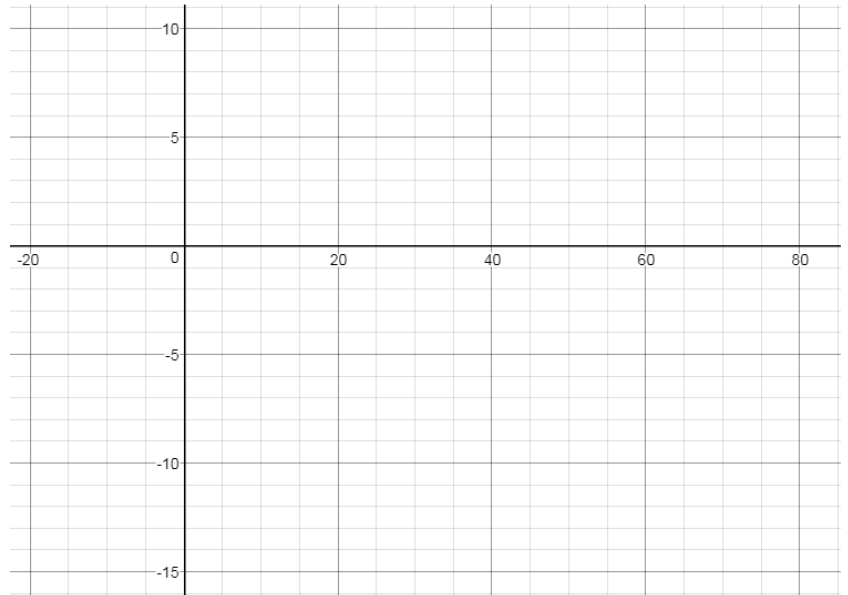
$$y = \frac{e^x - e^{-x}}{2}$$

17) Use Common Logarithms to solve for x in terms of y :

$$y = \frac{10^x + 10^{-x}}{2}$$

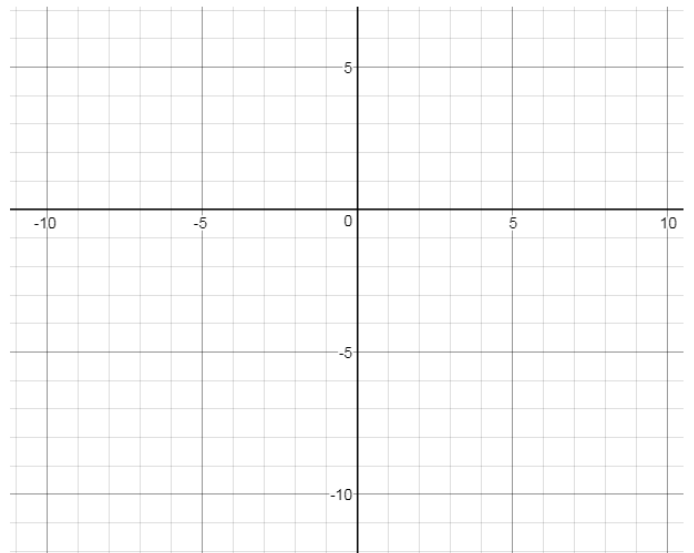
1) $y = \log_4 \frac{x+2}{64}$

graph the function, labeling any asymptotes and intercepts...



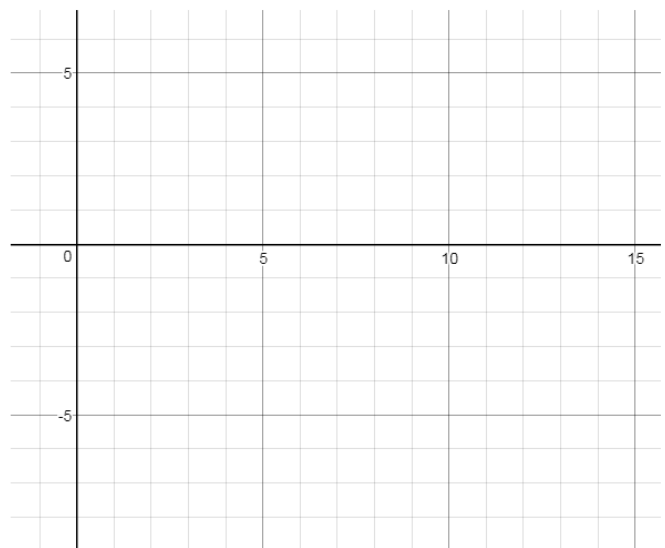
2) $f(x) = -4^{x+2}$

find the inverse $f^{-1}(x)$, and graph both functions



3) $y = -\log_3(81x)$

graph the function



1) Write an exponential equation that goes through (0, 7) and (6, 15).

2) Write an exponential equation that goes through (3, 10) and (7, 32).

3) An exponential function of form $f(x) = ab^x + c$

has these features: y-intercept is at 5

goes through (1, 7)

horizontal asymptote at $y = 1$

Identify the function.

