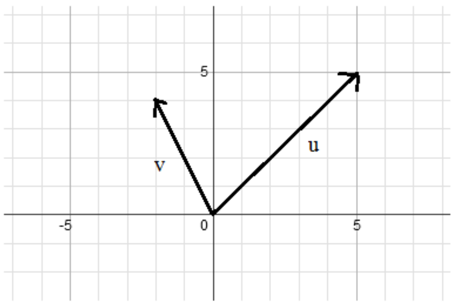
Vectors 2

Examples and Practice Questions (with solutions)

Topics include bearings, force, unit vectors, orthogonal vectors, graphing, angle vector theorem, and more.

Mathplane.com

Example: Given the graph of vectors u and v:

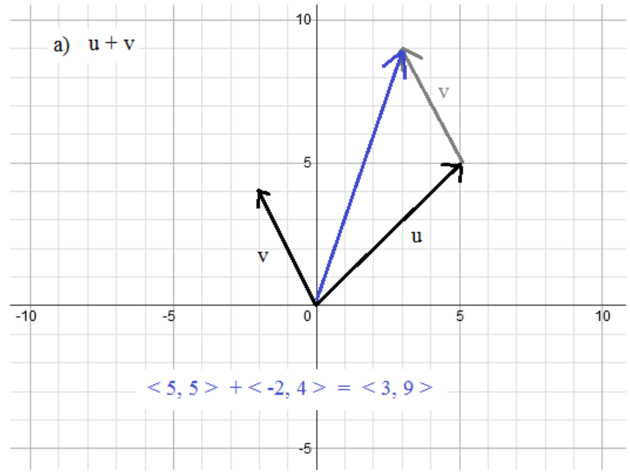


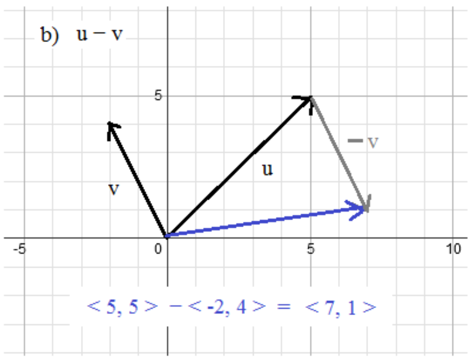
Find a) u + v

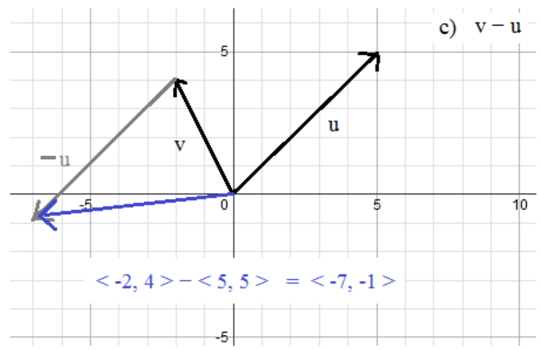
b) u − v

c) v − u

Show answer graphically
and using the components...



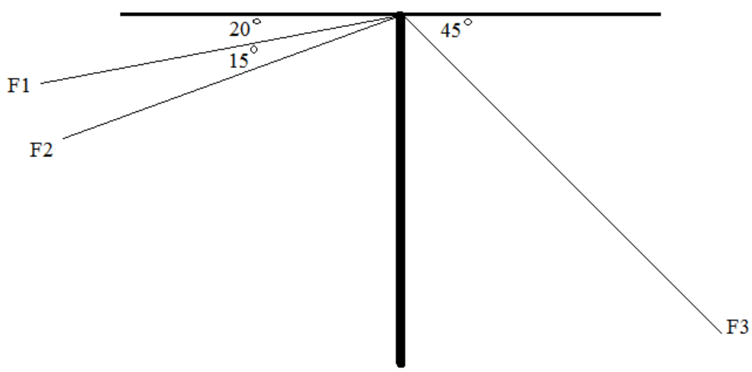




Example: 3 forces act at the top of a pole...
The magnitude of F1 is 250 lbs
and the magnitude of F2 is 300 lbs

If the resultant vector of the 3 forces
is directly vertically downward,

what is the magnitude of F3?



$x = r\cos\theta$

$y = r\sin\theta$

since it's a 45 degree angle, x and y will be the same...
Let's search for r, using the x values...

