Logarithms and Exponents Introduction

Notes, examples, puzzles, and exercises (with solutions)

$$4^{x} - 2^{x+1} = 3$$

What is x?

(Solution on the last page)

Topics include logarithm properties, exponent rules, and more.

Exponent Rules: Notes and Examples

Exponent *definition*:

$$X^A = X_1 \cdot X_2 \cdot X_3 \cdot \ \dots \ \cdot X_{A-2} \cdot X_{A-1} \cdot X_A$$

Examples: $4^5 = 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 = 1024$

$$\left(\frac{2}{3}\right)^3 = \frac{2}{3} \cdot \frac{2}{3} \cdot \frac{2}{3} = \frac{8}{27}$$

$$(-2)^7 = (-2) \cdot (-2) \cdot (-2) \cdot (-2) \cdot (-2) \cdot (-2) \cdot (-2) = -128$$

$$(-2)^6 = (-2) \cdot (-2) \cdot (-2) \cdot (-2) \cdot (-2) \cdot (-2) = 64$$

Rule #1 ('Addition Rule')

$$X^A \cdot X^B = X^{A+B}$$

Examples:

$$X^3 \cdot X^5 = X^8$$

$$5^3 \cdot 5^2 = 125 \cdot 25 = 3125 = 5^5$$

Note:

$$Y^2 \cdot Y^4 = Y^6$$

$$\underbrace{(Y \times Y)}_{2} \cdot \underbrace{(Y \times Y \times Y \times Y)}_{4} = \underbrace{Y \times Y \times Y \times Y \times Y \times Y}_{\text{6 total Y's}}$$

Rule #2: ('Multiplication Rule')

$$(X^A)^B = X^{AB}$$

Examples: $(X^4)^3 = X^{12}$

$$(4^2)^4 = 4^8 = 16^4 = 65536$$

Note:

$$(Y^5)^3 = \underbrace{(Y \times Y \times Y \times Y \times Y)}_{5} \cdot \underbrace{(Y \times Y \times Y \times Y \times Y)}_{5} \cdot \underbrace{(Y \times Y \times Y \times Y \times Y)}_{5} = Y^{15}$$

3 groups of 5 Y's
total: 3 x 5 = 15 Y's

Rule #3: ('zero exponent')

$$X^0 = 1$$

Examples: $Y^0 = 1$

$$8^0 = 1$$

$$(3cd)^0 = 1$$

What is 0^0 ? $0^A = 0$ because $0 \cdot 0 \cdot 0 \cdot 0 \dots = 0$
(if $A \neq 0$)

$X^0 = 1$ (zero exponent rule) $\boxed{0^0 = 1}$

Note: $Z^5 \cdot Z^{-5} = Z^0 = 1$

addition rule --- then, zero exponent rule

$$\frac{Z \times Z \times Z \times Z \times Z}{Z \times Z \times Z \times Z \times Z} = 1$$

Z^5

Z^{-5}

Rule #4: ('negatives' or 'reciprocal rule')

$$X^{(-A)} = \frac{1}{X^A}$$

Examples:

$$X^{-3} = \frac{1}{X^3}$$

$$5^{-2} = \frac{1}{25}$$ It is <u>not</u> equal to -25!!!

$$\left(\frac{1}{3}\right)^{-4} = 81$$

Note:

$$\boxed{Y^{(-A)} =} \quad Y^{(-A)} \cdot \frac{Y^A}{Y^A} = \frac{Y^{(-A)} \cdot Y^A}{Y^A} = \frac{Y^{(-A+A)}}{Y^A} = \frac{Y^0}{Y^A} = \boxed{\frac{1}{Y^A}}$$

multiply by one — exponent addition rule — zero exponent

Rule #5: ('base rule')

$$X^A \cdot Y^A = (XY)^A$$

Examples:

$$5^3 \cdot 7^3 = 125 \text{ x } 343 = 42875 = 35^3$$

$$= (5 \text{ x } 5 \text{ x } 5) \text{ x } (7 \text{ x } 7 \text{ x } 7) = (5 \text{ x } 7) \text{ x } (5 \text{ x } 7) \text{ x } (5 \text{ x } 7)$$

$$4^{\frac{1}{2}} \cdot 16^{\frac{1}{2}} = 64^{(1/2)} = 8$$

$$\sqrt{4} \text{ x } \sqrt{16} = \sqrt{4 \text{ x } 16} = \sqrt{64}$$

Rule #6: ('rational exponents')

$$X^{(1/2)} = \sqrt{X} \qquad X^{\left(\frac{A}{B}\right)} = \sqrt[B]{X^A}$$

Examples:

$$25^{(1/2)} = \sqrt{25} = 5$$

$$8^{(1/3)} = \sqrt[3]{8} = 2$$ ('cubed root of 8')

$$121^{(.5)} = 11$$

Note:

$$Y^{(1/2)} \cdot Y^{(1/2)} = Y^1 \qquad \sqrt{Y} \cdot \sqrt{Y} = Y$$

(addition exponent rule)

$$8^{(1/3)} \cdot 8^{(1/3)} \cdot 8^{(1/3)} = 8^1 = 8$$

Exponents/Roots/Addition Exercise

Solve the 14 problems. Then, add all the solutions.
What is the sum (rounded to 3 decimal places)?

1) $(4^2)^2$ = ______

2) $(3)^{-2}$ = ______

3) $8^3 + 8^{(1/3)}$ = ______

4) $5^2 + 5^{-2}$ = ______

5) $(.6)^4$ = ______

6) $\sqrt[3]{64} - \sqrt[3]{8}$ = ______

7) $1^0 - 2^1 + 3^2 - 4^3$ = ______

8) $(8)^{2/3}$ = ______

9) $\sqrt[3]{-343}$ = ______

10) $(1/2)^2 - (1/3)^2$ = ______

11) $122^{(1/3)} \cdot 122^{(2/3)}$ = ______

12) $231^{(3)} \times 231^{(-3)}$ = ______

13) $\sqrt{6^4}$ = ______

14) $\sqrt{8} \cdot \sqrt{2}$ = ______

Now add them up! The Total of ALL 14 solutions is ________

(rounded to 3 decimal places)

Exponents/Roots/Addition Exercise

Solve the 14 problems. Then, add all the solutions. What is the sum (rounded to 3 decimal places)?

SOLUTIONS!!