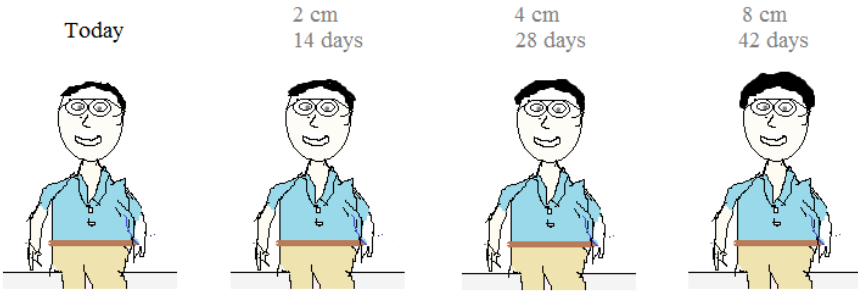
4) A used car is worth \$12,000 today, and \$3000 5 years from now.
What is the exponential model (of depreciation)?

If I try to sell the car 10 years from now,
what can I hope to get for it?

5) A piece of machinery cost 250,000 dollars...
After 5 years, it is worth 220,000 dollars...

What is the rate of depreciation?

For continuously compounding hair, Pe^{rt} shampoo will work it out...



Recommended Usage: Amount = Pe^{rt}



"contains growth factor and Ph balance....."

Look for Pert + or 2:1 in the math section of your department store...

Apply Faberge Organics Sequence:
Tell 2 friends, and they'll tell 2 friends,
and so on, and so on, and so on...

An exponential growth formula that is head and shoulders above the rest!

LanceAF #242 (6-7-16)
mathplane.com

Pert

1) John has \$2200 in an account that increases 7% annually...

A brand new sports car costs \$48,000, but it depreciates by 22% annually...

If John is willing to buy the sports car used, when would he be able to afford it?

2) A bank offers savings accounts that pay 4% interest compounded continuously,
OR accounts that pay 4.5% simple interest.

If you want to invest the amount for 5 years, which account should you use?

- 3) Jason opens an investment account with a 6.5% annual interest rate, compounding continuously. If he deposits \$1000, how much will he have in 9 months?

When will the account have \$2500?

- 4) Uranium has a 1/2 life of 2.7×10^5 years...

- a) How long does it take for 10 mg of Uranium to decay to 7 mg?
b) How much remains after 1,000,000 years?

- 3) A population in Algebratown is modeled by $P = 344e^{kt}$

where $t = 0$ (corresponds to 1990)

and P is the population in 1,000s

In 1975, the population was 189,000...

- a) Find k
b) Predict the population in 2030

1) $5 \cdot (.5)^x - 4 = 3 \cdot 2^{-x}$

2) $5^x + 125(5^{-x}) = 30$

3) Using exponents and/or logarithm properties, can you evaluate 2003^{97} ?

Is there more or less than 300 digits?

What is the estimate (in scientific notation)?

4) If it takes 7 years for an investment to double, how long would it take for the investment to triple?

5) A family's financial goal is to have \$20,000 in an account after 5 years....

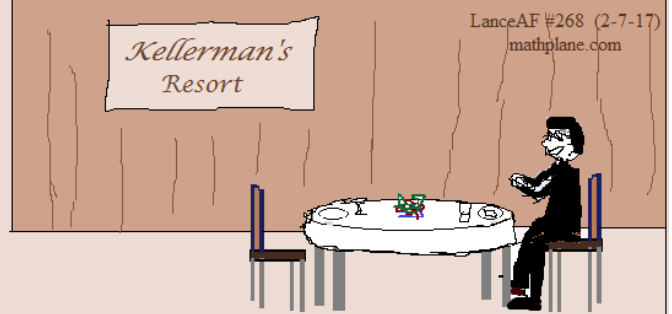
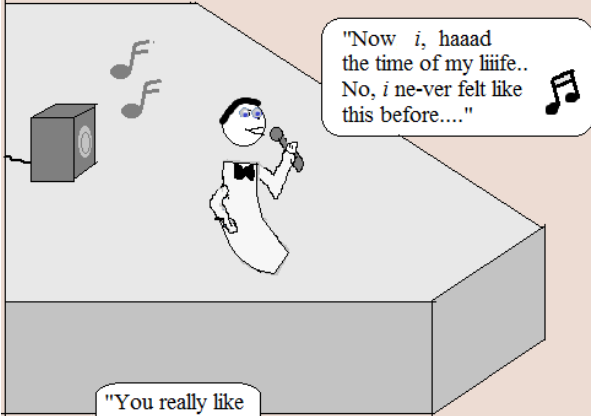
a) If the family has \$12,000, what yield will they need to reach the goal?

b) If rates are 6%, how much must they deposit to reach their goal?

(Assume the family does not add money later...)

Somewhere else in the Catskills....

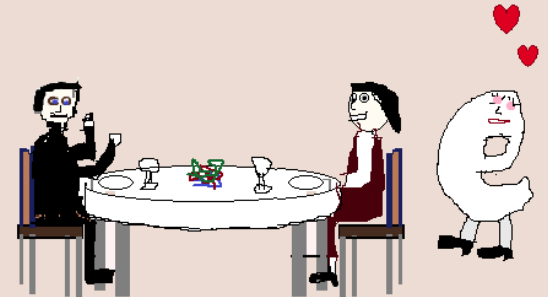
LanceAF #268 (2-7-17)
mathplane.com



"You really like to sway, Z!"

"Yes, I do, Max."

"Nobody puts Babe e in a corner."



The dancing numbers at the end of this talent show were memorable!