F1 + F2 + F3 must result in x = 0

F1 $x = r\cos\theta$ $x = 250\cos(200)$ $x = -234.9$

F2 $x = r\cos\theta$ $x = 300\cos(215)$ $x = -245.7$

F3 $x = r\cos\theta$ $x = r\cos(-45)$

(quadrant 2 is negative)

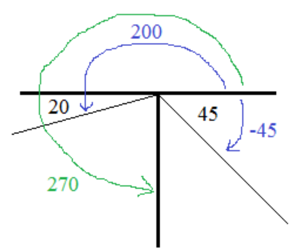
$$250\cos(200) + 300\cos(215) + r\cos(-45) = r\cos(270)$$

$$-234.9 \quad -245.7 \quad + r\cos(-45) = r(0)$$

$$-480.6 = -r\cos(-45)$$

$\boxed{r = 679.6}$

NOTE: we apply the forces in the
diagram to a coordinate plane

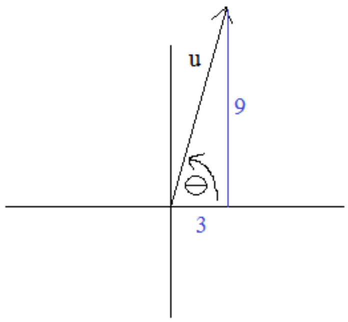


Finding the angle between 2 vectors in a plane

Example: Given vector u = < 3, 9 > and vector v has a magnitude of 8 and direction 60 degrees.
Find the angle between u and v.

Method 1: Find angle of vector u and compare to vector v

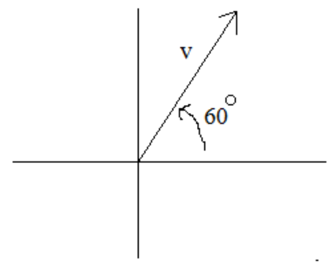
vector u = < 3, 9 >



$$\tan(\theta) = \frac{9}{3} = 3$$

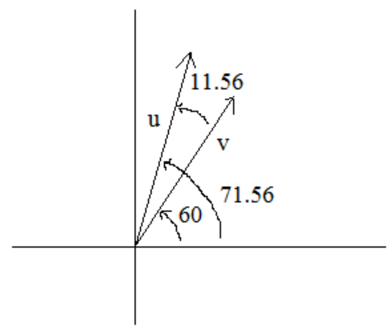
$$\tan^{-1}(3) = 71.56^\circ$$

angle of vector v is 60°



The angle between the vectors is

$71.56^\circ - 60^\circ = 11.56^\circ$



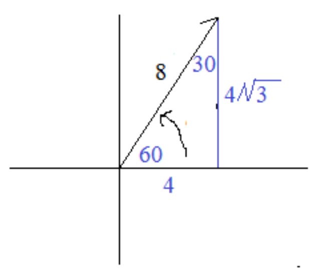
Method 2: Use angle formula $\cos\theta = \dfrac{u \cdot v}{\|u\|\,\|v\|}$ where θ is the angle between vectors u and v

u = < 3, 9 >

$\|u\| = \sqrt{3^2 + 9^2} = 3\sqrt{10}$

Since $\|v\| = 8$
and, the degree measure is 60,

vector v = $< 4, 4\sqrt{3} >$



$$\cos\theta = \frac{u \cdot v}{\|u\|\,\|v\|} = \frac{3 \cdot 4 + 9 \cdot 4\sqrt{3}}{3\sqrt{10} \cdot 8} = \frac{74.35}{75.89}$$

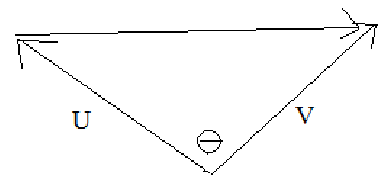
$$\theta = \cos^{-1}\frac{74.35}{75.89} = 11.56$$

Deriving the Angle Vector equation:

$$\cos\Theta = \frac{U \cdot V}{|U|\,|V|}$$

or

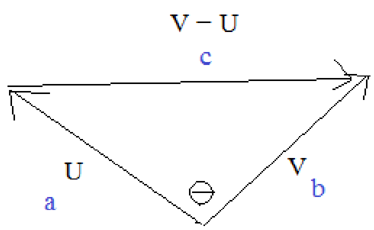
$$\frac{U \cdot V}{\|U\|\,\|V\|}$$



Law of Cosines: $c^2 = a^2 + b^2 - 2ab(\cos\Theta)$

Vector Dot Product & Length Property

$$\|\vec{w}\|^2 = \vec{w} \cdot \vec{w}$$



$$\|V - U\|^2 = \|U\|^2 + \|V\|^2 - 2\|U\| \cdot \|V\|(\cos\Theta)$$

$$(V - U) \cdot (V - U) = \|U\|^2 + \|V\|^2 - 2\|U\| \cdot \|V\|(\cos\Theta)$$

$$V \cdot V - U \cdot V - V \cdot U + U \cdot U = \|U\|^2 + \|V\|^2 - 2\|U\| \cdot \|V\|(\cos\Theta)$$

$$\cancel{\|V\|^2} - 2\,U \cdot V + \cancel{\|U\|^2} = \cancel{\|U\|^2} + \cancel{\|V\|^2} - 2\|U\| \cdot \|V\|(\cos\Theta)$$

$$-2\,U \cdot V = -2\|U\| \cdot \|V\|(\cos\Theta)$$

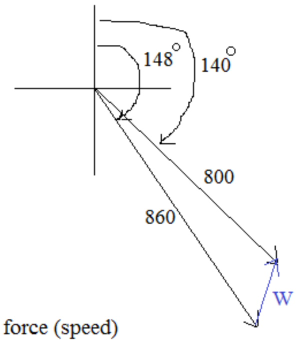
$$\frac{U \cdot V}{\|U\| \cdot \|V\|} = (\cos\Theta)$$