1) $(4^2)^2 = 16^2 = 256$ — 256

2) $(3)^{-2} = 1/9 = .111$ — .111

3) $8^3 + 8^{(1/3)} = 512 + 2 = 514$ — 514

4) $5^2 + 5^{-2} = 25 + 1/25 = 25 + .04 = 25.04$ — 25.04

5) $(.6)^4 = .6 \times .6 \times .6 \times .6 = .36 \times .36 = .1296$ — .130

795.281

6) $\sqrt[3]{64} - \sqrt[3]{8} = 4 - 2 = 2$ — 2

7) $1^0 - 2^1 + 3^2 - 4^3 = 1 - 2 + 9 - 64 = -56$ — -56

8) $(8)^{2/3} = (8^{1/3})^2 = 2^2 = 4$ — 4

9) $\sqrt[3]{-343} = -7$ (because $-7 \times -7 \times -7 = -343$) — -7

10) $(1/2)^2 - (1/3)^2 = 1/4 - 1/9 = .250 - .111 = .139$ — .139

— 56.861

11) $122^{(1/3)} \cdot 122^{(2/3)} = 122^1 = 122$ — 122

12) $231^{(3)} \times 231^{(-3)} = 231^0 = 1$ — 1

13) $\sqrt{6^4} = 6^{4/2} = 6^2 = 36$ — 36

14) $\sqrt{8} \cdot \sqrt{2} = 2\sqrt{2} \times \sqrt{2} = 4$ — 4

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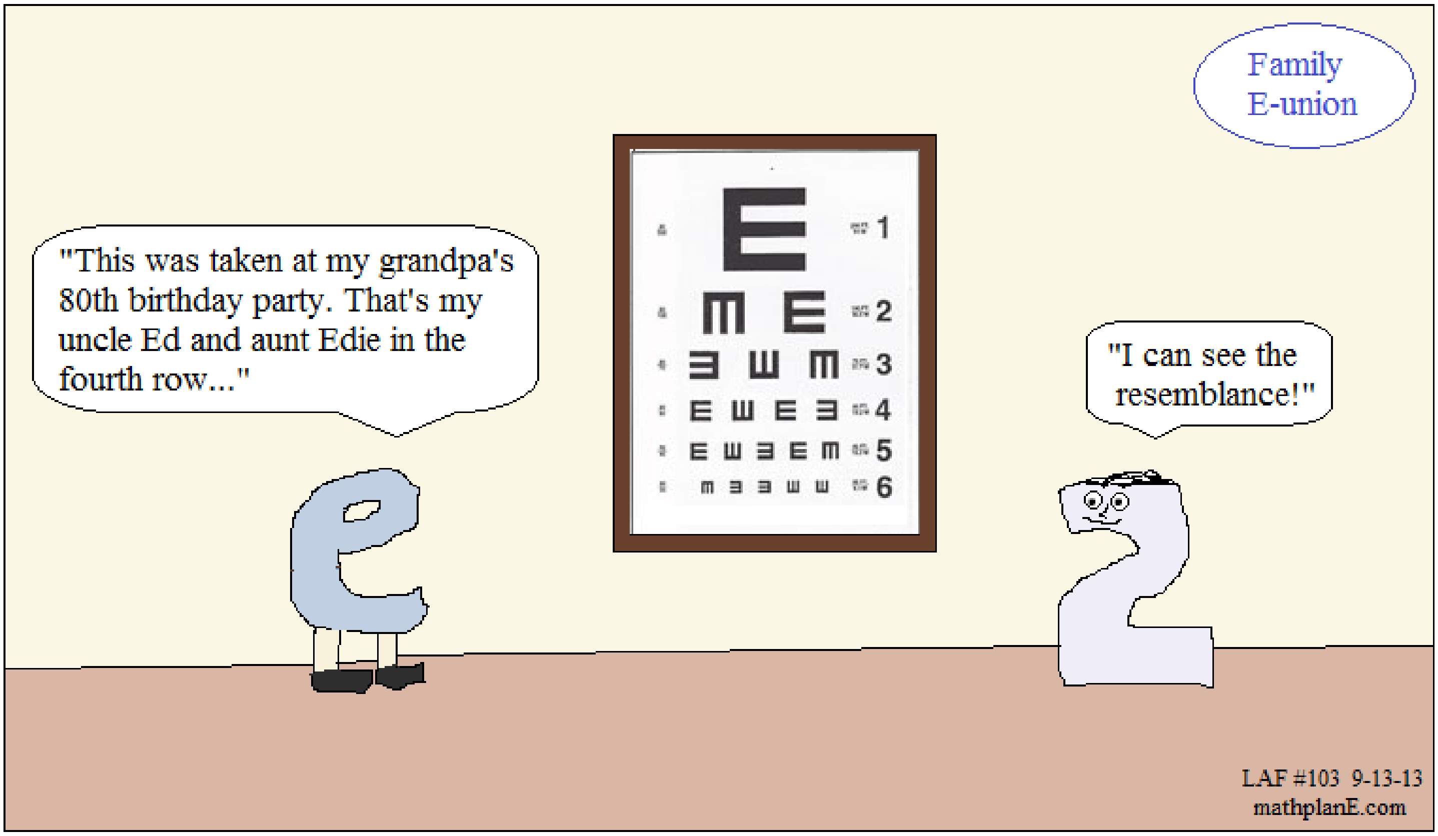
Now add them up! The Total of ALL 14 solutions is $\boxed{901.420}$

(rounded to 3 decimal places)


Family
E-union
"This was taken at my grandpa's 80th birthday party. That's my uncle Ed and aunt Edie in the fourth row..."
"I can see the resemblance!"
LAF #103 9-13-13
mathplanE.com

Logarithms -→

"Converting Logarithms to Exponents"

Notes to remember:

- $\log_b a = x$ if $b^x = a$
- $\log a = \log_{10} a$
- $\ln a = \log_e a$ "natural log of a"

Helpful memorizing trick for converting logarithms to exponents:

$\log 1000 = 3$

In this equation, you have 10, 1000, and 3... The only arrangement could be

$10^3 = 1000$

Basic Examples:

Logarithm	Exponent
1) $\log_b b = 1$	$b^1 = b$
$\log_6 6 = 1$	$6^1 = 6$
$\log 10 = 1$	$10^1 = 10$
$\ln e = 1$	$e^1 = e$
2) $\log_b 1 = 0$	$b^0 = 1$
$\log_5 1 = 0$	$5^0 = 1$
$\log 1 = \log_{10} 1 = 0$	$10^0 = 1$
$\ln 1 = \log_e 1 = 0$	$e^0 = 1$
3) $\log_b b^x = x$	$b^x = b^x$
$\log_6 36 = \log_6 (6)^2 = 2$	$6^2 = 36$

More examples:

Logarithm	Exponent
$\log_3 27 = 3$	$3^3 = 27$
$\log_3 9 = 2$	$3^2 = 9$
$\log_3 x = 2.5$ where $x \cong 15.58$	$3^{\frac{5}{2}} = x = \sqrt{(3)^5} = \sqrt{243}$ $\cong 15.58$
$\log_3 (1/81) = -4$	$3^{(-4)} = \frac{1}{81}$

Common Logarithm Rules and Examples

Product Rule: $\log_b(MN) = \log_b(M) + \log_b(N)$

Example: $\log 3 = .477$ Find $\log 12$
$\log 4 = .602$

Solution:

Logarithm	Comparable Exponent Expression
$\log_{10}(12) = X$	$10^X = 12$
$\log_{10}(3 \cdot 4) = \log_{10}(3) + \log_{10}(4)$	$10^{.477}$ and $10^{.602}$
$= .477 + .602$	$10^{.477} \cdot 10^{.602} = 10^{1.079}$
$\log_{10}(12) = 1.079$	$10^{1.079} = 12$

Quotient Rule: $\log_b\left(\frac{M}{N}\right) = \log_b(M) - \log_b(N)$

Example: $\log 3 = .477$ Find a) $\log(.75)$
$\log 4 = .602$ b) $\log(4/3)$

Solutions:

	Logarithm	Comparable Exponent Expression
a)	$\log_{10}(.75) = X$	$10^X = .75 = \left(\frac{3}{4}\right)$
	$\log_{10}\left(\frac{3}{4}\right) = X$	$10^{.477}$ and $10^{.602}$
	$\log_{10}\left(\frac{3}{4}\right) = \log_{10}(3) - \log_{10}(4)$	$\frac{10^{.477}}{10^{.602}} = 10^{-.125}$
	$= .477 - .602 = -.125$	$10^{-.125} = .75$
	$\log_{10}(.75) = -.125$	
b)	$\log_{10}(4/3) = X$	$10^X = (4/3)$
	$= \log_{10}(4) - \log_{10}(3)$	$\frac{10^{.602}}{10^{.477}} = 10^{.125}$
	$= .602 - .477 = .125$	
	$\log_{10}(4/3) = .125$	$10^{.125} = 1.33$

$10^{-1} = 1/10 = .10$

$10^{-.125} = .75$

$10^0 = 1$

$10^{.125} = 1.33$

$10^1 = 10$

Common Logarithm Rules and Examples (continued)

Power Rule:

$$\log_b M^X = x \log_b M$$

Example: $\log 3 = .477$ Find $\log 27$

Solution:

Logarithm	Comparable Exponent Expression
$Y = \log_{10} 27 = \log_{10}(3)^3$	$10^Y = 27$
$= 3 \log_{10}(3)$	$= 3 \times 3 \times 3$
$= 3\,(.477)$	$= 10^{.477} \cdot 10^{.477} \cdot 10^{.477}$
$Y = \log_{10} 27 = 1.431$	$= 10^{3 \times .477} = 10^{1.431}$

Note: The Power Rule is an extension of the Product Rule..
Example:

$\log 16 = \log(2 \times 2 \times 2 \times 2) = \log 2 + \log 2 + \log 2 + \log 2$ (product rule)

or

$\log 16 = \log 2^4 = 4 \log 2$ (power rule)

Change of Base Formula

$$\log_a X = \frac{\log_b X}{\log_b a}$$

Assume a, b, and X are positive and $a \neq 1$ $b \neq 1$

Example:

$\log 3 = .477$ Find $\log_2 3$

$\log 2 = .301$

Solutions:

Logarithm	Comparable Exponent Expression
$Y = \log_2 3$ $a = 2$, let $b =$ (base) 10, $X = 3$	$2^Y = 3$
$Y = \dfrac{\log_{10} 3}{\log_{10} 2} = \dfrac{.477}{.301} = 1.585$	$2^{1.585} \cong 3$

Evaluate $(\log_4 125)(\log_5 16)$ (without a calculator)

Solution

Change of Base $\left(\frac{\log 125}{\log 4}\right)\left(\frac{\log 16}{\log 5}\right)$

Algebra $\left(\frac{\log 125}{\log 5}\right)\left(\frac{\log 16}{\log 4}\right)$

(Change of Base) $(\log_5 125)(\log_4 16)$

$\log_5 125 = 3$ 5^3

3×2

$\log_4 16 = 2$ 4^2

6

Check (with a calculator)

$(\log_4 125)$ 3.483

$3.483 \times 1.723 = 6.001$ ✓

(approximately)

$(\log_5 16)$ 1.723

Example: Graph the function $y = 3\log_2(x-1) + 6$

$y = 2^x \Longrightarrow y = \log_2(x)$

inverse

Step 1: recognize the parent function $\log_2(x)$

x	2^x
-2	1/4
-1	1/2
0	1
1	2
2	4

domain
and
range
are flipped

x	$\log_2(x)$
1/4	-2
1/2	-1
1	0
2	1
4	2

Step 2: transform the parent function

$\Longrightarrow y = \log_2(x-1)$

x's are shifted
1 unit to the right

x	$\log_2(x-1)$
1 1/4	-2
1 1/2	-1
2	0
3	1
5	2

$\Longrightarrow 3\log_2(x-1)$

y's are stretched
by a factor of 3

x	$3\log_2(x-1)$
1 1/4	-6
1 1/2	-3
2	0
3	3
5	6

$\Longrightarrow 3\log_2(x-1) + 6$

y's are shifted
up 6 units

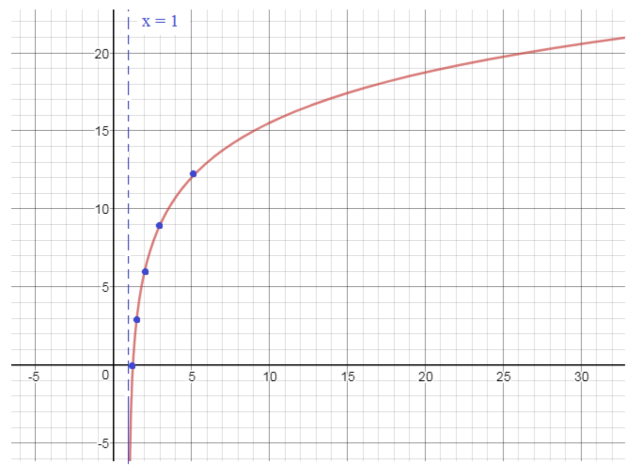
x	$3\log_2(x-1) + 6$
1 1/4	0
1 1/2	3
2	6
3	9
5	12

Step 3: Recognize vertical asymptote and key points...

vertical asymptote: $x = 1$

domain: $(1, \infty)$

range: all real numbers



Example: Graph $y = \log_5(2x+6) + 7$ and show the transformations...

change to $y = \log_5 2(x+3) + 7$

Parent Function

x	$\log_5(x)$
1/25	-2
1/5	-1
1	0
5	1
25	2

Horizontal Compression (by a factor of 1/2)

x		$\log_5 2(x)$
1/50	1/25	-2
1/10	1/5	-1
1/2	1	0
5/2	5	1
25/2	25	2

Horizontal Shift (3 units to the left)

x		$\log_5 2(x+3)$
-2 49/50	1/50	-2
-2 9/10	1/10	-1
-2 1/2	1/2	0
-1/2	5/2	1
19/2	25/2	2

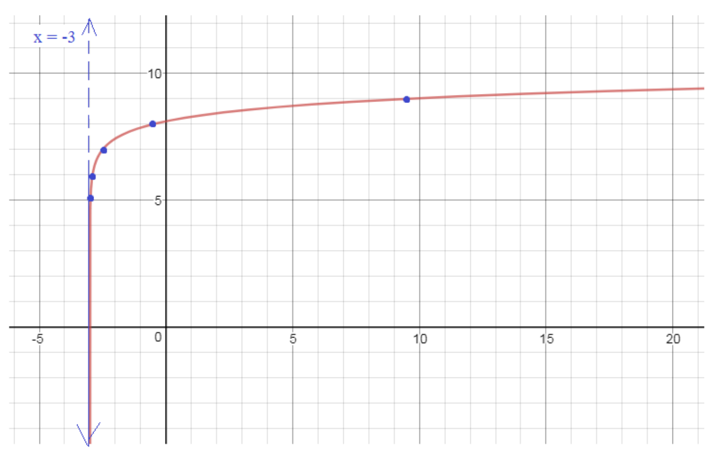
Vertical Shift (7 units up)

x		$\log_5 2(x+3) + 7$
-2 49/50	-2	5
-2 9/10	-1	6
-2 1/2	0	7
-1/2	1	8
19/2	2	9

vertical asymptote: $x = -3$

domain: $(-3, \infty)$

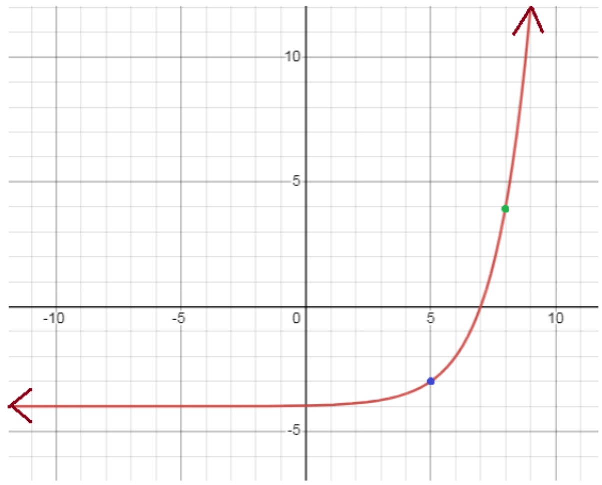
range: all real numbers



Finding Exponential and Logarithmic Equations

Example: Find the following exponential equation

$$y = \pm (r)^{x - A} + B$$



Step 1: Find B

This is the horizontal asymptote $y = -4$

$$y = (r)^{x - A} - 4$$

Step 2: Find A

This is the horizontal shift....

Ordinarily, for a parent function with horizontal asymptote at $y = 0$, there is a point (0, 1) ---> 1 unit above the asymptote...

In this graph, the point (5, -3) is 1 unit above the asymptote... therefore, the horizontal shift is 5 to the right....

$$y = (r)^{x - 5} - 4$$

Step 3: Find the common ratio (r)

Plug in one of the points.... We'll choose (8, 4)...

$$4 = (r)^{8 - 5} - 4$$

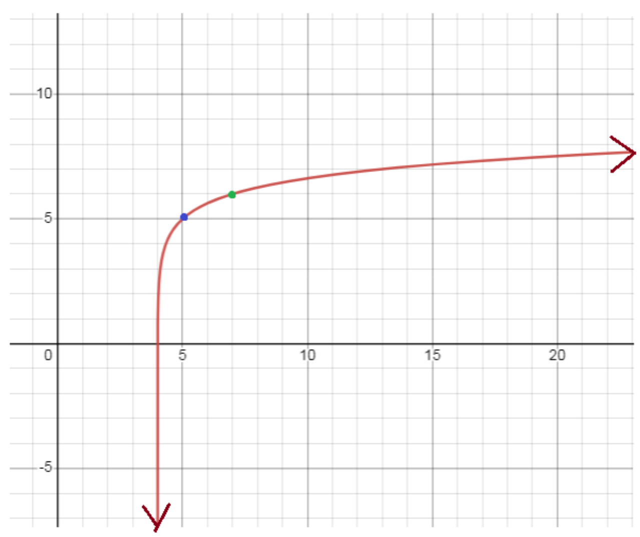
$$8 = r^3$$

$$r = 2$$

$$\boxed{y = 2^{x - 5} - 4}$$

Example: Find the following logarithmic equation

$$y = \pm \log_a (x - A) + B$$



Step 1: Find A

This is the vertical asymptote $x = 4$

$$y = \log_a (x - 4) + B$$

Step 2: Find B

This is the vertical shift.

