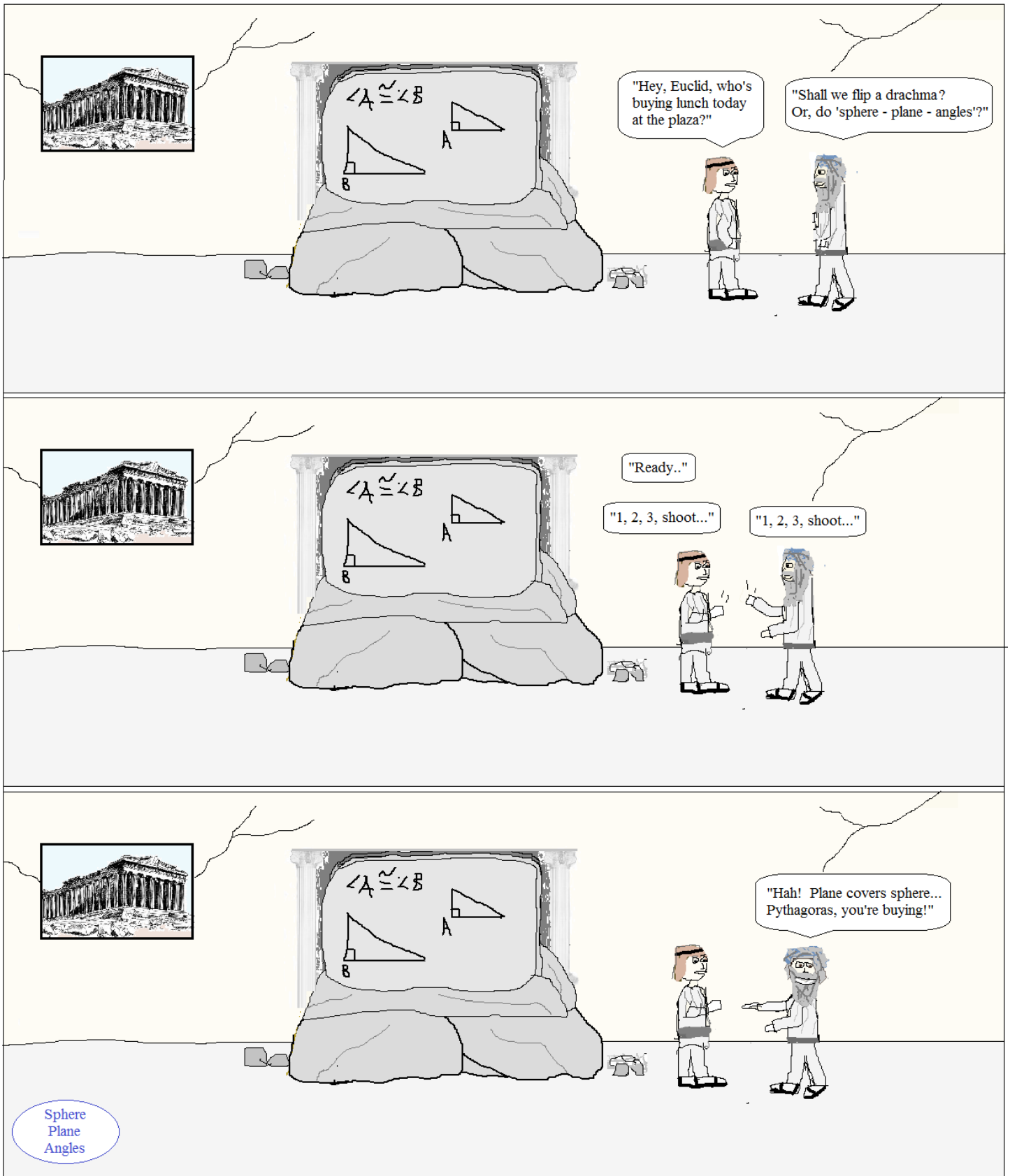
Similarity and Proportions

Review Test

Topics include Angle Bisector Theorem, Altitude to hypotenuse, Quadrilaterals, Proofs, and more.

Mathplane.com



Before the invention of scissors and paper...

LanceAF #356 (5-25-19)
mathplane.com

Similarity Review Test

1) $\triangle ABC \sim \triangle DEF$

AB = 10

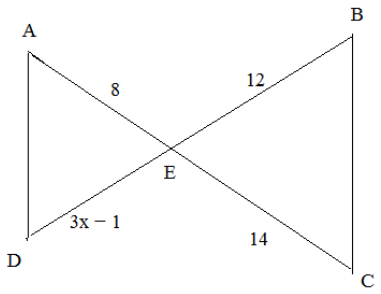
BC = x + 12

DE = x + 4

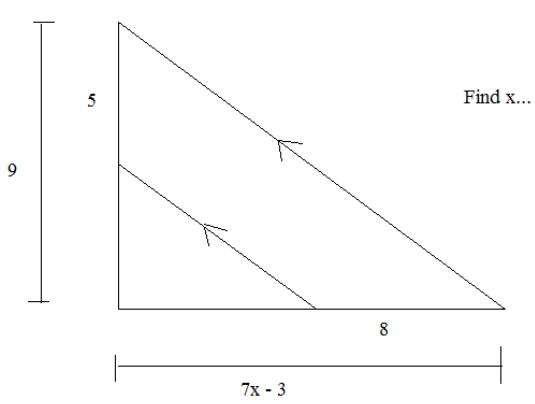
EF = 2

Find x...