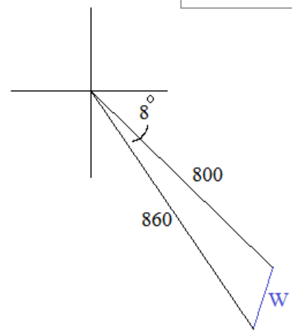
Example: A plane flying at 860 km per hour at a bearing of 148° has a ground speed of 800 km per hour and a bearing of 140°. What is the wind direction and force?

Air speed + Wind Speed = Ground Speed

Method 1: Using Law of Cosines/Sines

Step 1: Draw a diagram





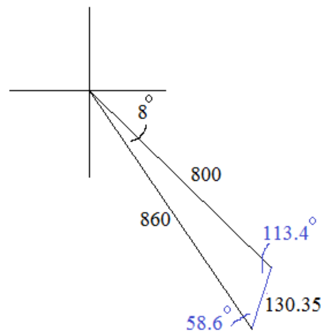
Step 2: Use law of cosines to find wind force (speed)

$$W^2 = 800^2 + 860^2 - 2(800)(860)\cos(8^\circ)$$

$$W^2 = 1379600 - 1376000(.990)$$

$$W^2 = 16991$$

$$\boxed{W = 130.35}$$



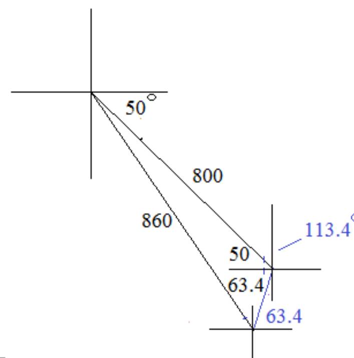
Step 3: Use law of sines to get angles (to find bearing)

$$\frac{\sin(8)}{130.35} = \frac{\sin(x)}{800} \qquad \sin(x) = \frac{800\sin(8)}{130.35} = .854 \qquad x = 58.6^\circ$$

Then, $58.6 + 8 + y = 180$ so, the other angle is 113.4°

Using geometry properties and theorems, the angle is 63.4 degrees...

Therefore, the bearing is $\boxed{26.6^\circ}$

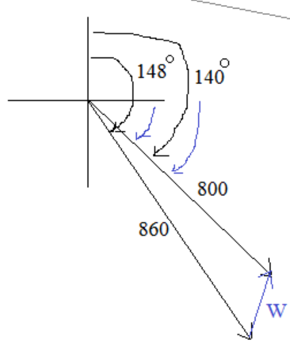


Method 2: Use component vectors

Step 1: Change bearings into standard angle measures...

140 degrees ----> −50 degrees (or 310 degrees)

148 degrees ----> −58 degrees (or 302 degrees)



Step 2: Find the component vectors

$\langle r\cos\theta, r\sin\theta \rangle$

Ground speed: $\langle 800\cos310, 800\sin310 \rangle$

$\langle 514.2, -612.8 \rangle$

Air speed: $\langle 860\cos302, 860\sin302 \rangle$

$\langle 455.7, -729.3 \rangle$

Step 3: Solve Air speed + Wind Speed = Ground Speed

$$\langle 455.7, -729.3 \rangle + \langle i, j \rangle = \langle 514.2, -612.8 \rangle$$

Wind speed $= \langle 58.5, 116.5 \rangle$

Force (magnitude) $= \sqrt{58.5^2 + 116.5^2} = \boxed{130.36}$

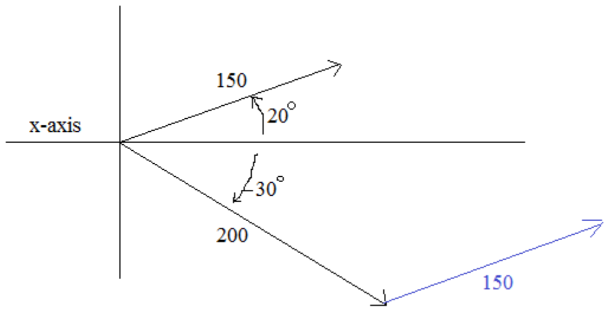
direction: $\tan^{-1}\dfrac{116.5}{58.5} = 63.33$ ** then, convert back to bearings..