Thirty Dancing

SOLUTIONS-→

1) Between what 2 consecutive integers are the following:

SOLUTIONS

a) $\log 500$ $\log(5) + \log(100) = \log(5) + 2$

$\log_{10}(5) = x$ $10^0 = 1$ so, $\log(5)$ is between 0 and 1
 $10^1 = 10$

b) $\log_5(.5)$ $5^x = \frac{1}{2}$

$10^x = 5$ $\log(500)$ is between 2 and 3
 $5^{-1} = 1/5$
 $5^0 = 1$ $\log_5(.5)$ is between -1 and 0
 $5^1 = 5$

2) Solve: $2^{5x+3} = 3^{2x+1}$

Method 1: Lift both sides with logs

Method 2: Split the exponents

$\log 2^{5x+3} = \log 3^{2x+1}$

$2^{5x} \cdot 2^3 = 3^{2x} \cdot 3^1$

$(5x+3)\log 2 = (2x+1)\log 3$

$(2^5)^x \cdot 8 = (3^2)^x \cdot 3$

$\frac{(5x+3)}{(2x+1)} = \frac{\log 3}{\log 2}$

$32^x \cdot 8 = 9^x \cdot 3$

$\frac{(5x+3)}{(2x+1)} = \log_2 3$

$\left(\frac{32}{9}\right)^x = \frac{3}{8}$

$\frac{(5x+3)}{(2x+1)} = 1.585$

$\log_{(32/9)}(.375) = x$

$3.17x + 1.585 = 5x + 3$

$-1.415 = 1.83x$

$x = -.7732$ (approx)

$x = -.7732$ (approx)

3) $\log_2 x + \log_4 x + \log_8 x = 11$

Find x

$\frac{\log x}{\log 2} + \frac{\log x}{\log 4} + \frac{\log x}{\log 8} = 11$

change of base

$\frac{\log x}{\log 2} + \frac{\log x}{\log(2^2)} + \frac{\log x}{\log(2^3)} = 11$

find common log denominator

$\frac{\log x}{\log 2} + \frac{\log x}{2\log 2} + \frac{\log x}{3\log 2} = 11$

$\frac{6\log x}{6\log 2} + \frac{3\log x}{6\log 2} + \frac{2\log x}{6\log 2} = 11$

$\frac{11\log x}{6\log 2} = 11$

$\frac{11\log x}{\log 2^6} = 11$

$\frac{11\log x}{\log(64)} = 11$

$11\log x = 11\log(64)$

$x = 64$

4) $|\log_4 x| = 3$

SOLUTIONS

$\log_4 x = 3$ OR $\log_4 x = -3$

$x = 64$

$x = 1/64$

5) $(\log_5 x)^3 - (\log_5 x)^2 - \log_5 x^9 + 9 = 0$

$(\log_5 x)^3 - (\log_5 x)^2 - 9\log_5 x + 9 = 0$ logarithm power rule

Let $A = \log_5 x$

using substitution

$A = -3, 3, 1$

$A^3 - A^2 - 9A + 9 = 0$

$(\log_5 x) = -3$

$(\log_5 x) = 3$

$(\log_5 x) = 1$

$A^2(A-1) - 9(A-1) = 0$

factor by grouping

$x = 1/125$

$x = 125$

$x = 5$

$(A-1)(A^2-9) = 0$

6) $\log_2(\log_3(\log_4 x)) = 0$

$(\log_3(\log_4 x)) = 2^0$

$\log_3(\log_4 x) = 1$

$x = 64$

$\log_4 x = 3^1$

$x = 4^3$

7) $x^{\sqrt{\log x}} = 10^8$

$\sqrt{\log x}(\log x) = 8 \log 10$

$\log x^{\sqrt{\log x}} = \log 10^8$

$(\log x)^{\frac{3}{2}} = 8$

$\log x = 4$

$x = 10,000$

8) $f(x) = \log_6(3x)$

$\log_6(3 \cdot 2 \cdot 6^{5x})$

$g(x) = 2 \cdot 6^{5x}$

$\log_6(6 \cdot 6^{5x})$

$5x + 1$

find $f(g(x))$

$\log_6(6^{5x+1})$

9) $\ln(x) = 1 + \ln(3x - 4)$

$\ln(x) = \ln(e) + \ln(3x - 4)$ change 1 to $\ln(e)$

$\ln(x) = \ln(3xe - 4e)$ logarithm product rule

SOLUTIONS

$x = 3xe - 4e$ drop the \ln 's

$4e = x(3e - 1)$ collect like terms and factor

$x = \frac{4e}{(3e - 1)}$

10) $\log(x+1)^4 = 20$

$10^{20} = (x+1)^4$

$(10^5)^4 = (x+1)^4$

$10^5 = (x+1)$

$x = 99,999$

11) $\log_3 27^{x-1}$

$y = \log_3 27^{x-1}$

$27^{x-1} = 3^y$

$(3^3)^{x-1} = 3^y$

$3x - 3 = y$

12) $\log_x \frac{81}{x^3} = -1$

$x^{-1} = \frac{81}{x^3}$

$\frac{1}{x} = \frac{81}{x^3}$

$x^3 = 81x$

$x^3 - 81x = 0$

$x(x^2 - 81) = 0$

since x cannot be 0 or negative, the only solution is 9

$x = 0, -9, 9$

13) $(\log_3 x)(\log_x 2x)(\log_{2x} y) = \log_x x^2$

Use Change of Base...