2) The triangles are similar: find x

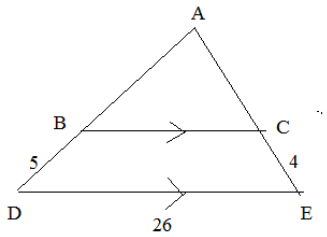


3)



Find x...

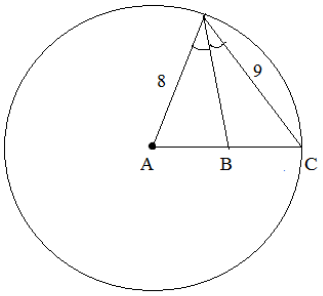
4)



If the perimeter of $\triangle ADE$ is 60, then what are the lengths of $\overline{AB}$ and $\overline{AC}$?

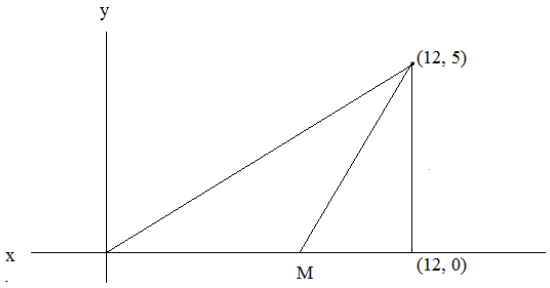
Similarity Review Test

5)



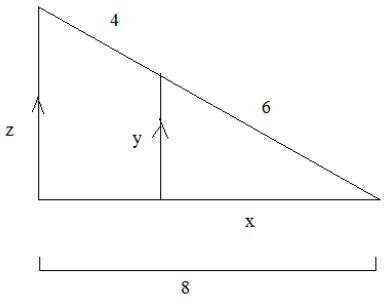
What is the length of $\overline{AB}$?

6)

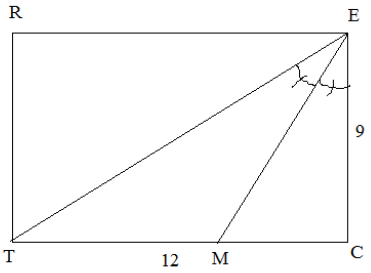


What is the coordinate of point M?

7) Find x, y, z...



8) Inside the rectangle RECT, find the length of $\overline{EM}$



TC = 12

Similarity Review Test

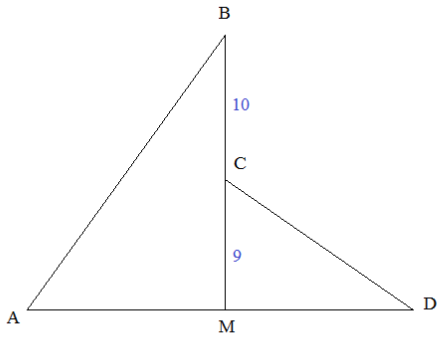
9) Given: $\angle B = \angle D$

M is the midpoint of $\overline{AD}$

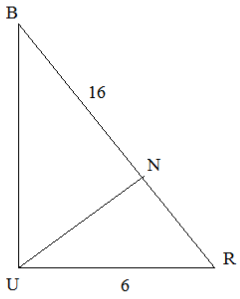
BM $\perp$ AD

Find: The length of $\overline{AD}$

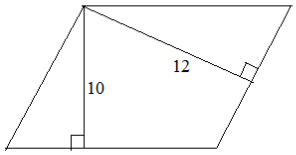
The length of $\overline{AB}$



10) Solve the triangle by finding $\overline{BU}$, $\overline{UN}$, and $\overline{RN}$

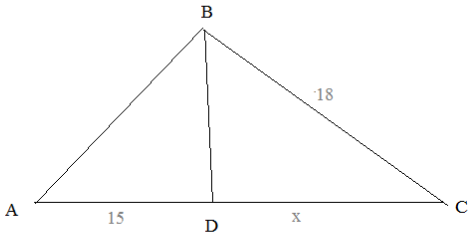


11) The parallelogram has perimeter 154 feet. Find the area.



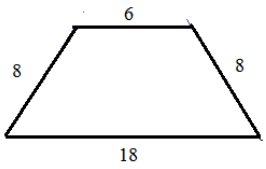
12) Given: $\angle DBC = \angle A$

Find: the length of $\overline{DC}$



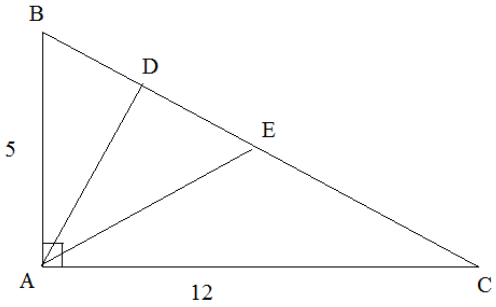
Similarity Review Test

13) An isosceles trapezoid has bases 6 and 18, and legs with length 8.
If the legs were extended upward, how long before the legs would meet?



14) ABC is a right triangle where $\overline{AD}$ and $\overline{AE}$ are angle trisectors.

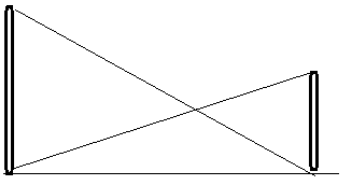
Find the length of $\overline{AE}$



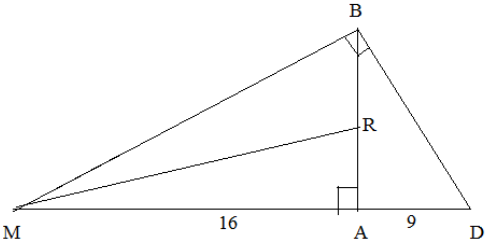
15) Two telephone poles are 80 feet apart.
One pole is 50 feet high. The second pole is 30 feet high.

