Triangle Theorems and Restrictions

Notes, Examples, and Practice Questions (and Solutions)

Triangle Theorems and Restrictions

Isosceles Triangles -- "If sides, then angles" theorem:

If two sides of a triangle are congruent, then the opposite angles are congruent.

"Triangle Inequality" theorem:

The length of any side of a triangle must exceed the sum of the other 2 sides.

"Remote Exterior Angle" theorem:

The measure of an exterior angle of a triangle equals the sum of the 2 remote interior angles.
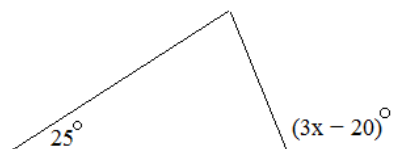
implies ⟹ *The measure of an exterior angle must be greater than each remote interior angle.*

Keep in mind, there are 2 restrictions to look for:

Comparison restriction: if an angle A is larger than angle B,
then opposite side a is larger than opposite side b

Absolute restriction: angles and sides must fit these constraints:
a) angle measures between 0 and 180
b) sides must be greater than 0
c) no side can exceed the sum of the other 2 sides

Example: What are the restrictions of x?

Comparative Restriction

Exterior Angle = Sum of remote interior angles

⟹ The measure of the *exterior angle* of a triangle is *greater than* the measure of either *remote interior angle*.

$$3x - 20 > 25$$
$$3x > 45$$
$$x > 15°$$

But, x can't be 200! So, there must be a maximum value, too...

Absolute Restriction

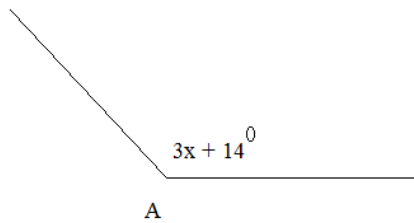The measure of an interior angle of a triangle must be less than 180 degrees.
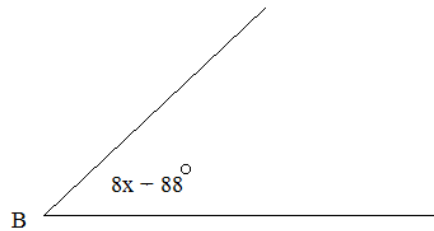
$$3x - 20 < 180$$
$$3x < 200$$
$$x < 200/3°$$

$$15 < x < 66.\overline{6}$$

I. Restrictions questions

   1) What values of x make $\angle$A obtuse?

$$3x + 14^{0}$$

A

   2) What values of y make $\angle$B acute?

$$8x - 88^{\circ}$$

B

   3) What values of z make $\angle$C greater than $\angle$D?

$$2x + 9$$

C

$$5x - 6$$

D

   4) What are the restrictions of x? (i.e. What are possible values of x?)

$$100^{\circ}$$

$$(4x + 8)^{\circ}$$

5) What are the restrictions of y?

K

79°   y°   34°

L   P   M

6) If angle TIR is obtuse, what is the measure?

T

3x + 55

2
x

25 - x

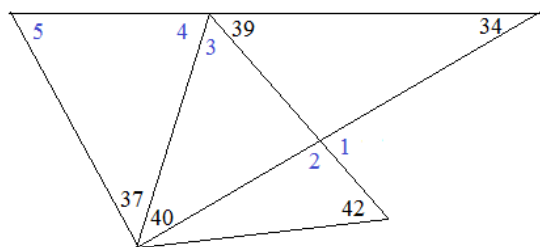I   R

II. Exterior Angles and Triangles

1) Find the measures of the angles

5   4   39   34

3

1

2

37
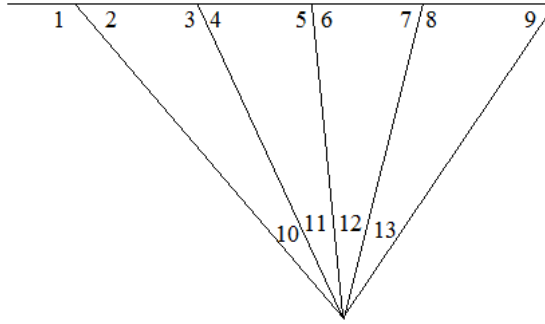40   42

Angle 1:

Angle 2:

Angle 3:

Angle 4:

Angle 5:

2) < or > ?   (justify your answers)

∠6 _____ ∠4

∠8 _____ ∠12

∠1 _____ ∠5

∠3 _____ ∠9
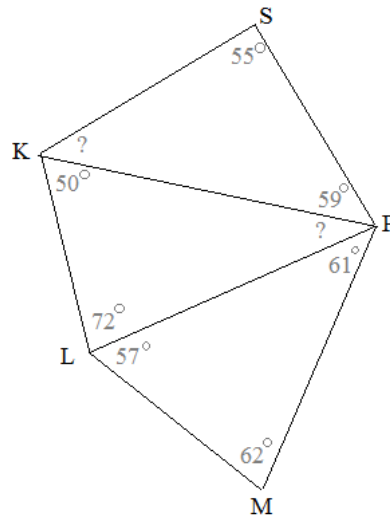
```
 1  2      3 4      5 6      7 8          9
```
(angles 10, 11, 12, 13 at the vertex below)

3) Which segment is largest? Shortest?