$90 + 63.33 = \boxed{26.67^\circ}$

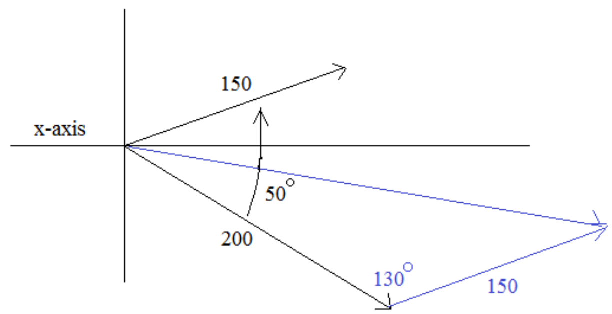
Example: Two forces pull on an object.
The first force pulls at 150 lbs at a 20 degree angle (relative to the x-axis)
The second force pulls at 200 lbs at a −30 degree angle (relative to the x-axis)

What is the total force and which direction is the object pulled?

Method 1: Using Law of cosines and Law of Sines



Use "tip to tail"...



Property of parallelogram:
consecutive angles are supplementary...

$(50^\circ + 130^\circ = 180^\circ)$

To find force, use law of cosines...

$a^2 = 200^2 + 150^2 - 2(200)(150)\cos(130^\circ)$

$a^2 = 62500 - 60000(-.643)$

$a^2 = 101{,}067$

$a = 317.9$ lbs

To find direction, we can use law of sines...

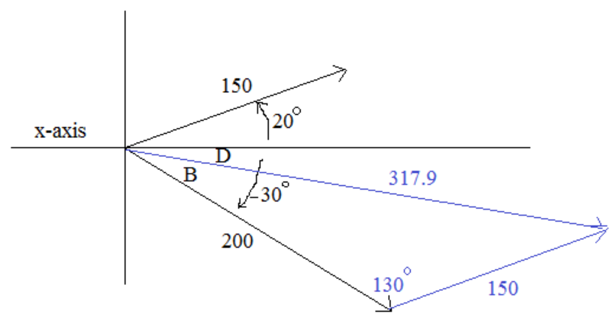
$$\frac{\sin(130)}{317.9} = \frac{\sin B}{150}$$

$$\sin B = \frac{150\sin(130)}{317.9} = .361$$

$B = 21.2$ degrees

Since $D + B = {}^{-}30$,

direction $D = -8.8^\circ$



Method 2: Use Component Vectors

$< r\cos\theta, r\sin\theta >$

$< 150\cos(20), 150\sin(20) >$ $< 200\cos(-30), 200\sin(-30) >$

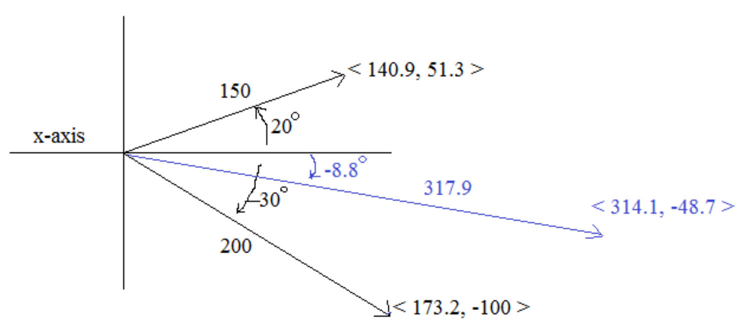
$< 140.9, 51.3 >$ $< 173.2, -100 >$

Add components to get resultant vector:

$< 140.9 + 173.2, 51.3 + (-100) > = < 314.1, -48.7 >$

magnitude of vector: $\sqrt{314.1^2 + (-48.7)^2} = 317.9$

direction of vector: $\tan^{-1}\left(\frac{-48.7}{314.1}\right) = -8.8^\circ$



Example: The sum of 2 vectors is $\langle 3, 7 \rangle$.

If one vector is parallel to $\langle 4, 1 \rangle$
and
one vector is perpendicular to $\langle 4, 1 \rangle$,

what are the 2 vectors?

If a vector is parallel to $\langle 4, 1 \rangle$, it will have the same ratio (i.e. slope)... $\langle 4a, a \rangle$

If a vector is perpendicular to $\langle 4, 1 \rangle$, it will have the opposite reciprocal... $\langle -1b, 4b \rangle$

$$\langle 4a, a \rangle + \langle -1b, 4b \rangle = \langle 3, 7 \rangle$$

$$4a - b = 3$$
$$a + 4b = 7$$

$$16a - 4b = 12$$
$$a + 4b = 7$$

$$17a = 19$$

$$a = 19/17$$

$$b = 25/17$$

Since $a = 19/17$ and $b = 25/17$,

$\boxed{\text{the 2 vectors are } \langle 76/17, 19/17 \rangle \text{ and } \langle -25/17, 100/17 \rangle}$

Notice the sum of the vectors is $\langle 3, 7 \rangle$

$\langle 76/17, 19/17 \rangle$ is parallel to $\langle 4, 1 \rangle$

(same ratio..)