A wire runs from the top of the first pole to the bottom of the second.
And, another wire runs from the top of the second pole to the bottom of the first.

How far above the ground do the wires pass each other?



16) If $\overline{RM}$ bisects angle BMA,
what is the length of $\overline{BR}$?



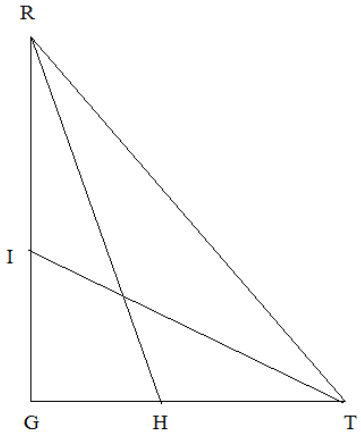
Similarity Review Test

17) Given: $\overline{TI}$ is an angle bisector

$\overline{RH}$ is an angle bisector

$\overline{RI} = 4 \quad \overline{RT} = 8 \quad \overline{HT} = 3$

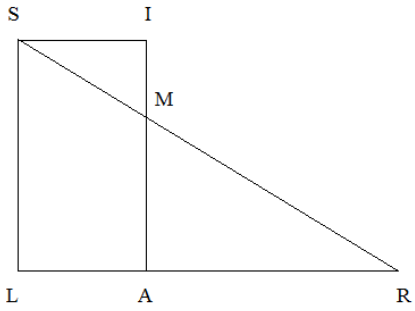
Find: $\overline{GI}$ and $\overline{GH}$



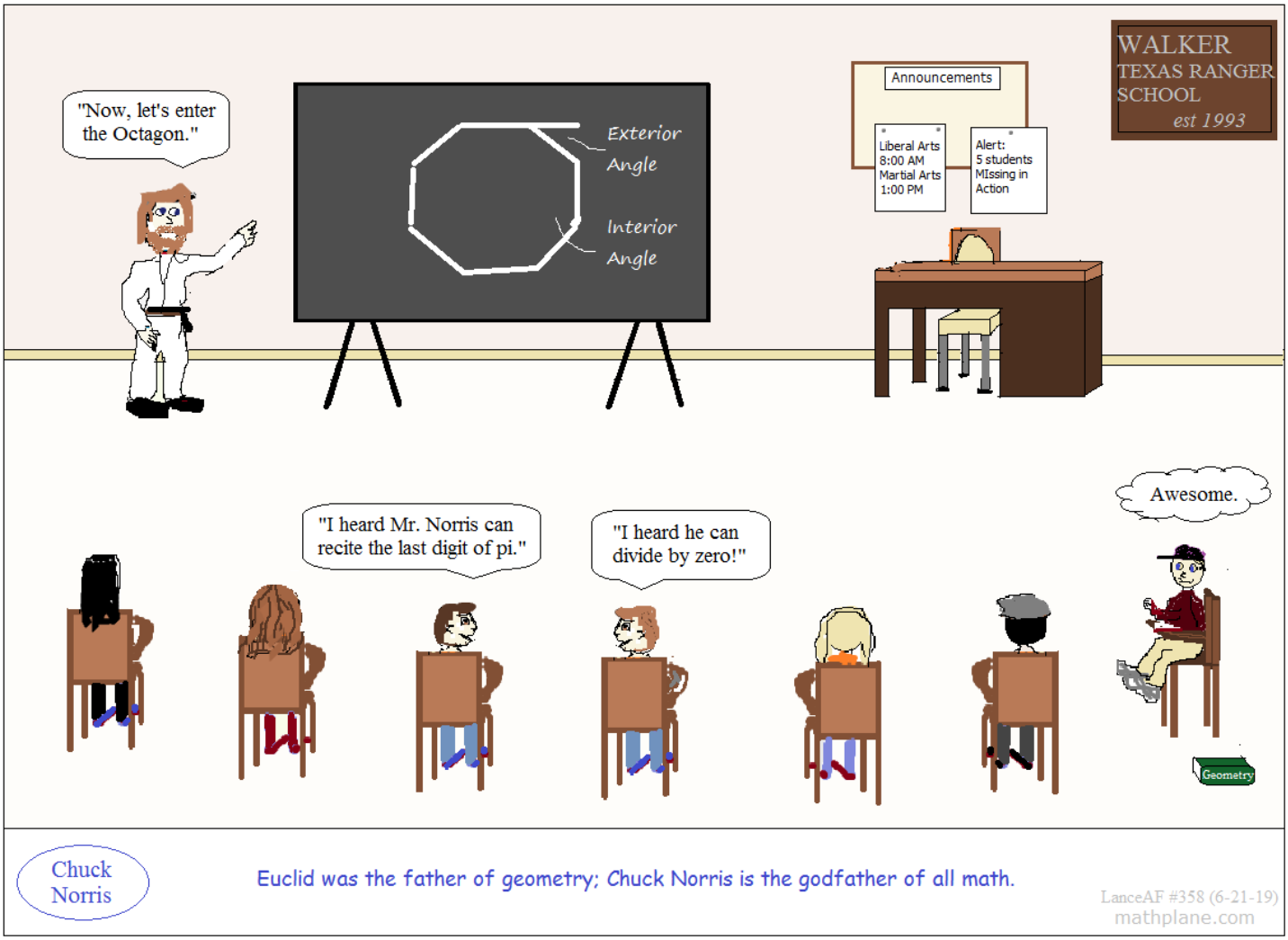
18) Given: $\frac{SR}{MR} = \frac{LR}{AR}$