   (The figure is not drawn to scale)

S
55°

K
?
50°

59°
?
P
61°

72°
L 57°

62°
M

4) All the triangles are isosceles... What is the measure of angle A?

A
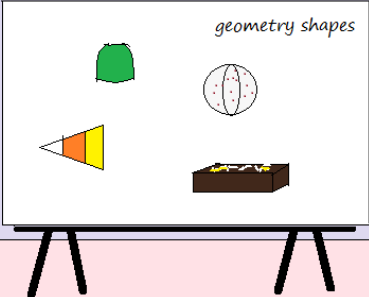
SOLUTIONS→

I. Restrictions questions

1) What values of x make $\angle A$ obtuse?

$90 < 3x + 14 < 180$

$25.33 < x < 55.33$

$3x + 14^{0}$

A
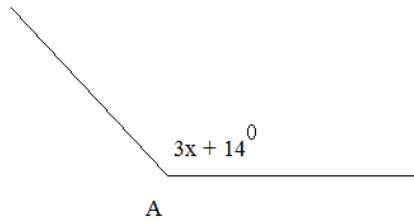
2) What values of y make $\angle B$ acute?

$0 < 8x - 88 < 90$

$11 < x < 22.25$

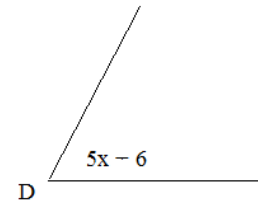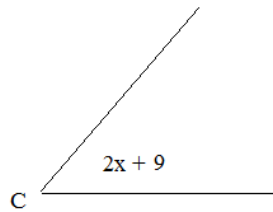$8x - 88^{\circ}$

B
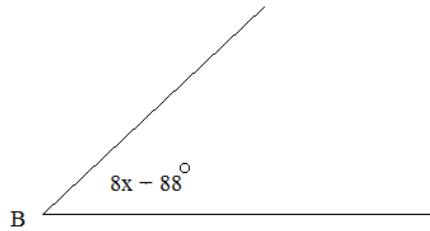
3) What values of z make $\angle C$ greater than $\angle D$?

$2x + 9 > 5x - 6$ "comparative restriction"

$-3x > -15$

$x < 5$

$2x + 9$

C

$5x - 6$

D

Then, $2x + 9 > 0$

angle must be between 0 and 180   "absolute restrictions"

$2x > -9$

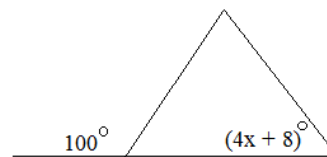$x > -9/2$ So, $x < 5$ and $x > -9/2$ and $x > 6/5$

$5x - 6 > 0$

$5x > 6$ $6/5 < x < 5$

$x > 6/5$

4) What are the restrictions of x? (i.e. What are possible values of x?)

"comparative restriction"   angle must be less than 100
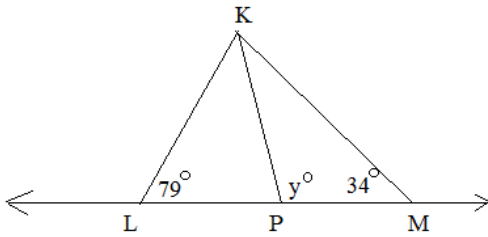   (triangle inequality theorem)

$(4x + 8) < 100$

100° $(4x + 8)^{\circ}$

"absolute restriction"   angle must be greater than 0

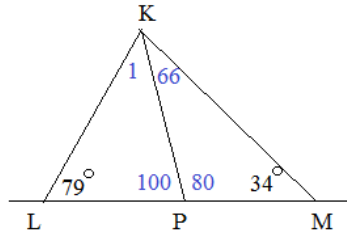$0 < (4x + 8)$ $-2 < x < 23$

5) What are the restrictions of y?

y is <u>exterior</u> angle of $\triangle$ KLP.
So, $y > 79°$

Then, y is <u>interior</u> angle of $\triangle$ KPM.
So, $y + 34 < 180$
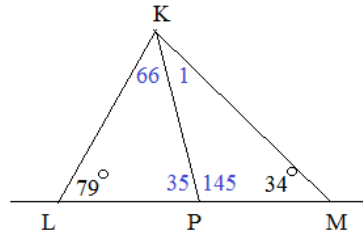
$y < 146°$