$\langle -25/17, 100/17 \rangle$ is perpendicular to $\langle 4, 1 \rangle$

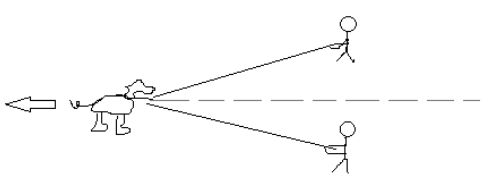
(the dot product is 0)

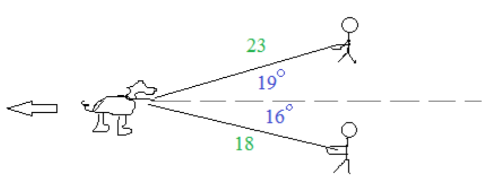
Example: My dog Norway is tugging on two ropes held by me and my friend.

If I'm pulling on a rope with a force of 23 at an angle of 19 degrees,
and my friend is pulling on the other rope with a force of 18 at an angle of 16 degrees,

what is the force my dog is pulling?

Step 1: Draw a diagram





Step 2: Convert the angles and force into vectors

$x = r\cos\theta$ $\quad \langle 23\cos(19), 23\sin(19) \rangle$

$y = r\sin\theta$ $\quad \langle 21.75, 7.49 \rangle$

$\langle 18\cos(-16), 18\sin(-16) \rangle$

$\langle 17.30, -4.96 \rangle$

Step 3: Find the resultant vector

$$\langle 21.75, 7.49 \rangle + \langle 17.30, -4.96 \rangle = \langle 39.05, 2.53 \rangle$$

The resultant vector of me and my friend combined...

Step 4: Find the force of the dog

Since the vector of me and my friend is $\langle 39.05, 2.53 \rangle$,

the force ('magnitude') is $\sqrt{39.05^2 + 2.53^2} = 39.13$

therefore, the force of Norway the dog is $\boxed{39.13...}$

$\langle 0, -39.13 \rangle$ is the vector

Vectors and Angles

$$\cos\theta = \frac{a \cdot b}{\|a\|\,\|b\|}$$ where θ is the angle between vectors a and b.

Dot Product		
	> 0	ACUTE
	< 0	OBTUSE
	= 0	RIGHT (Orthogonal Vectors)

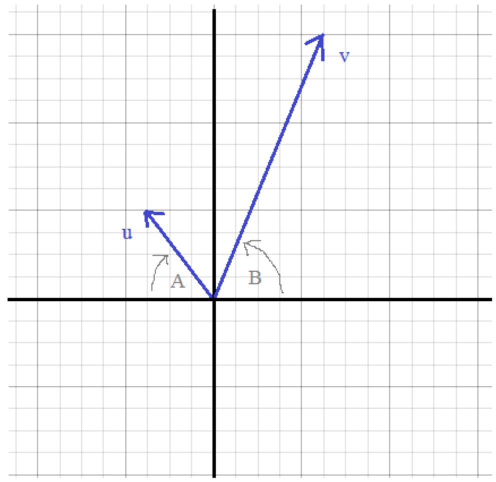
Example: v = < 5, 12 >

u = < -3, 4 >

Find the angle between vectors u and v,

a) using the vector formula

b) using geometry and trigonometry



Using the formula, $$\cos\theta = \frac{u \cdot v}{\| u \| \| v \|}$$

$$\cos\theta = \frac{33}{(5)(13)}$$

$\boxed{59.5 \text{ degrees}}$

Using geometry and trigonometry, $\tan A = \frac{4}{3}$ $\quad A = 53.1$ $\quad \tan B = \frac{12}{5}$ $\quad B = 67.4$