$\overline{SL} = \overline{IA}$

Prove: SIAL is a parallelogram



Statements	Reasons


"Now, let's enter the Octagon."
Exterior Angle
Interior Angle
Announcements
Liberal Arts 8:00 AM Martial Arts 1:00 PM
Alert: 5 students MIssing in Action
WALKER TEXAS RANGER SCHOOL est 1993
"I heard Mr. Norris can recite the last digit of pi."
"I heard he can divide by zero!"
Awesome.
Geometry
Chuck Norris
Euclid was the father of geometry; Chuck Norris is the godfather of all math.
LanceAF #358 (6-21-19)
mathplane.com

SOLUTIONS-→

Similarity Review Test

1) $\triangle ABC \sim \triangle DEF$

AB = 10

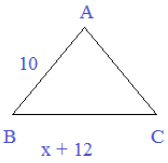
BC = x + 12

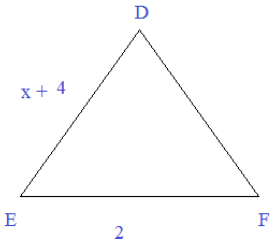
DE = x + 4

EF = 2

Find x...

SOLUTIONS





$\frac{10}{x+12} = \frac{x+4}{2}$

$x^2 + 16x + 48 = 20$

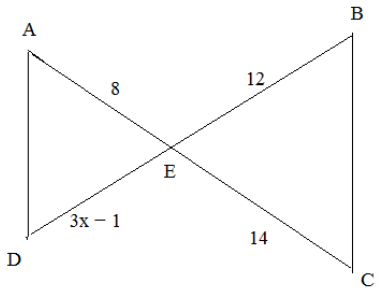
$(x + 2)(x + 14) = 0$

$x = -2, -14$

x must equal 2

(sides cannot be negative)

2) The triangles are similar: find x



If AD || BC, then there is one solution..

(angle A = angle C)

$\triangle AED \sim \triangle CEB$

$\frac{8}{14} = \frac{3x-1}{12}$

$96 = 42x - 14$

$110 = 42x$

$x = 2.62$

But, if angle A = angle B,

$\triangle AED \sim \triangle BEC$

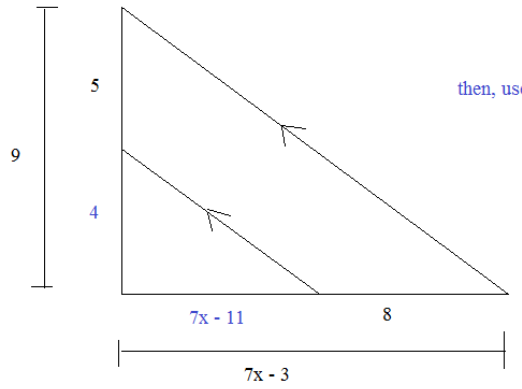
$\frac{8}{12} = \frac{3x-1}{14}$

$112 = 36x - 12$

$124 = 36x$

$x = 3.44$

3)



then, use side-splitter

$\frac{4}{7x-11} = \frac{5}{8}$

$35x - 55 = 32$

$35x = 87$

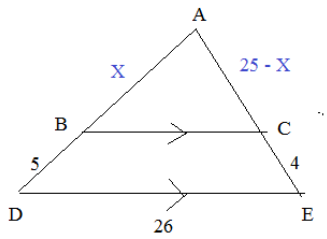
$x = \frac{87}{35}$

$\frac{5}{4} = \frac{8}{7(\frac{87}{35}) - 11}$

$32 = 35(\frac{87}{35}) - 55$

$87 = 35(\frac{87}{35})$ ✓

4)



If the perimeter of $\triangle ADE$ is 60, then what are the lengths of $\overline{AB}$ and $\overline{AC}$?

perimeter is 60...

BD + CE + DE + AB + AC = 60

5 + 4 + 26 + X + (25 - X) = 60

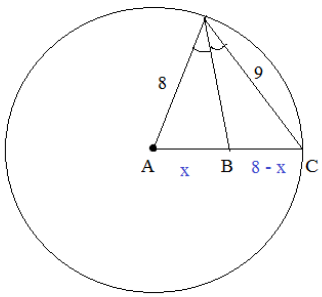
side-splitter theorem: $\frac{X}{25-X} = \frac{5}{4}$

$125 - 5X = 4X$

$X = \frac{125}{9}$ AB

$25 - X = \frac{100}{9}$ AC

5)



What is the length of $\overline{AB}$?

AC is 8 (because all radii are congruent)

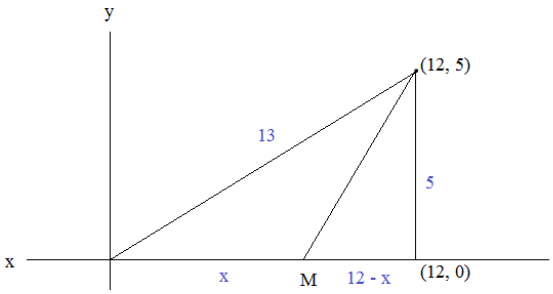
angle bisector theorem: $\frac{8}{9} = \frac{x}{8 - x}$

$9x = 64 - 8x$

$x = 64/17$

6)

What is the coordinate of point M?