Ordinarily, for a log parent function with vertical asymptote at $x = 0$, there is a point (1, 0) ---> 1 unit away from the asymptote.

In this graph, the point (5, 5) is 1 unit from the asymptote...

$$y = \log_a (x - 4) + 5$$

Step 3: find the base a

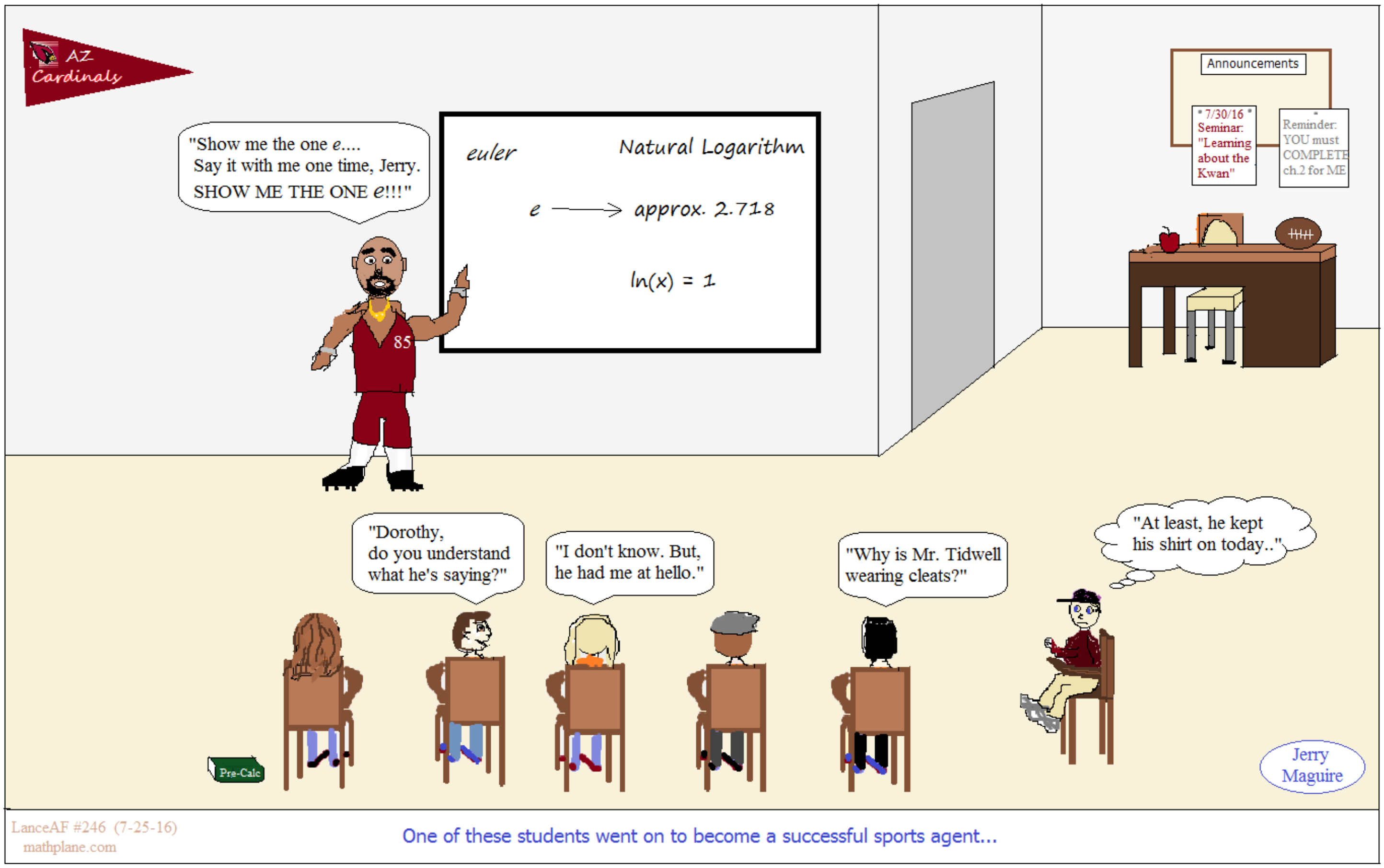
Plug in a point from the curve.... We'll use (7, 6)...

$$6 = \log_a (7 - 4) + 5$$

$$1 = \log_a (3)$$

$$a = 3$$

$$\boxed{y = \log_3 (x - 4) + 5}$$



Practice Exercises-→

Using Logarithms and Exponents: Quick Quiz

Part I Find x:

1) $\log_2 8 = x$ 2) $\log_x(1/9) = -2$ 3) $\log_5(x) = 3$ 4) $\log_8(-2) = x$ 5) $\ln 1 = x$

Part II $\boxed{\log 4 = .602}$ Find the following:
(w/o calculator)

1) log 400 2) log .004 3) log 16 4) log 2.5

Part III

Find x:

1) $\log_5 x + \log_5 (x-4) = 1$

2) $\log_2 (x+3) - 2\log_2 x = \log_2 4$

3) $2^{6-x} = 4^{2+x}$

4) $3^x \sqrt{27} = 9^{x-1}$

5) $3^x = 8$

6) $e^{3x} = 12$

Part IV (Miscellaneous)

1) A local bank savings account compounds interest annually.
   If $1500 would increase to $2000 in 7 years, what is the annual rate of interest the bank offers?

2) Find x:

a) $1 + 7\log_2 x = 8$

b) $5(2)^{x+5} - 7 = 13$

c) $\ln e^{4X+1} = \ln(9)$

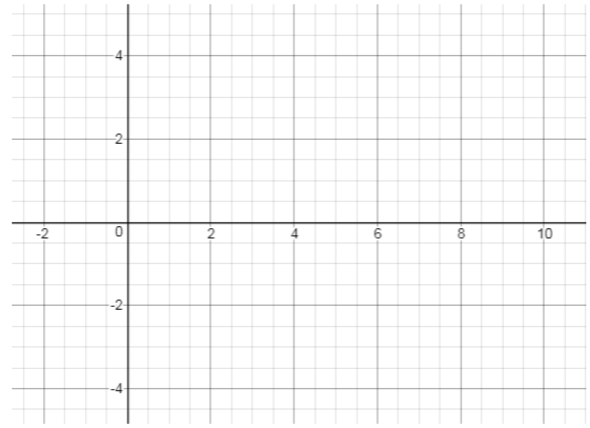
3) Find the inverses:

a) $y = 3^{X+2} + 4$

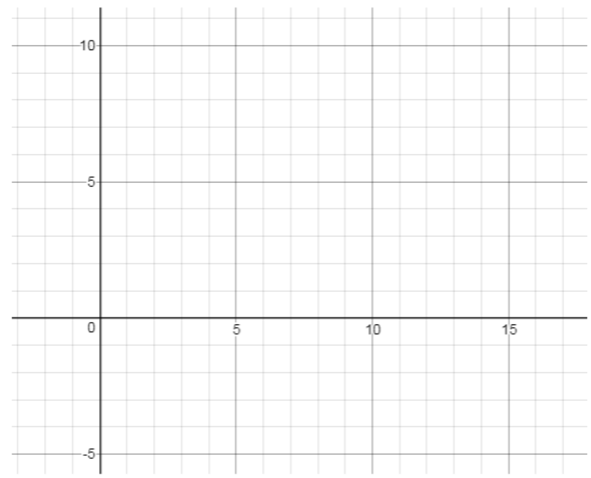
b) $y = \log_6 x + 2$

Graphing Logarithm Functions   Sketch the following functions.   Identify the intercepts and asymptotes. Determine the domain and range.