A + B + Angle = 180

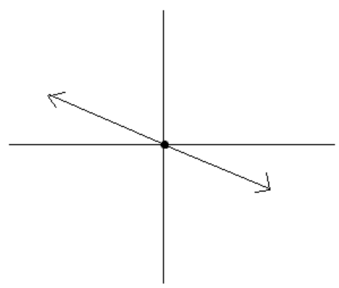
53.1 + 67.4 + Angle = 180

$\boxed{\text{Angle} = 59.5}$

Example: Find the angle between < 3, -1 > and < -3, 1 >

$$\cos\theta = \frac{-10}{10} = -1$$

$\boxed{\theta = 180 \text{ degrees}}$



Example. | u | = 8 "Bearing direction" 115 degrees

| v | = 10 "Bearing direction" 50 degrees

Draw the vectors. Then find | u + v |, using a) component vectors

b) law of cosines and geometry

a) using component vectors

u: $x = 8\cos(-25^\circ)$ 7.25 < 7.25, -3.4 >

$y = 8\sin(-25^\circ)$ -3.4

v: $x = 10\cos(40^\circ)$ 7.66 < 7.66, 6.4 >

$y = 10\sin(40^\circ)$ 6.4

u + v = < 14.91, 3 >

$\boxed{| u + v | = 15.2}$

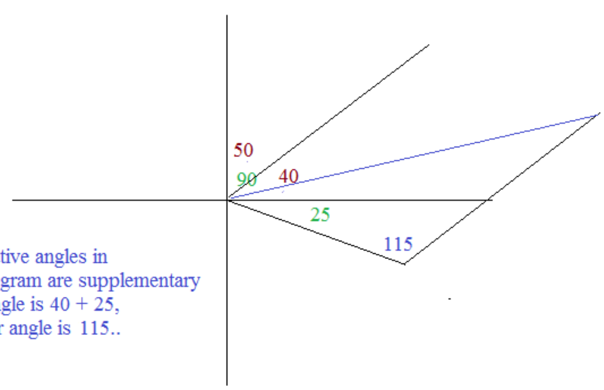
b) using law of cosines and geometry

$$c^2 = 8^2 + 10^2 - 2(8)(10)\cos(115)$$

$$c^2 = 164 - 160\cos(115)$$

$$c^2 = 164 - -67.6$$

$\boxed{c = 15.2}$



consecutive angles in parallelogram are supplementary
Since angle is 40 + 25,
the other angle is 115..

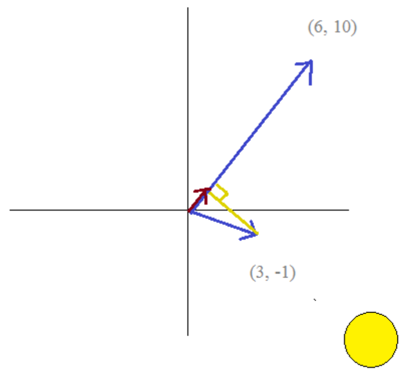
Example: Find the projection of vector u $< 3, -1 >$ onto vector v $< 6, 10 >$

Projection of vectors

The projection formula:

The projection of vector u onto vector v is

$$\text{proj}_v\, u = \frac{u \cdot v}{|v|^2}\,\vec{v}$$



The sun shines on vector u and creates a shadown on vector v

$u \cdot v = 18 - 10 = 8$

$|v| = \sqrt{3^2 + 11^2} = \sqrt{130}$

$|v|^2 = 130$

$\text{proj}_v\, u = \frac{8}{130} < 6, 10 > \Rightarrow < .37 , .61 >$

$< \frac{24}{65} , \frac{8}{13} >$

Red projected vector $< .37, .61 >$ has the same direction as $< 6, 10 >$

$< .37, .61 >$ + perpendicular vector $= < 3, -1 >$

perpendicular vector $= < 2.63, -1.61 >$

$< .37, .61 > \cdot < 2.63, -1.61 > = 0$

dot product is 0, so perpendicular

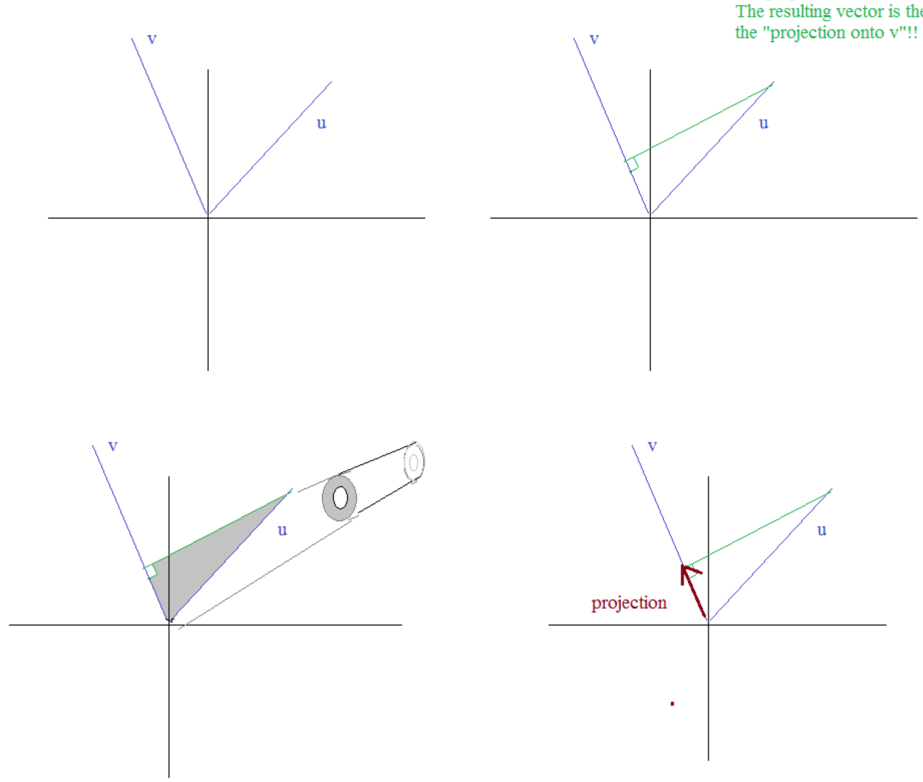
Projection of u onto v....

$$\text{proj}_v\, u = \frac{u \cdot v}{|v|^2}\,\vec{v}$$

$$= \frac{u \cdot v}{|v|}\,\frac{\vec{v}}{|v|}$$

(unit vector)

imagine a flashlight shining in a perpendicular direction... The resulting vector is the the "projection onto v"!!