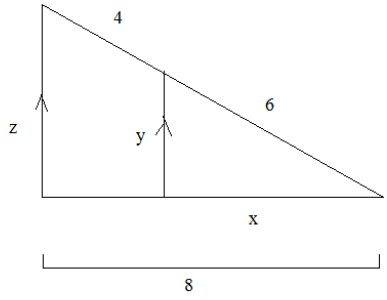
Using angle bisector theorem $\frac{13}{5} = \frac{x}{12 - x}$

$5x = 156 - 13x$

$18x = 156$

$x = 8\frac{2}{3}$

7) Find x, y, z...



Side-Splitter Theorem $\frac{6}{x} = \frac{4}{8 - x}$

$48 - 6x = 4x$

$x = 4.8$

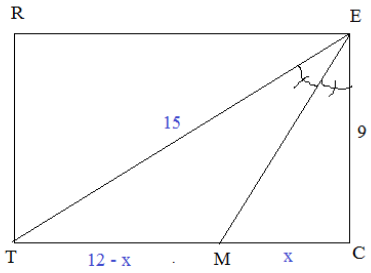
Pythagorean Theorem $4.8^2 + y^2 = 6^2$

$y = 3.6$

Similar Triangles $\frac{z}{3.6} = \frac{10}{6}$

$z = 6$

8) Inside the rectangle RECT, find the length of $\overline{EM}$



$\overline{ET}$ is 15 (Pythagorean Theorem)

$9^2 + 12^2 = 15^2$

Using Angle Bisector Theorem, we can find MC

$\frac{15}{12 - x} = \frac{9}{x}$

$15x = 108 - 9x$

$24x = 108$

$x = 4.5$

Since MC = 4.5 and EC = 9,

Pythagorean Theorem

$\overline{EM} = 10.06$

Similarity Review Test

SOLUTIONS

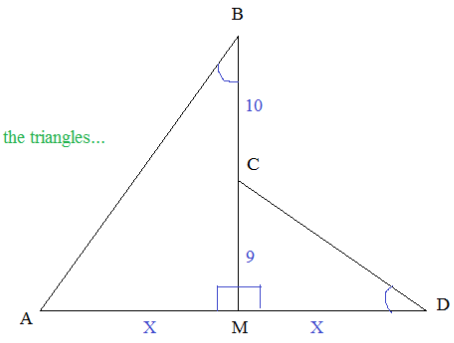
9) Given: $\angle B = \angle D$

M is the midpoint of $\overline{AD}$

$BM \perp AD$

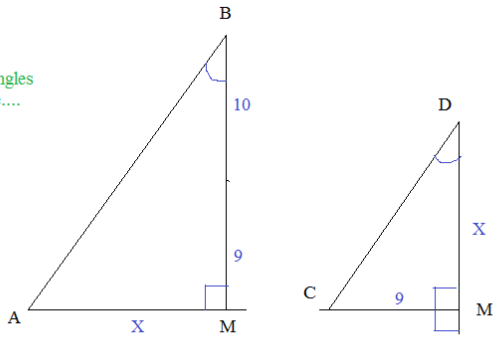
Find: The length of $\overline{AD}$

The length of $\overline{AB}$



Label the triangles...

Separate, reorient the triangles and set them side-by-side....



Apply similarity ratios... $\frac{19}{X} = \frac{X}{9}$ $X = \sqrt{171}$

$AD = 2\sqrt{171}$

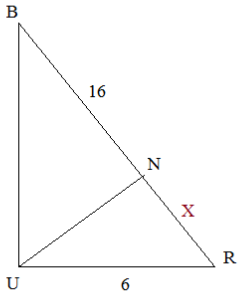
... and, Pythagorean Theorem..

$X^2 + 19^2 = AB^2$

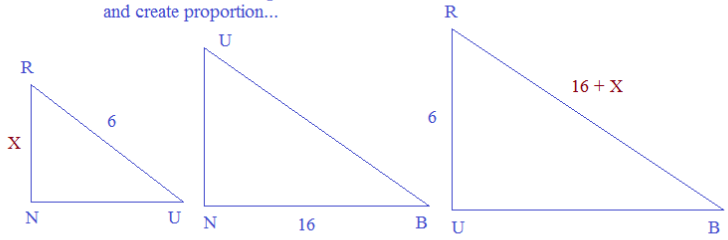
$171 + 361 = AB^2$

$AB = \sqrt{532}$

10) Solve the triangle by finding $\overline{BU}$, $\overline{UN}$, and $\overline{RN}$



method 1: break into similar triangles and create proportion...



"left" / "hypotenuse" $\frac{X}{6} = \frac{6}{X + 16}$ "left" / "hypotenuse"

$X^2 + 16X = 36$

$X^2 + 36X - 36 = 0$

$(x + 18)(x - 2) = 0$

$x = 2$ or -18

cannot have negative sides

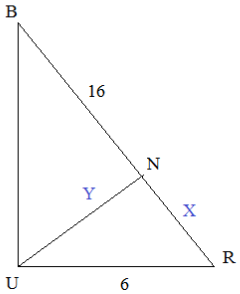
Since $X = 2$, RN is 2

Then, UN is $\sqrt{32}$

Pythagorean Theorem..

And, BU is $\sqrt{288}$

method 2: Use Pythagorean Theorem and Altitude to Hypotenuse to create a system of equations...



$X^2 + Y^2 = 6^2$ Pythagorean Theorem

$16 \cdot X = Y^2$ Altitutde to Hypotenuse

Solve system of equations..