$79 < y < 146$

Random Check:

and,

6) If angle TIR is obtuse, what is the measure?
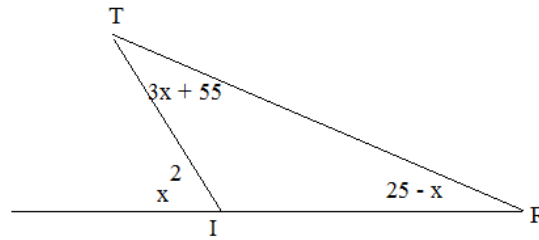
$x^2 = 3x + 55 + 25 - x$

$x^2 = 2x + 80$

$(x - 10)(x + 8) = 0$

$x = -8$ and 10
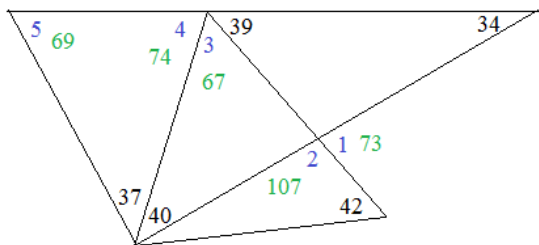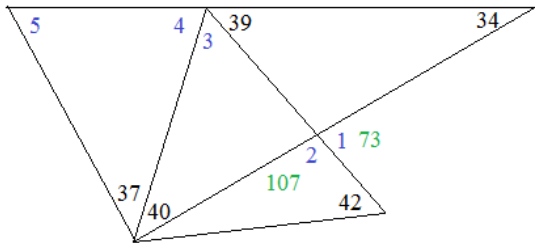
If $x = 10$, then TIR $= 80$

If $x = -8$, then TIR $= 116$

II. Exterior Angles and Triangles

1) Find the measures of the angles

Angle 1: Remote exterior angle theorem
73 degrees $39 + 34 = 73$

Angle 2: Supplementary angles
107 degrees $73 + 107 = 180$

Angle 3: Remote exterior angle theorem
67 degrees $67 + 40 = 107$

Angle 4: Straight angle sum is 180 degrees
74 degrees $39 + 67 + 74 = 180$

Angle 5: Interior angle sum of triangle is 180 degrees
69 degrees $69 + 74 + 37 = 180$

2) < or > ?   (justify your answers)

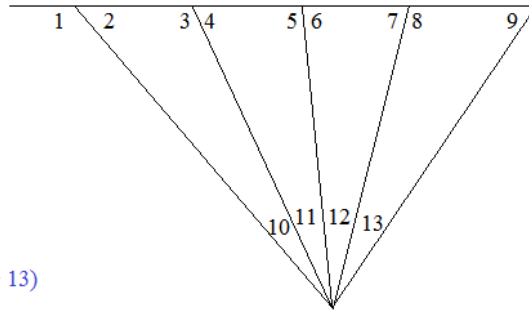(ext. angle theorem)

∠6 __>__ ∠4   ∠6 = ∠4 + ∠11

∠8 __>__ ∠12   ∠8 = ∠6 + ∠12

∠1 __>__ ∠5   ∠1 = ∠5 + ∠ (10 + 11)

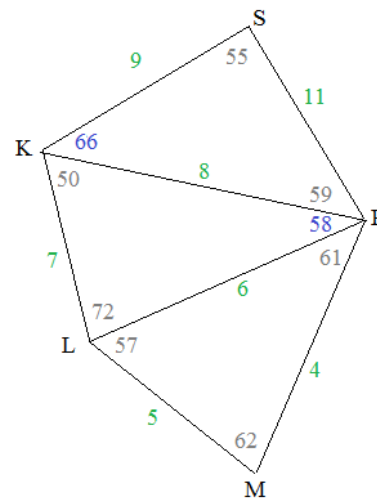∠3 __>__ ∠9   ∠3 = ∠9 + ∠ (11 + 12 + 13)

3) Which segment is largest? Shortest?

(The figure is not drawn to scale)
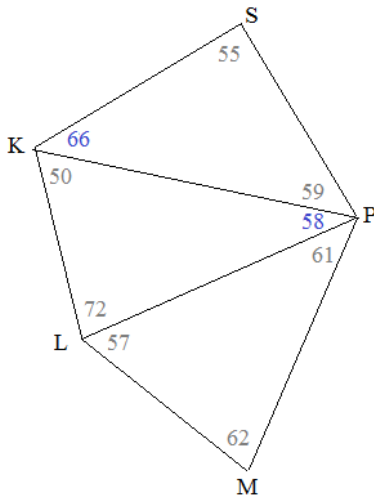
$\overline{PS}$ is the largest segment!