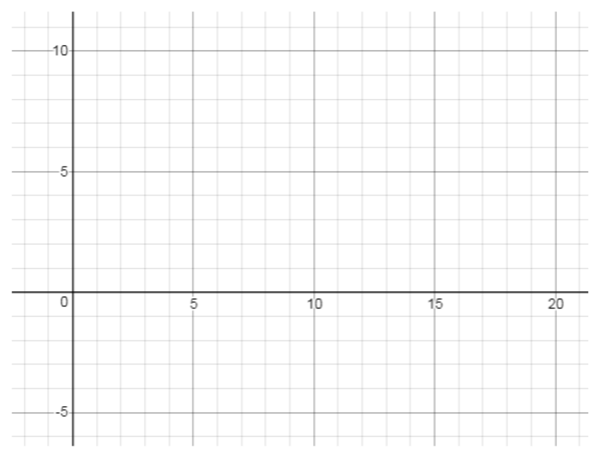
1) $y = \log_3 x$



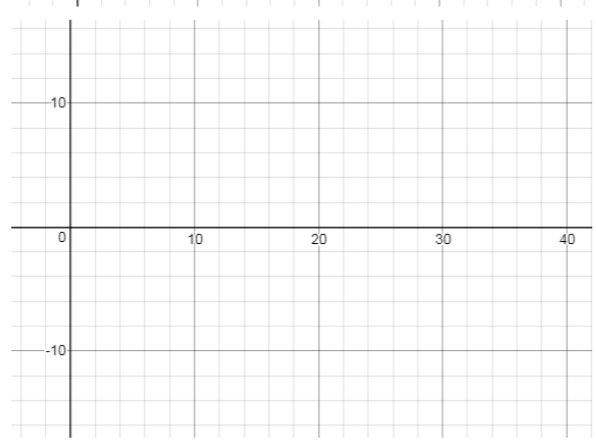
2) $y = \log_2 x + 5$



3) $g(x) = 2\log_3(x - 1) + 4$



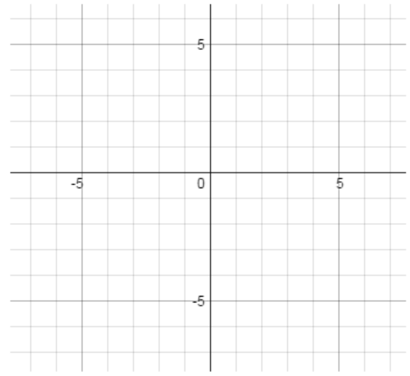
4) $y = -4\log_2(\frac{1}{3}x)$



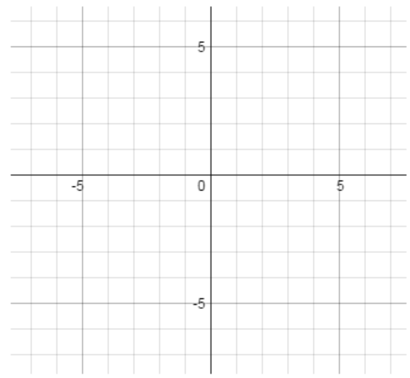
Graphing: *e* and the natural log (ln)

Sketch the following functions. Identify the intercepts and asymptotes. Determine the domain and range.

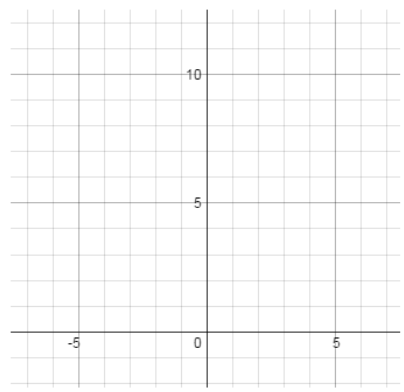
1) $y = e^{x} - 4$



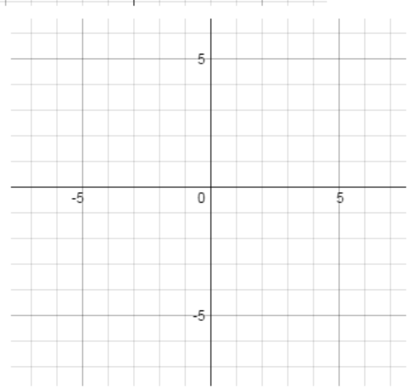
2) $y = \ln(x) + 2$



3) $f(x) = e^{x-2} + 5$



4) $g(x) = \ln(x + 1)$



Using Logarithms and Exponents: Quick Quiz

SOLUTIONS

Part I Find x:

1) $\log_2 8 = x$

$2^x = 8$

$\boxed{x = 3}$

2) $\log_x (1/9) = -2$

$x^{-2} = 1/9$

$\boxed{x = 3}$

3) $\log_5 (x) = 3$

$5^3 = x$

$\boxed{x = 125}$

4) $\log_8 (-2) = x$

$8^x = -2$

$\boxed{\text{No solution!!}}$

5) $\ln 1 = x$

$\log_e 1 = x$

$e^x = 1$

$\boxed{x = 0}$

Part II $\boxed{\log 4 = .602}$ Find the following:
(w/o calculator)

1) $\log 400$

$\log (4 \cdot 100)$

$\log 4 + \log 100 =$

$.602 + 2 = \boxed{2.602}$

2) $\log .004$

$\log \frac{4}{1000}$

$\log 4 - \log_{10} 1000$

$.602 - 3 = \boxed{-2.398}$

3) $\log 16$

$\log (4)^2$

$2 \log (4)$

$2 \cdot .602 = \boxed{1.204}$

4) $\log 2.5$

$\log \frac{10}{4}$

$\log_{10} 10 - \log 4$

$1 - .602 = \boxed{.398}$

(note: $10^{2.6} \cong 400$)

Part III

Find x:

1) $\log_5 x + \log_5 (x-4) = 1$

$\log_5 x(x-4) = 1$

$5^1 = x(x-4)$

$5 = x^2 - 4x$

$x^2 - 4x - 5 = 0$

$(x - 5)(x + 1) = 0$

$\boxed{x = 5}, \cancel{-1}$ log (-1) does not exist

2) $\log_2 (x+3) - 2\log_2 x = \log_2 4$

$\log_2 (x+3) - \log_2 x^2 = \log_2 4$

$\log_2 \frac{(x+3)}{x^2} = \log_2 4$

$\frac{(x+3)}{x^2} = 4$

$4x^2 = x + 3$

$4x^2 - x - 3 = 0$

$(4x + 3)(x - 1) = 0$

$x = \cancel{-3/4}$ or $\boxed{1}$

quick check: (plug 1 into the original equation)

$\log_2 (1+3) - 2\log_2 (1) = \log_2 4$

$2 - 2(0) = 2$ ✓

3) $2^{6-x} = 4^{2+x}$

$2^{6-x} = (2^2)^{2+x}$

$2^{6-x} = 2^{4+2x}$

$6 - x = 4 + 2x$

$\boxed{x = \frac{2}{3}}$

quick check:

$2^{5.33} = 4^{2.67}$

$40.3 = 40.3$ ✓

4) $3^x \sqrt{27} = 9^{x-1}$

$3^x \cdot 3\sqrt{3} = (3^2)^{x-1}$

$3^x \cdot 3^1 \cdot 3^{1/2} = 3^{2x-2}$

$3^{(x+1+1/2)} = 3^{2x-2}$

$x + 3/2 = 2x - 2$

$\boxed{x = \frac{7}{2}}$

check: $3^{3.5}\sqrt{27} = 9^{2.5}$

$46.76 \cdot 5.19 = 243$

$242.7 \cong 243$ ✓

5) $3^x = 8$

$\log 3^x = \log 8$

$x \log 3 = \log 8$

$x = \frac{\log 8}{\log 3}$

$= \frac{.903}{.477} = \boxed{1.89}$

quick check:

$3^{1.89} = 8$ ✓

6) $e^{3x} = 12$

$\ln e^{3x} = \ln 12$

$3x \ln e = \ln 12$

$3x(1) = \ln 12$

$3x = 2.48$

$\boxed{x = .83}$

quick check:

$e^{3(.83)} = 2.71^{2.49} = 11.97$ ✓

Part IV (Miscellaneous)

SOLUTIONS

1) A local bank savings account compounds interest annually.
If $1500 would increase to $2000 in 7 years, what is the annual rate of interest the bank offers?