$X^2 + 16X = 36$

$X^2 + 36X - 36 = 0$

$(x + 18)(x - 2) = 0$

$x = 2$ or -18

RN UN

since $X = 2$, $Y = \sqrt{32}$

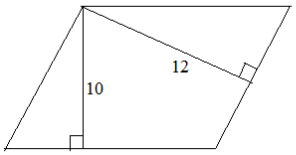
Then, apply Pythagorean Theorem to get BU

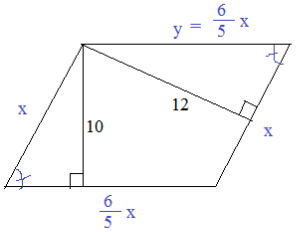
$16^2 + \sqrt{32}^2 = BU^2$

$BU = 12\sqrt{2}$

Similarity Review Test

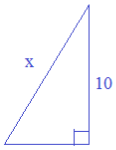
11) The parallelogram has perimeter 154 feet. Find the area.

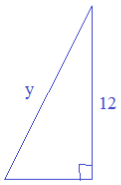




SOLUTIONS

Similar right triangles (because opposite angles of parallelogram are congruent)





$\frac{x}{10} = \frac{y}{12}$

$y = \frac{12x}{10}$

$y = \frac{6}{5}x$

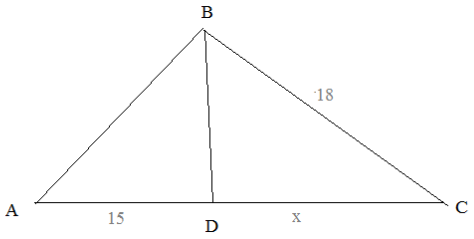
Since the perimeter is 154, we'll add all the sides...

$4.4x = 154$

$x = 35$

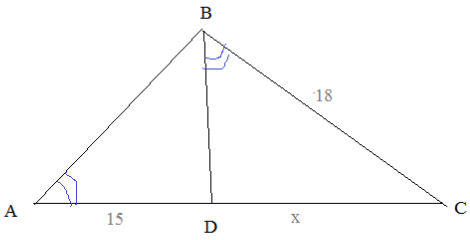
base of parallelogram: 42 height: 10 area: 420	base of parallelogram: 35 height: 12 area: 420 ✓

12)

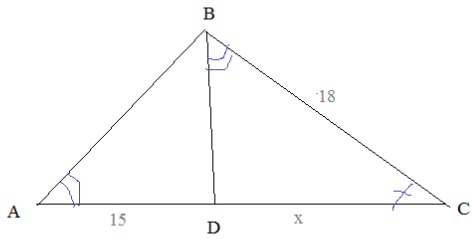


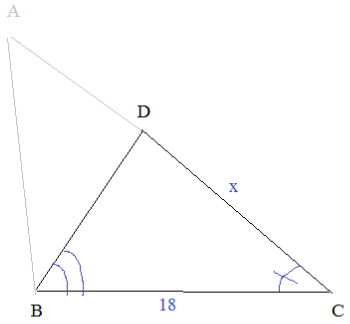
Given: $\angle DBC = \angle A$

Find: the length of $\overline{DC}$



Reorient to show the similar triangles (Angle-Angle)





$\frac{x}{18} = \frac{18}{x+15}$

$x^2 + 15x = 324$

$x^2 + 15x - 324 = 0$

$(x - 12)(x + 27) = 0$

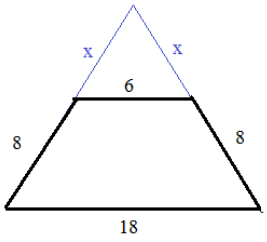
$x = 12$ or -27

since x is a length, then 12 is the solution!

Similarity Review Test

13) An isosceles trapezoid has bases 6 and 18, and legs with length 8.
If the legs were extended upward, how long before the legs would meet?

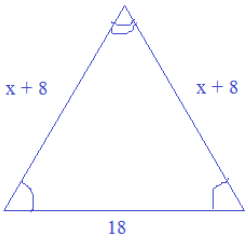
SOLUTIONS



$$\frac{x}{(x+8)} = \frac{6}{18}$$

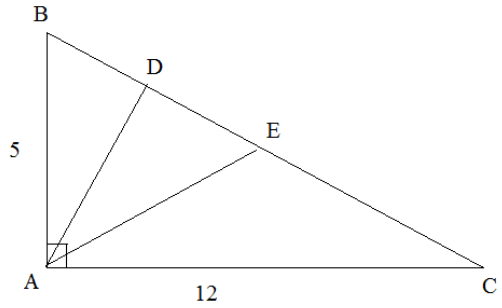
$$18x = 6x + 48$$

$$x = 4$$

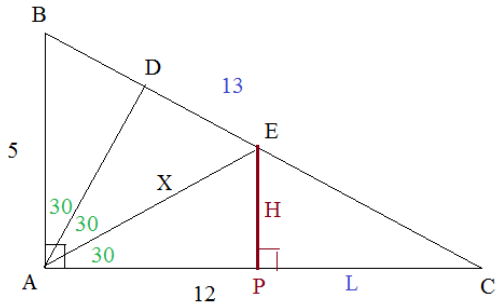


14) ABC is a right triangle where $\overline{AD}$ and $\overline{AE}$ are angle trisectors.