$\overline{PM}$ is the shortest segment!

Sum of interior angles of triangle are 180
(also, sum of interior angles of quadrilaterals is 360)

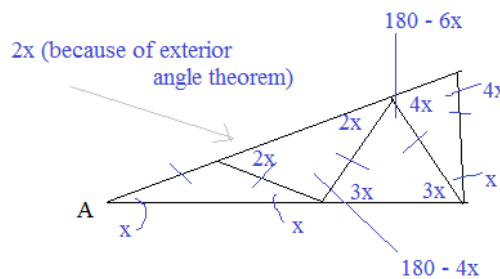then, in each triangle, the largest side is opposite
the largest angle.

(for comparison, add segment values)

In each triangle, the order of angle
size is matched by the order of
opposite side lengths

4) All the triangles are isosceles... What is the measure of angle A?

$x + 4x + 4x = 180$
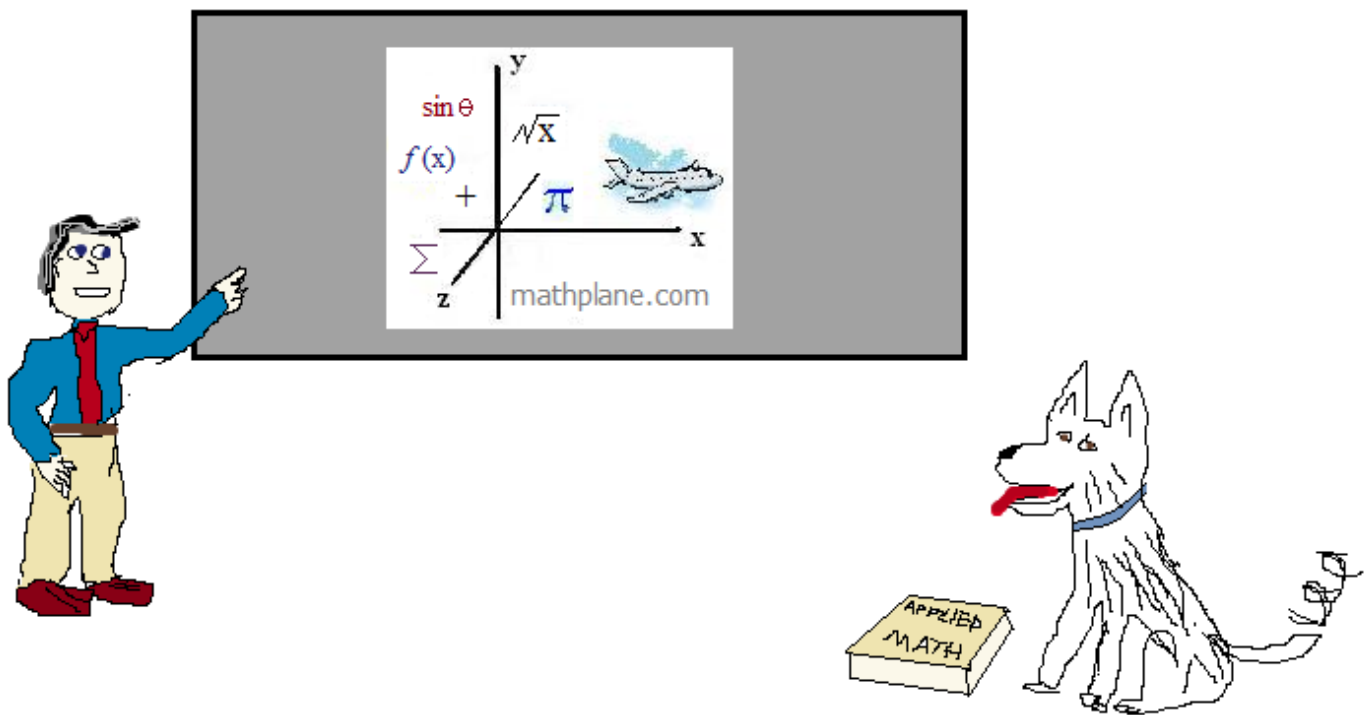
$x = 20$

2x (because of exterior
       angle theorem)
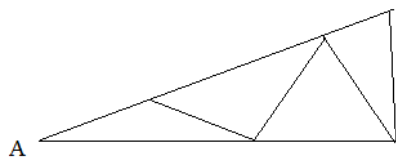
Thanks for visiting. (Hope it helped!)

If you have questions, suggestions, or requests, let us know.

Cheers

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