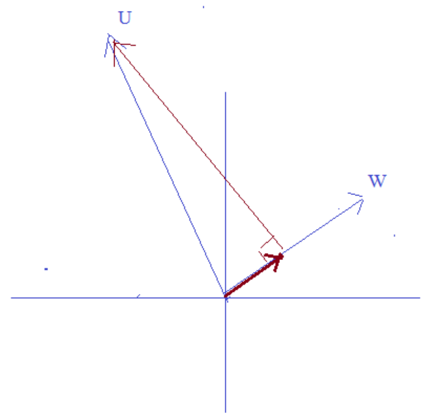
Example: Vectors $\vec{U} = \langle -2, 8 \rangle$ and $\vec{W} = \langle 4, 3 \rangle$

Find the projetion of U onto W.

Then, identify the component vectors, where the resultant is U

$$\text{Proj}_W U = \frac{U \cdot W}{\| W \|^2} W$$

$$= \frac{16}{25} \langle 4, 3 \rangle = \left\langle \frac{64}{25}, \frac{48}{25} \right\rangle$$



$$P_1 + P_2 = U$$

$$\left\langle \frac{64}{25}, \frac{48}{25} \right\rangle + P_2 = \langle -2, 8 \rangle$$

$$P_2 = \left\langle \frac{-114}{25}, \frac{152}{25} \right\rangle$$

$$P_1 \cdot P_2 = \left\langle \frac{64}{25}, \frac{48}{25} \right\rangle \cdot \left\langle \frac{-114}{25}, \frac{152}{25} \right\rangle = -7296/625 + 7296/625 = 0 \checkmark$$

the components are perpendicular...

Example: The angle between 2 vectors is 45 degrees...

If one vector is $\langle 3, 4 \rangle$ and the other vector is $\langle 2, k \rangle$,

then what is k?

$$\cos(45^\circ) = \frac{\langle 3, 4 \rangle \cdot \langle 2, k \rangle}{\sqrt{25}\,\sqrt{(k^2 + 4)}} \Rightarrow \frac{\sqrt{2}}{2} = \frac{4k + 6}{5\sqrt{(k^2+4)}}$$

angle between vectors:

$$\cos \theta = \frac{u \cdot v}{|u|\,|v|}$$

$$5\sqrt{2k^2 + 8} = 8k + 12$$

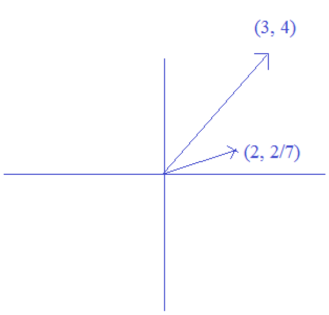
$$50k^2 + 200 = 64k^2 + 192k + 144$$

$$14k^2 + 192k - 56 = 0$$

$$7k^2 + 96k - 28 = 0$$

$$(7k - 2)(k + 14) = 0$$

$$\boxed{k = 2/7} \text{ or } \cancel{-14}$$



check the answers...