Find the length of $\overline{AE}$



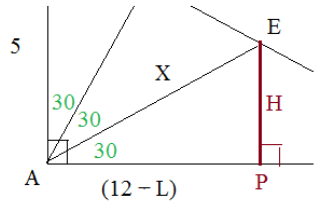
5-12-13 right triangle



Draw altitude H

△BAC and △EPC are similar triangles

$$\frac{5}{12} = \frac{H}{L} \Rightarrow L = \frac{12H}{5}$$



Since this is a 30-60-90 triangle,

$2H = X$

$$(12 - L)^2 + H^2 = X^2 \quad \text{Pythagorean Theorem}$$

$$(12 - \frac{12H}{5})^2 + H^2 = (2H)^2 \quad \text{Substitution}$$

$$(12 - \frac{12H}{5})^2 = 3H^2$$

$$(12 - \frac{12H}{5}) = \sqrt{3}\,(H)$$

$$12 = (\sqrt{3} + \frac{12}{5})H$$

$$H = \frac{12}{(\sqrt{3} + \frac{12}{5})} = 2.90$$

Therefore, $X = 2 \times 2.90 = 5.8$

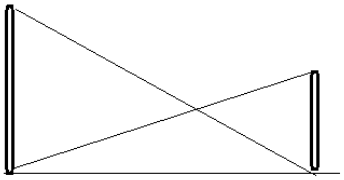
Similarity Review Test

15) Two telephone poles are 80 feet apart.
One pole is 50 feet high. The second pole is 30 feet high.

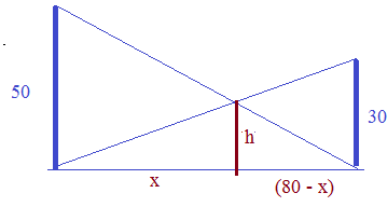
SOLUTIONS

A wire runs from the top of the first pole to the bottom of the second.
And, another wire runs from the top of the second pole to the bottom of the first.

How far above the ground do the wires pass each other?

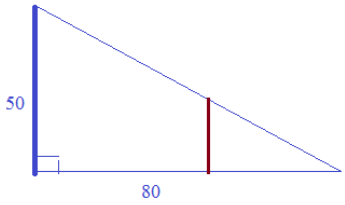


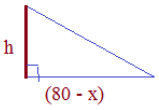
Step 1: Draw and label diagram; Recognize problem.



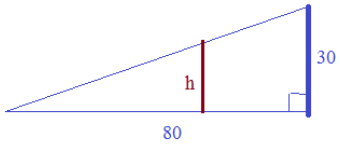
These are similar overlapping triangles!

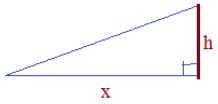
Step 2: Split the triangles and create proportions





$$\frac{50}{80} = \frac{h}{(80 - x)} \Rightarrow h = \frac{50(80 - x)}{80}$$





$$\frac{30}{80} = \frac{h}{x} \Rightarrow h = \frac{30x}{80}$$

Solve the system:

$$h = \frac{50(80 - x)}{80}$$

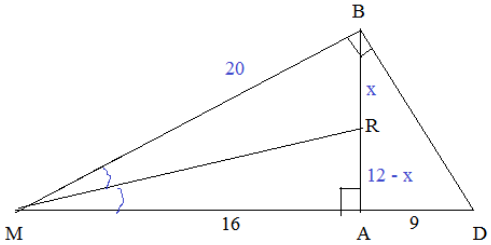
$$h = \frac{30x}{80}$$

$$4000 - 50x = 30x$$

$$x = 50$$

If $x = 50$, then $h = 18.75$

16) If $\overline{RM}$ bisects angle BMA,
what is the length of $\overline{BR}$?



$\overline{BA}$ is 12 (altitude to hypotenuse / geometric mean)

Then, we split $\overline{BA}$ into segments $\overline{BR} = x$ and $\overline{RA} = (12 - x)$

We recognize $\overline{BM} = 20$ (Pythagorean Theorem)

Finally, using Angle bisector theorem...

$$\frac{20}{x} = \frac{16}{12 - x}$$

$$16x = 240 - 20x$$

$$36x = 240$$

$\overline{BR} = 6.66$

$\overline{RA} = 5.33$

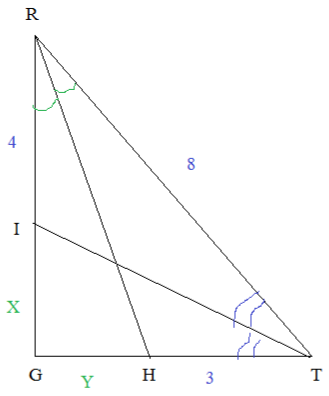
SOLUTIONS

17) Given: $\overline{TI}$ is an angle bisector

$\overline{RH}$ is an angle bisector

$\overline{RI} = 4 \quad \overline{RT} = 8 \quad \overline{HT} = 3$

Find: $\overline{GI}$ and $\overline{GH}$



Step 1: mark the diagram. look for clues

Angle Bisector Theorem!