$\frac{\log x}{\log 3} \cdot \frac{\log 2x}{\log x} \cdot \frac{\log y}{\log 2x} = \log_x x^2$

~~$\frac{\log x}{\log 3} \cdot \frac{\log 2x}{\log x} \cdot \frac{\log y}{\log 2x} = \log_x x^2$~~

$\frac{\log y}{\log 3} = \log_x x^2$

$\frac{\log y}{\log 3} = 2 \log_x x$ (power rule)

$\frac{\log y}{\log 3} = 2(1)$

$\log_3 y = 2$

$y = 9$

14) $(\log_3 x)^3 + (\log_3 x)^2 = \log_3 x^{17} - 15$

recognize the difference between $\log_3 x^{17}$ and $(\log_3 x)^{17}$

using the power rule, rewrite...

$(\log_3 x)^3 + (\log_3 x)^2 - 17 \log_3 x + 15 = 0$

Let $A = \log_3 x$

$\log_3 x = 1$

$\log_3 x = 3$

$\log_3 x = -5$

$x = 3, 27, 1/243$

$A^3 + A^2 - 17A + 15 = 0$

Now it's a factoring polynomials question... We'll use the rational root theorem --- 'p's and 'q's....

possible rational roots: 1, -1, 3, -3, 5, -5, 15, -15...

since $f(1) = 0$, we know 1 is a root..

$$\begin{array}{r|rrrr} 1 & 1 & 1 & -17 & 15 \\ & & 1 & 2 & -15 \\ \hline & 1 & 2 & -15 & 0 \end{array}$$

$(A - 1)(A^2 + 2A - 15) = 0$

$(A + 1)(A + 5)(A - 3) = 0$

$A = 1, 3, -5$

15) Solve for t using logarithms with base a

SOLUTIONS

Logarithms Practice Test

a) $2a^{t/3} = 11$

$$\frac{11}{2} = a^{t/3}$$

$$\log_a \left(\frac{11}{2} \right)^+ = t/3$$

$$3\log_a \left(\frac{11}{2} \right) = t$$

b) $4a^{2t} = B + 5$

$$a^{2t} = \frac{B+5}{4}$$

$$\log_a \left(\frac{B+5}{4} \right) = 2t$$

$$\frac{1}{2} \log_a \left(\frac{B+5}{4} \right) = t$$

c) $M = Sa^{ct} + D$

$$\frac{M-D}{S} = a^{ct}$$

$$\log_a \left(\frac{M-D}{S} \right) = ct$$

$$\frac{1}{c} \log_a \left(\frac{M-D}{S} \right) = t$$

16) Use Natural Logarithms to solve for x in terms of y:

$$y = \frac{e^x - e^{-x}}{2}$$

$$2y = e^x - e^{-x}$$

$$e^x (e^x - e^{-x} - 2y = 0)$$

$$e^{2x} - 2ye^x - 1 = 0$$

$$A^2 - 2yA - 1 = 0$$

$$A = \frac{2y \pm \sqrt{4y^2 + 4}}{2}$$

$$A = y \pm \sqrt{y^2 + 1}$$

$$e^x = y \pm \sqrt{y^2 + 1}$$

$$\ln(y \pm \sqrt{y^2 + 1}) = x$$

$$\ln(y + \sqrt{y^2 + 1}) = x$$

since $y < \sqrt{y^2 + 1}$ and
ln cannot be negative....

17) Use Common Logarithms to solve for x in terms of y:

$$y = \frac{10^x + 10^{-x}}{2}$$

$$2y = 10^x + 10^{-x}$$

$$10^x (2y = 10^x + 10^{-x})$$

$$10^{2x} - 2y10^x + 1 = 0$$

$$A^2 - 2yA + 1 = 0$$

$$a=1 \quad b = -2y \quad c = 1$$

$$A = \frac{2y \pm \sqrt{4y^2 - 4}}{2}$$

$$A = y \pm \sqrt{y^2 - 1}$$

$$10^x = y \pm \sqrt{y^2 - 1}$$

$$\log(y \pm \sqrt{y^2 - 1}) = x$$

Since we're solving for x in terms of y, we are essentially finding the inverse.

Therefore, the domain and ranges switch...

**Since range of above is $y \geq 1$
the domain of this equation is $y \geq 1$

1) $y = \log_4 \frac{x+2}{64}$

graph the function, labeling any asymptotes and intercepts...

$y = \log_4(x+2) - \log_4(64)$

$y = \log_4(x+2) - 3$

vertical asymptote: $x = -2$

y-intercept
(occurs when $x = 0$)

x-intercept
(occurs when $y = 0$)