$A = P(1 + r)^t$

substitute $2000 = 1500(1 + r)^7$

solve $\frac{4}{3} = (1 + r)^7$

$\left(\frac{4}{3}\right)^{1/7} = (1 + r)$

(approximately) $1.042 = 1 + r$

$\boxed{r = .042 \text{ or } 4.2\%}$

Check solution:

$1500(1 + .042)^7 = 1500 \text{ x } 1.33$

$= 2000.6$ ✓

(approximately)

2) Find x:

a) $1 + 7\log_2 x = 8$

Isolate the log term (variable)

$7\log_2 x = 7$

$\log_2 x = 1$

Change to exponent form and solve

$2^1 = x$

$\boxed{x = 2}$

b) $5(2)^{x+5} - 7 = 13$

Isolate exponent term (variable)

$5(2)^{x+5} = 20$

$(2)^{x+5} = 4$

"Change to common base" to solve

$(2)^{x+5} = 2^2$

$\boxed{x = -3}$

c) $\ln e^{4X+1} = \ln(9)$

use 'power rule' of logs

$(4X + 1)\ln e = \ln(9)$

solve and simplify

$(4X + 1)(1) = 2.197$

$4X = 1.197$

$\boxed{X = .299}$

3) Find the inverses:

a) $y = 3^{X+2} + 4$ "flip the variables"

$x = 3^{y+2} + 4$ solve for y

$x - 4 = 3^{y+2}$

$\log(x - 4) = \log 3^{y+2}$

$\log(x - 4) = (y + 2)\log 3$

$y + 2 = \frac{\log(x - 4)}{\log(3)}$ "Change of base"

$\boxed{y = \log_3 (x - 4) - 2}$

b) $y = \log_6 x + 2$ "switch the variables"

$x = \log_6 y + 2$ solve for y

$x - 2 = \log_6 y$

isolate log term w/ y variable. then, change to exponent form

$\boxed{y = 6^{x-2}}$

Quick Check:
If $x = 2$, then $y = 6^{(2)-2} = 1$

(2, 1)

Then, the inverse would be (1, 2)

$(2) = \log_6(1) + 2$

$2 = 0 + 2$ ✓

Graphing Logarithm Functions

Sketch the following functions. Identify the intercepts and asymptotes. Determine the domain and range.

SOLUTIONS

1) $y = \log_3 x$

Make a table of values, utilizing the exponential form... $x = 3^y$

y	x
-2	1/9
-1	1/3
0	1
1	3
2	9

so, the coordinates are

(1/9, -2)

(1/3, -1)

(1, 0)

(3, 1)

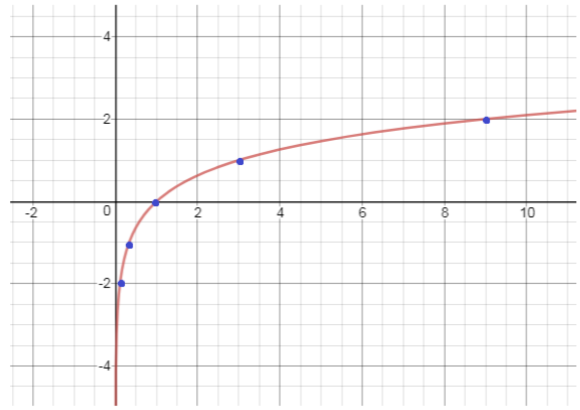
(9, 2)

domain is $x > 0$

range is all real numbers

y-intercept: none

x-intercept: (1, 0)



2) $y = \log_2 x + 5$

First, focus on the parent function $\log_2 x$

In exponential form: $x = 2^y$

y	x
-2	1/4
-1	1/2
0	1
1	2
2	4

$\Rightarrow$

x	y
1/4	-2
1/2	-1
1	0
2	1
4	2

Then, recognize the vertical shift up 5 units...

$\Rightarrow$

x	y
1/4	3
1/2	4
1	5
2	6
4	7

$0 = \log_2 x + 5$

$-5 = \log_2 x$

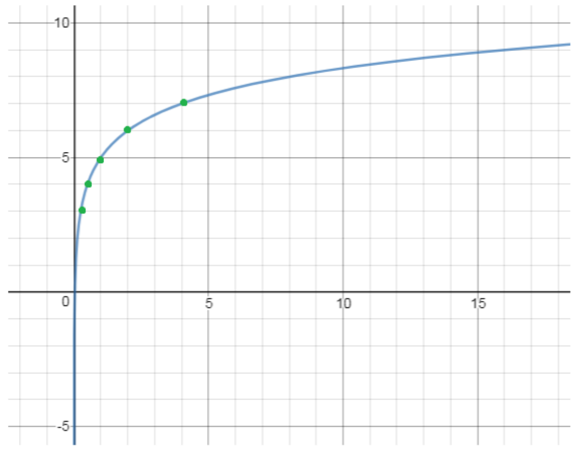
$x = 2^{-5}$

x-intercept: (1/32, 0)

y-intercept: none

domain is $x > 0$

range is all real numbers



3) $g(x) = 2\log_3(x - 1) + 4$

Since logs cannot be zero or negative,

vertical asymptote at $x = 1$

domain: $x > 1$

range: all real numbers

y-intercept occurs when $x = 0$
so, does not exist...

x-intercept occurs when $y = 0$
so,

$0 = 2\log_3(x - 1) + 4$

$-2 = \log_3(x - 1)$

$(x - 1) = 3^{-2}$ (10/9, 0)

$x = 10/9$

This time, we'll graph using inverse function

first, find the inverse:

$x = 2\log_3(y - 1) + 4$

$\frac{(x - 4)}{2} = \log_3(y - 1)$

$y = 3^{\frac{(x-4)}{2}} + 1$

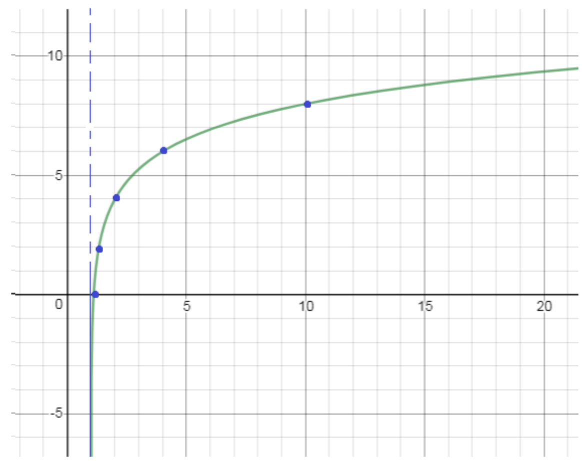
second, set up a table

x	y
0	1 1/9
2	1 1/3
4	2
6	4
8	10

(inverse table of values)

finally, flip the x and y coordinates...

x	g(x)
1 1/9	0
1 1/3	2
2	4
4	6
10	8



4) $y = -4\log_2(\frac{1}{3}x)$

domain: $x > 0$
range: all real numbers
x-intercept: (3, 0)
y-intercept: none

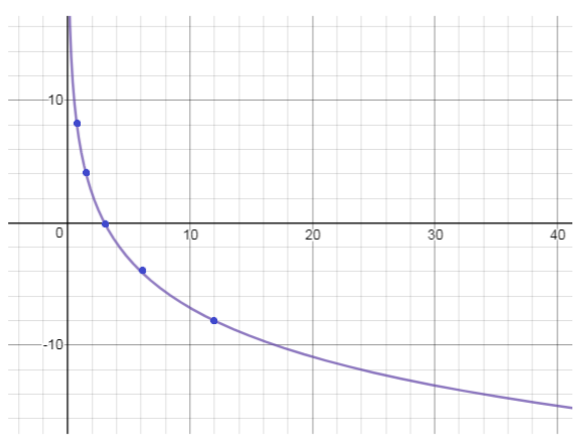
inverse table

x	2^x	
-2	1/4	2^{-2}
-1	1/2	2^{-1}
0	1	2^0
1	2	2^1
2	4	2^2

x	$\log_2 x$
1/4	-2
1/2	-1
1	0
2	1
4	2

x	$\log_2(1/3x)$
3/4	-2
3/2	-1
3	0
6	1
12	2

x	$-4\log_2(1/3x)$
3/4	8
3/2	4
3	0
6	-4
12	-8



Graphing: e and the natural log (ln)

Sketch the following functions. Identify the intercepts and asymptotes. Determine the domain and range.