Step 2: Set up proportions

$\triangle RGT \quad \frac{4+X}{8} = \frac{Y}{3} \quad 8Y = 3X + 12$

$\triangle TRG \quad \frac{8}{Y+3} = \frac{4}{X} \quad 8X = 4Y + 12$

Step 3: Solve the System

$-3X + 8Y = 12$

$8X - 4Y = 12$

$-3X + 8Y = 12$

$16X - 8Y = 24$

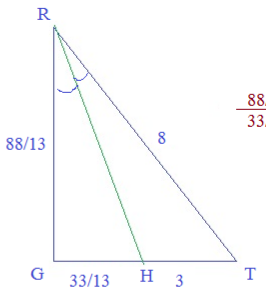
$13X = 36$

$X = 36/13 \quad \overline{GI}$

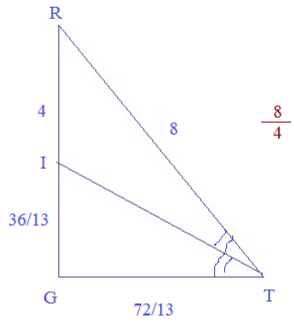
$Y = 33/13 \quad \overline{GH}$

NOTE: RGT is not a right triangle.
Pythagorean Theorem has no application

Check Answer:



$\frac{88/13}{33/13} = \frac{8}{3}$ ✓

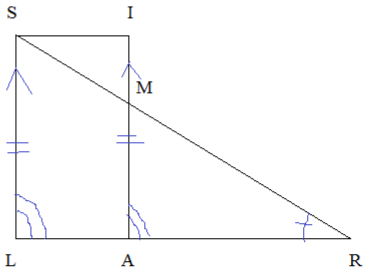


$\frac{8}{4} = \frac{72/13}{36/13}$ ✓

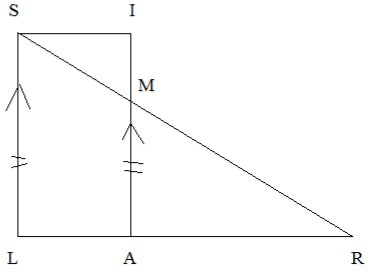
18) Given: $\frac{SR}{MR} = \frac{LR}{AR}$

$\overline{SL} = \overline{IA}$

Prove: SIAL is a parallelogram



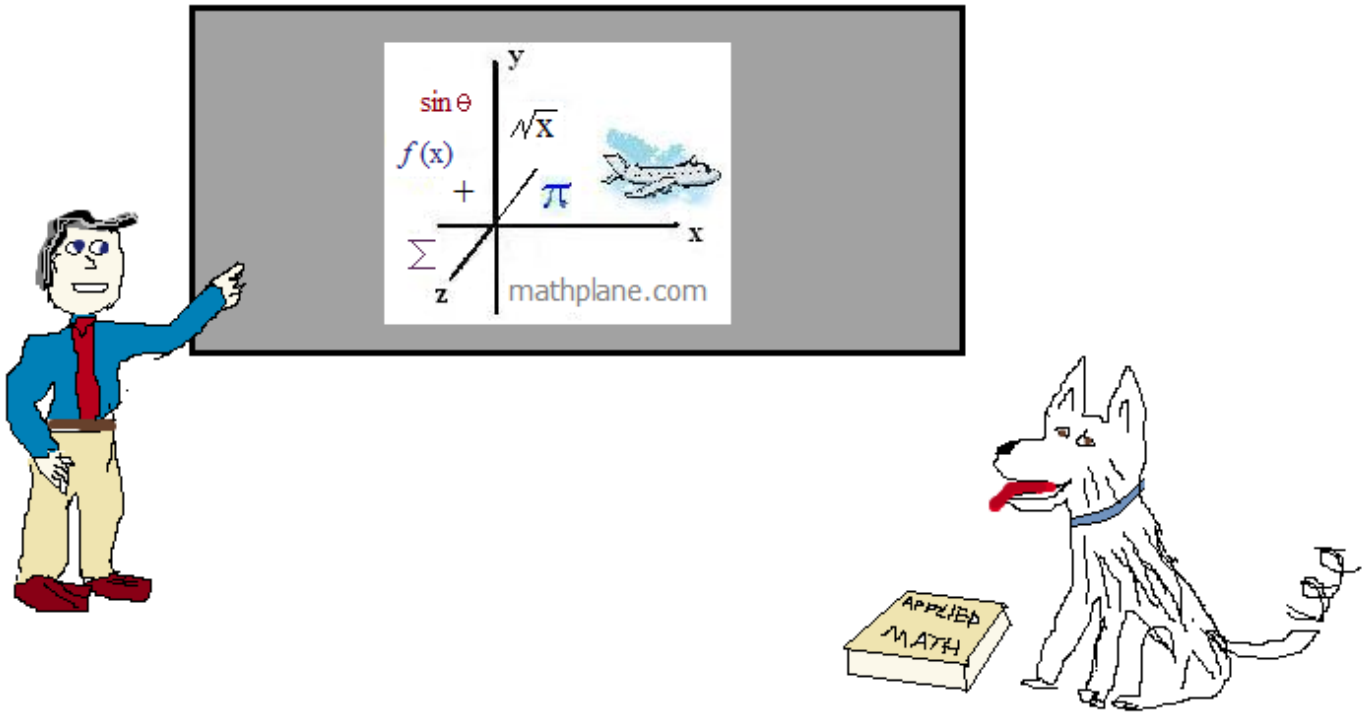
Statements	Reasons
1) SL = IA	1) Given
2) $\angle MRA = \angle SRL$	2) Reflexive property
3) $\frac{SR}{MR} = \frac{LR}{AR}$	3) Given
4) $\triangle SRL \sim \triangle MRA$	4) SAS~ (Side-Angle-Side Similarity)
5) $\angle MAR = \angle SLR$	5) CASTC (Corresponding Angles in Similar Triangles are Congruent)
6) SL || IA	6) If corresponding angles congruent, then, lines are parallel
7) SIAL is parallelogram	7) Definition of parallelogram (if one pair of sides are parallel AND congruent, then quadrilateral is parallelogram)



Thanks for visiting!

Hope it helped.

If you have questions, suggestions, or requests, let us know.



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