$y = \log_4 \frac{(0)+2}{64}$

$0 = \log_4 \frac{x+2}{64}$

$4^y = \frac{1}{32}$

$4^0 = \frac{x+2}{64}$

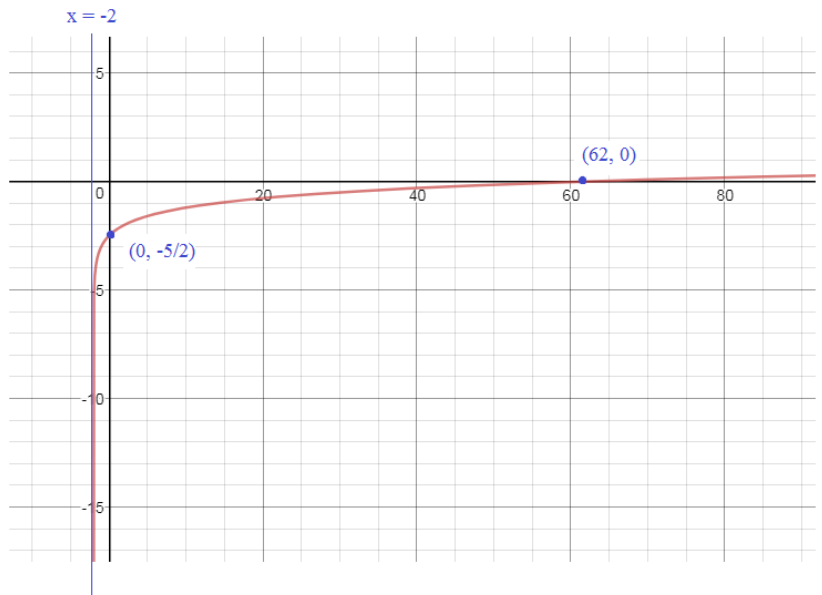
$2^{2y} = 2^{-5}$

$x = 62$

$(0, -5/2)$

$(62, 0)$

SOLUTIONS



2) $f(x) = -4^{x+2}$

find the inverse $f^{-1}(x)$, and graph both functions

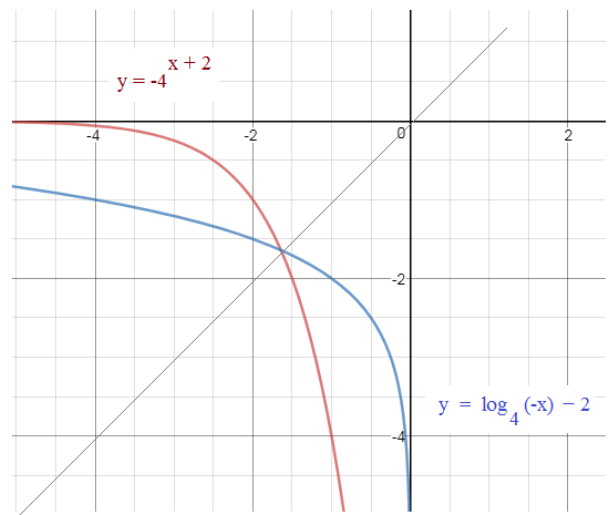
$x = -4^{y+2}$ switch x and y

$-x = 4^{y+2}$ change the negative root

$\log_4(-x) = y+2$ switch to log form

$y = \log_4(-x) - 2$

$f^{-1}(x) = \log_4(-x) - 2$



3) $y = -\log_3(81x)$

graph the function

$y = -[\log_3(81) + \log_3(x)]$

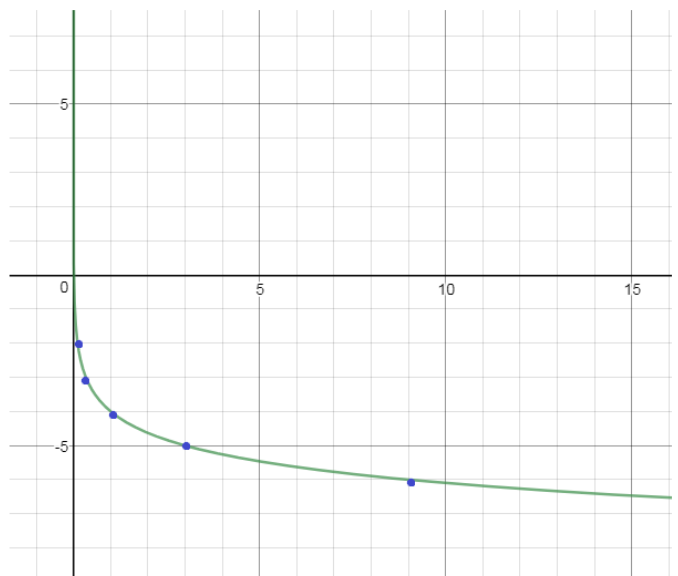
$y = -4 - \log_3(x)$

vertical asymptote: $x = 0$

x	$\log_3(x)$		x	$-\log_3(x)$		x	$-\log_3(x) - 4$
1/9	-2	reflect over x-axis	1/9	2	vertical shift down 4 units	1/9	-2
1/3	-1		1/3	1		1/3	-3
1	0		1	0		1	-4
3	1		3	-1		3	-5
9	2		9	-2		9	-6

Or, using original equation....

x	$\log_3(x)$		x	$\log_3(81x)$		x	$-\log_3(81x)$
1/9	-2	horizontal squeeze by 81	1/729	-2	reflect over x-axis	1/729	2
1/3	-1		1/243	-1		1/243	1
1	0		1/81	0		1/81	0
3	1		3/81	1		3/81	-1
9	2		9/81	2		9/81	-2
						27/81	-3
						81/81	-4



1) Write an exponential equation that goes through (0, 7) and (6, 15).