1) $y = e^x - 4$

x	e^x	$e^x - 4$
0	1	-3
1	e	e - 4 (-1.29)
2	e^2	$e^2 - 4$ (3.34)
-1	1/e	1/e - 4 (-3.6)
-2	$1/e^2$	$1/e^2$ - 4 (-3.86)

The domain is all real numbers.
So, there is no vertical asymptote

Since e^x will never equal zero or be negative, the horizontal asymptote is $y = -4$

the range is $(-4, \infty)$

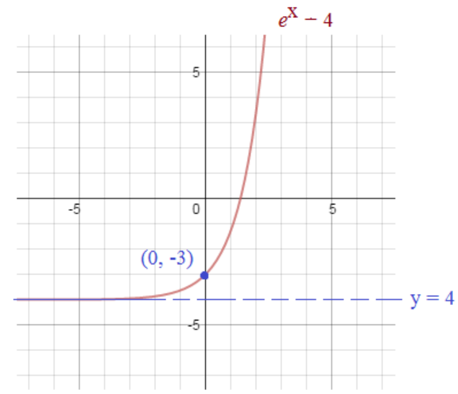
y-intercept: (0, -3)

$0 = e^x - 4$

$4 = e^x$

$x = \ln(4) = 1.39$ (approx.)

x-intercept: (1.39, 0)



2) $y = \ln(x) + 2$

(start with 'easy' points)

x	ln(x)	ln(x) + 2
1	0	2
e	1	3
e^2	2	4
1/e	-1	1
$1/e^2$	-2	0

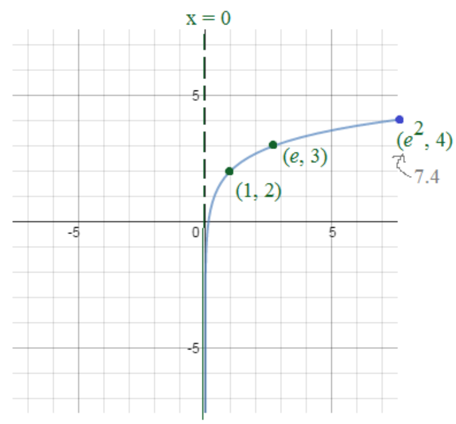
Since natural log cannot be 0 or negative, the domain is $(0, \infty)$

the range is all real numbers..

(Note: ln(x) and e^x are inverses... so, the domain/range of one is range/domain of the other!)

vertical asymptote: x = 0
horizontal asymptote: none

y-intercept: none
x-intercept: (.135, 0) or $(1/e^2, 0)$



3) $f(x) = e^{x-2} + 5$

x	x - 2	e^{x-2}	$f(x)$
2	0	1	6
3	1	e	e + 5
4	2	e^2	12.39
1	-1	1/e	5.37
0	-2	$1/e^2$	5.135

exponential function,
no vertical asymptote
domain: all real numbers

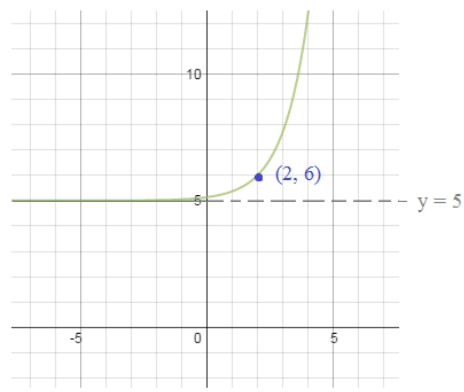
horizontal asymptote: y = 5 (the vertical shift)
range: $(5, \infty)$

x-intercept: none (the function never crosses the x-axis)

$e^{x-2} + 5 = 0$ $\quad e^{x-2} = -5$ No real solution

y-intercept: $(0, 5 + 1/e^2)$

i.e. (0, 5.135)



4) $g(x) = \ln(x + 1)$

	x	x + 1	ln(x + 1)
	0	1	0
1.72	e - 1	e	1
6.39	e^2 - 1	e^2	2
	1/e - 1	1/e	-1
	$1/e^2$ - 1	$1/e^2$	-2

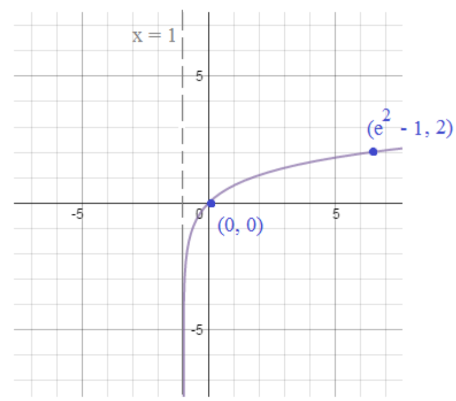
horizontal shift is 1 to the left...
vertical asymptote: x = -1

domain: $(-1, \infty)$

horizontal asymptote: none

range: all real numbers

x-intercept and y-intercept: (0, 0)



Logarithm Puzzle →

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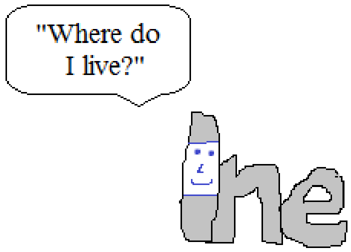

Letter/Number Key

1	2	3	4	5	6	7	8	9	0
A	B	C	G	I	L	M	N	O	Y

Solve the 12 problems below....
Then, convert the numbers into letters to reveal the answer!

---

1) $\log_2 32 =$

2) If $\log_X(1/64) = -2$, then $X = ?$

3) $f(x) = \log_5(2x - 14) + 6$ The domain is (z, ∞)
What is z?

4) $\log(1) =$

5) $216^{\frac{1}{3}} =$

6) $X^{.25} = 1.73$

7) $2^{(x+5)} = 8^{(7-x)}$ What is x?

8) $\log_3 x + \log_3(x - 2) = 1$ Find x.

9) $\ln e =$

10) $\log(400) - \log(4) =$

11) $2 + 9\log_3(8 - X) = 11$

12) $3^x = 8$ Find x (to the nearest hundredth)

1. ☐ 9 →

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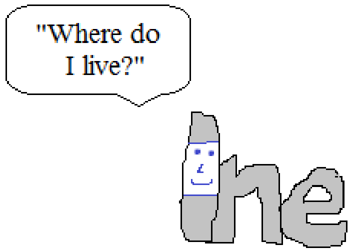


Letter/Number Key									
1	2	3	4	5	6	7	8	9	0
A	B	C	G	I	L	M	N	O	Y

Solve the 12 problems below....
Then, convert the numbers into letters to reveal the answer!

SOLUTIONS

---

1) $\log_2 32 =$ change to exponent form: $2^x = 32$ $\quad x = 5$ $\quad$ 5 → I

2) If $\log_X(1/64) = -2$, then $X = ?$ $\quad X^{-2} = \frac{1}{64}$ $\quad X^2 = 64$ $\quad X = 8$ $\quad$ 8 → N

3) $f(x) = \log_5(2x - 14) + 6$ $\quad$ The domain is (z, ∞). What is z? $\quad$ all log values must be greater than 0... therefore, the domain is $x > 7$ $\quad$ 7 → M

4) $\log(1) =$ $\quad \log_{10} 1 = X$ $\quad 10^X = 1$ $\quad X = 0$ $\quad$ 0 → Y

5) $216^{\frac{1}{3}} =$ $\quad \sqrt[3]{216} = 6$ $\quad 6^3 = 216$ $\quad$ 6 → L

6) $X^{.25} = 1.73$ $\quad (X^{1/4})^4 = (1.73)^4$ $\quad X \cong 9$ $\quad$ 9 → O

In My "Log" Cabin

7) $2^{(x+5)} = 8^{(7-x)}$ What is x? $\quad 2^{(x+5)} = 2^{3(7-x)}$ $\quad x + 5 = 21 - 3x$, $4x = 16$, $x = 4$ $\quad$ 4 → G

8) $\log_3 x + \log_3(x - 2) = 1$ Find x. $\quad \log_3[(x)(x-2)] = 1$ (log product rule) $\quad x^2 - 2x = 3$, $(x-3)(x+1) = 0$, $x = \cancel{-1}$ or 3 (-1 is extraneous!) $\quad$ 3 → C

9) $\ln e =$ "ln" is "log base e" $\quad \log_e(e) = x$ $\quad e^x = e$ $\quad X = 1$ $\quad$ 1 → A

10) $\log(400) - \log(4) =$ (use log property of division) $\quad \log_{10}\frac{400}{4}$ $\quad \log_{10}(100) = 2$ $\quad$ 2 → B