SOLUTIONS

$$y = ab^x \quad 7 = ab^0 \quad y = 7b^x$$

$$a = 7 \quad 15 = 7b^6$$

$$b^6 = \frac{15}{7}$$

$$b = \sqrt[6]{\frac{15}{7}}$$

$$y = 7 \sqrt[6]{\frac{15}{7}}^x$$

$$y = 7 \left(\frac{15}{7} \right)^{\frac{x}{6}}$$

or approx. $y = 7(1.13544)^x$

2) Write an exponential equation that goes through (3, 10) and (7, 32).

$$10 = ab^3 \quad 32 = ab^7$$

$$a = 10b^{-3} \quad a = 32b^{-7}$$

$$b^4 = \frac{32}{10} \quad b = 1.34$$

$$32 = a(1.34)^7 \quad a = 4.13$$

$$y = 4.13(1.34)^x$$

3) An exponential function of form $f(x) = ab^x + c$

has these features: y-intercept is at 5

goes through (1, 7)

horizontal asymptote at $y = 1$

Identify the function.

there is no horizontal shift...
since horizontal asymptote is $y = 1$,
there is a vertical shift of 1...

ordinarily the intercept would be at (0, 2)
(up 1 unit.)
But, instead it is at (0, 5)
(up 4 units!)

$$y = 4(b)^x + 1$$

plug in (1, 7)....

$$y = 4 \left(\frac{3}{2} \right)^x + 1$$

4) A used car is worth \$12,000 today, and \$3000 5 years from now.
What is the exponential model (of depreciation)?

If I try to sell the car 10 years from now,
what can I hope to get for it?

$$y = 12,000 \left(\frac{3000}{12000} \right)^{\frac{x}{5}} \quad y = 12,000 \left(\sqrt[5]{.25} \right)^x$$

$$y = 12,000 \left(\frac{3000}{12000} \right)^{\frac{10}{5}} = 750 \text{ dollars}$$

5) A piece of machinery cost 250,000 dollars...
After 5 years, it is worth 220,000 dollars...

What is the rate of depreciation?

$$220,000 = 250,000(1+r)^5$$

$$\sqrt[5]{\frac{22}{25}} = 1+r$$

$$.974757 = 1+r$$

$$r = -.02524 \text{ or approx. } 2.5\%$$

SOLUTIONS

1) John has \$2200 in an account that increases 7% annually...

A brand new sports car costs \$48,000, but it depreciates by 22% annually...

If John is willing to buy the sports car used, when would he be able to afford it?

Exponential Model of John's account...

7% growth

t = time in years

initial value: \$2200

$$A_J = 2200(1.07)^t$$

Exponential Model of the Sport's car's value...

22% decay

t = time in years

initial value: \$48,000

$$A_C = 48,000(.78)^t$$

When are the values equal?

$$2200(1.07)^t = 48000(.78)^t$$

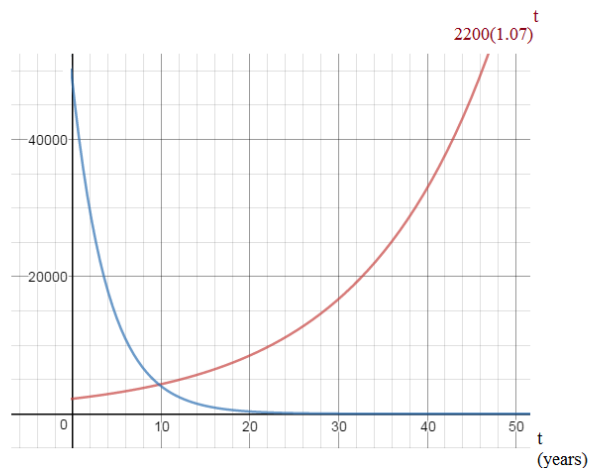
$$(1.07)^t = 21.8182(.78)^t$$

$$(1.37179)^t = 21.8182$$

$$\log_{(1.37179)} 21.8182 = t \quad \frac{\log(21.8182)}{\log(1.37179)} = t$$

$$t = 9.752 \text{ (approximately)}$$

John would have to wait almost 10 years before he could buy that car!



the intersection is when John has enough money to buy the sports car...

2) A bank offers savings accounts that pay 4% interest compounded continuously, OR accounts that pay 4.5% simple interest.

If you want to invest the amount for 5 years, which account should you use?

4% compounded continuously: $10000e^{.04(5)} = 12,214$

4.5% simple interest.... $10000(.045) = 450$

5 years of interest = $450 \times 5 = 2250$

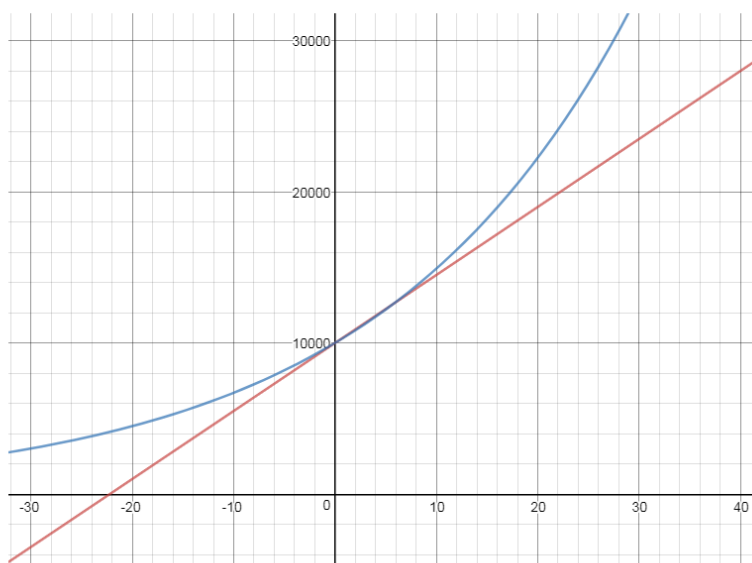
total: 12,250

The simple interest is slightly better...

What time frame are the accounts equal?

$$10000e^{.04(t)} = 10000 + t(450) \quad 5.77 \text{ years (or, 0 years!)}$$

After 5.77 years, the compounding interest path is better..



- 3) Jason opens an investment account with a 6.5% annual interest rate, compounding continuously. If he deposits \$1000, how much will he have in 9 months?

SOLUTIONS

When will the account have \$2500?

since it is "compounding continuously",

$$A = Pe^{rt}$$

$$r = .065 \text{ (NOT .65)}$$

$$t = 9/12 = .75 \text{ (NOT 9)}$$

$$P = 1000$$

$$A = 1000e^{.065(.75)}$$

$$A = \$1049.96$$

$$2500 = 1000e^{.065(t)}$$

$$2.5 = e^{.065(t)}$$

$$\ln(2.5) = .065(t)$$

$$t = 14.09 \text{ years (approximately)}$$

quick check:
using "rule of 72",
amount will double every
 $\frac{72}{6.5}$ years (approx.)

\$1000 year 0
\$2000 year 11
\$2500 year 14 ✓
\$4000 year 22

- 4) Uranium has a 1/2 life of 2.7×10^5 years...

- a) How long does it take for 10 mg of Uranium to decay to 7 mg?
b) How much remains after 1,000,000 years?

Approach 1: $A = 10\left(\frac{1}{2}\right)^{\frac{t}{270,000}}$

this is a quick method...
However, it doesn't show the rate of decay...

$$7 = 10\left(\frac{1}{2}\right)^{\frac{t}{270,000}}$$

$$.7 = \left(\frac{1}{2}\right)^{\frac{t}{270,000}}$$

$$\log_{.5}(.7) = \frac{t}{270,000}$$

$$t = 138,935 \text{ years}$$

$$A = 10\left(\frac{1}{2}\right)^{\frac{1,000,000}{270,000}}$$

$$A = .767492 \text{ (approx.)}$$

Approach 2: $A = Pe^{rt}$

this is a 2-step process: a) find the rate of decay r
b) find the time t

$$5 = 10e^{r(270,000)}$$

$$\frac{1}{2} = e^{270,000r}$$

$$\ln(1/2) = 270,000r$$

$$r = \frac{\ln(.5)}{270,000} \text{ (approx. } -.000003)$$

$$7 = 10e^{\left(\frac{\ln(.5)}{270,000}\right)t}$$

$$\ln(.7) = \frac{\ln(.5)}{270,000}t \text{ or } \ln(.7) = -.000003t$$

approx.

$$t = 138,935 \text{ years}$$

$$A = 10e^{\left(\frac{\ln(.5)}{270,000}\right)1,000,000}$$

Or, use

$$A = A_0 e^{kt}$$

(same equation with
different descriptive variables)

$$A = 10e^{\left(\frac{\ln(.5)}{270,000}\right)t}$$

$$A = .767492 \text{ (approx.)}$$

quick check:

years	amount
0	10
270,000	5
540,000	2.5
810,000	1.25
1,000,000	.767 ✓
1,080,000	.625

- 3) A population in Algebratown is modeled by $P = 344e^{kt}$

where $t = 0$ (corresponds to 1990)
and P is the population in 1,000s

In 1975, the population was 189,000...

a) Find k

b) Predict the population in 2030

To find k, we use the info that is given...

$$1975 \text{ ----> } t = -15$$

$$189,000 \text{ ----> } P = 189$$

$$189 = 344e^{k(-15)}$$

$$\ln\left(\frac{189}{344}\right) = -15k$$

$$k = .039926$$

(almost 4% growth)

$$P = 344e^{kt}$$

Population in 2030, corresponds to $t = 40$

$$P = 344e^{(.039926)(40)}$$

$$P = 1698.83 \text{ ----> } \text{approx. } 1,698,830$$

quick check:

$$\frac{72}{4} = 18 \text{ years to double}$$

1990	344,000
2008	688,000
2026	1,376,000
2030	1,698,830 ✓

SOLUTIONS

1) $5 \cdot (.5)^x - 4 = 3 \cdot 2^{-x}$

$5\left(\frac{1}{2}\right)^x - 4 = 3 \frac{1}{2^x}$

$2\left(\frac{1}{2}\right)^x = 4$

$x = -1$

$\left(\frac{1}{2}\right)^x = 2$

2) $5^x + 125(5^{-x}) = 30$

method 1: Substitute $A = 5^x$

$A + 125\left(\frac{1}{A}\right) = 30$

$A^2 + 125 = 30A$

$(A - 5)(A - 25) = 0$

$A = 5 \text{ or } 25$

Method 2: multiply by 5^x

$5^x \left(5^x + 125(5^{-x}) = 30 \right)$

$5^{2x} + 125 = 30(5^x)$

$5^{2x} - 30(5^x) + 125 = 0$

$(5^x - 5)(5^x - 25) = 0$

$x = 1, 2$

3) Using exponents and/or logarithm properties, can you evaluate 2003^{97} ?