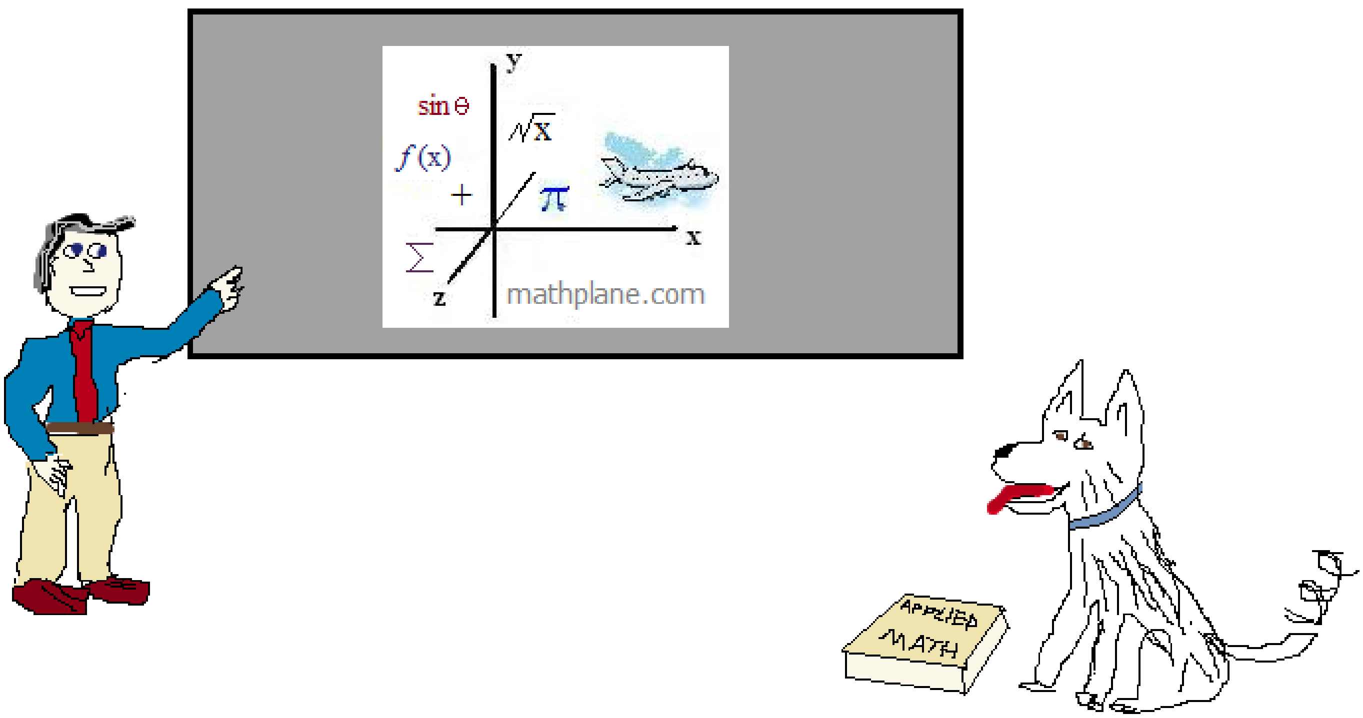
11) $2 + 9\log_3(8 - X) = 11$ $\quad 9\log_3(8 - X) = 9$ $\quad \log_3(8 - X) = 1$, $8 - X = 3$, $X = 5$ (isolate the log; then, solve) $\quad$ 5 → I

12) $3^x = 8$ Find x (to the nearest hundredth) $\quad \log 3^x = \log 8$ $\quad x\log 3 = \log 8$ $\quad x = \frac{\log 8}{\log 3} \cong 1.8928$ $\quad$ 1.89 → N

Thanks for visiting. (Hope it helped!)

If you have questions, suggestions, or requests, let us know.

Enjoy



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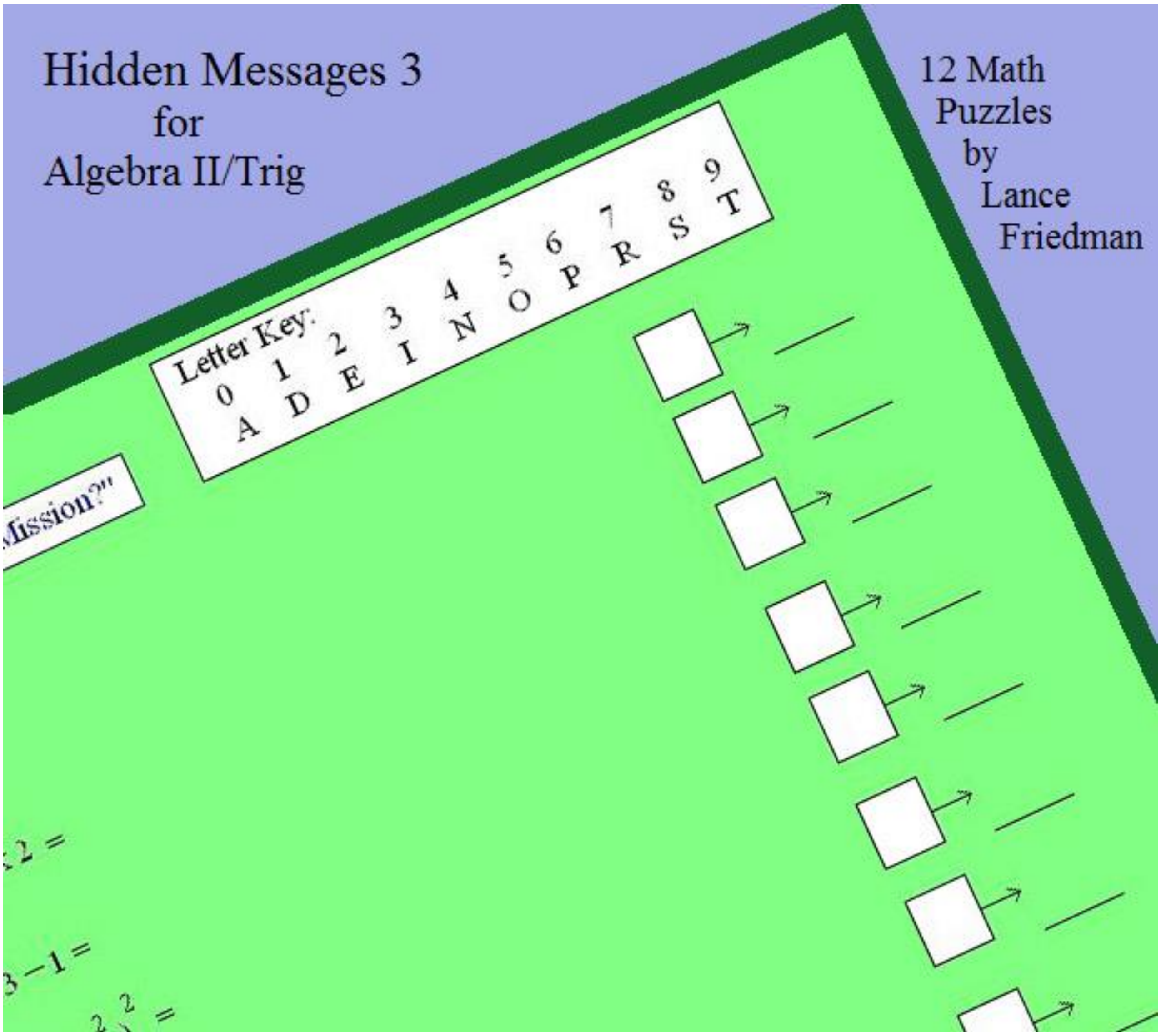

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Logarithm and Exponent Challenge Question

$4^{x} - 2^{x+1} = 3$

$4^{x} - 2^{x+1} - 3 = 0$

$(2^{2})^{x} - (2^{x})(2^{1}) - 3 = 0$

$(2^{x})^{2} - (2^{x})(2^{1}) - 3 = 0$

Let $y = 2^{x}$

$y^{2} - 2y - 3 = 0$

$(y - 3)(y + 1) = 0$

$y = -1, 3$

therefore,

$2^{x} = -1$ and 3

-1 is extraneous!

$2^{x} = 3$

approximately 1.585

$2^{x} = 3$	$2^{x} = -1$
X log 2 = log 3	x log 2 = log (-1)
X = Log3/log 2	x = log (-1)/ log (2)
X = 1.5849625	x does not exist

Check:

$4^{1.585} - 2^{2.585} =$

$9 - 6 = 3$ ✓