Is there more or less than 300 digits?

What is the estimate (in scientific notation)?

$x = 10^{320} \cdot 10^{-263}$

There are more than 300 digits...

1.8323×10^{320}

$x = 2003^{97}$

$\log x = \log 2003^{97}$

$\log x = 97 \log 2003$

$\log x = 97 \cdot (3.3017)$

$\log x = 320.263$

$\log_{10} x = 320.263$

$x = 10^{320.263}$

$x = 10^{320} \cdot 10^{-263}$

4) If it takes 7 years for an investment to double, how long would it take for the investment to triple?

$A = Pe^{rt}$

First find the growth rate (r)...

$200 = 100e^{7r}$

$\ln(2) = 7r$

$r = \ln(2)/7$

$r = .099 \text{ approx.}$

Then, to find how long it triples....

$300 = 100e^{.099t}$

$3 = e^{.099t}$

$\ln(3) = .099t$

$t = 11.1 \text{ years (approximately)}$

5) A family's financial goal is to have \$20,000 in an account after 5 years....

a) If the family has \$12,000, what yield will they need to reach the goal?

b) If rates are 6%, how much must they deposit to reach their goal?

(Assume the family does not add money later...)

$20000 = 12000e^{r(5)}$

$\frac{5}{3} = e^{5r}$

$\ln(1.667) = 5r$

$r = .1022$

the yield must be at least 10.2%

$20000 = Pe^{.06(5)}$

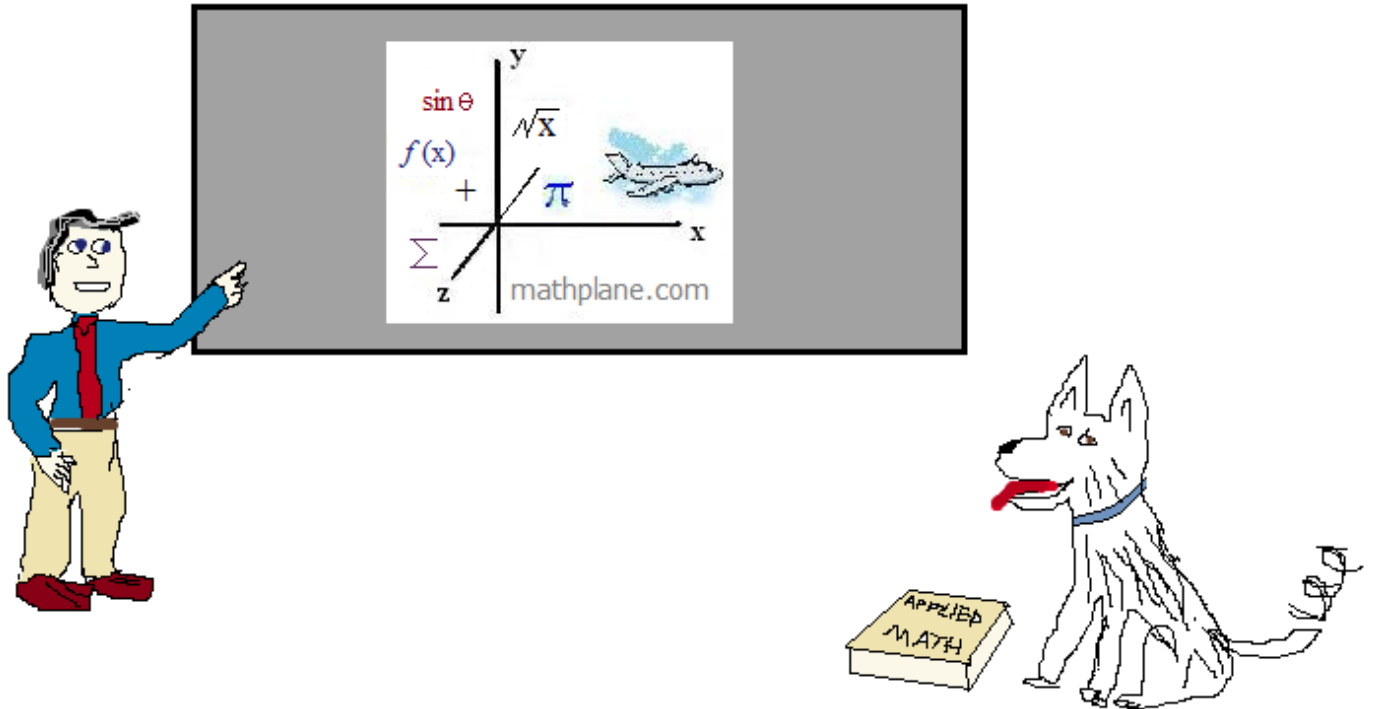
$20000 = Pe^{.3}$

$P = \$14,816.4 \text{ or more}$

Thanks for visiting. (Hope it helps!)

If you have questions, suggestions, or requests, let us know.

Cheers



Also, Mathplane *Express* for mobile and tablets at Mathplane.ORG

Or, visit the mathplane stores at TES and TeachersPayTeachers