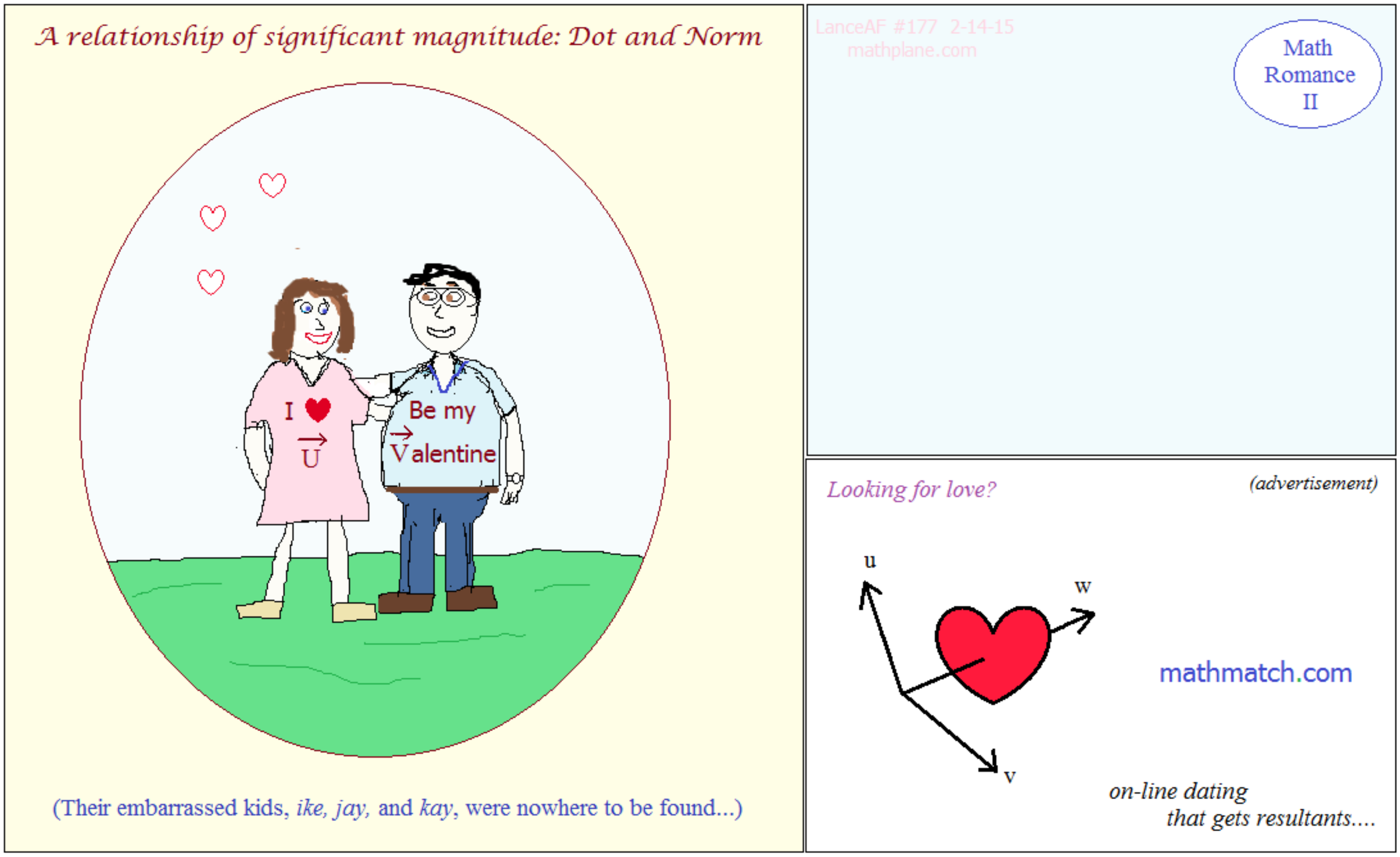
angle of $\langle 3, 4 \rangle$

$$\tan^{-1}(4/3) = 53.13^\circ$$

angle of $\langle 2, 2/7 \rangle$

$$\tan^{-1}\left(\frac{2/7}{2}\right) = 8.13^\circ$$

45 degree difference ✓



Practice Test-→

Vectors Questions

1) Find the unit vector of $\vec{v} = 2i - 5j$

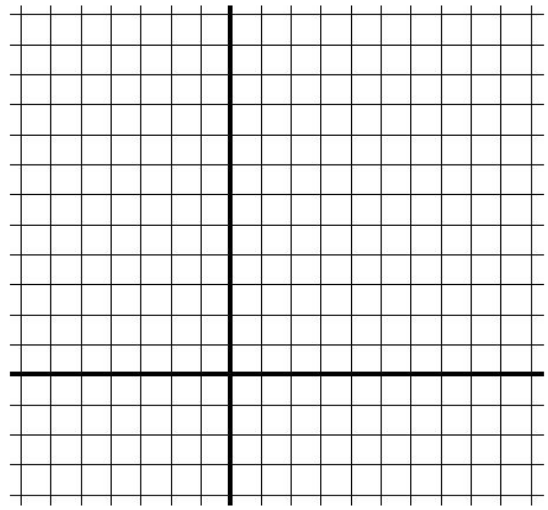
2) For the given vector $\vec{u} = < -4, 9 >$

a) What is the length of the vector?

b) Find the unit vector of $\vec{u}$

c) Determine a vector in the same direction with length 5...

d) Graph the vectors...



3) For the given vectors $v = < 3, -4 >$ $w = < 1, K >$

a) If v and w are parallel, what is K?

b) If v and w are orthogonal (or perpendicular), what is K?

4) Find the vector with the given magnitude and direction:

$\| v \| = 8$ in the direction $(5, 6)$

5) Given the following magnitudes:

$\| F_1 \| = 72$ $\| F_2 \| = 38$

$\| F_1 + F_2 \| = 93$

what is the angle between the vectors?

Vectors Questions

6) Given the vectors in component form

$$u = \langle -4, -4 \rangle \quad v = \langle 4, 2 \rangle$$

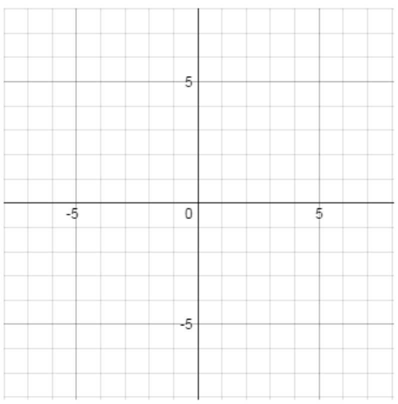
Find a) $u + v$

b) $u + 3v$

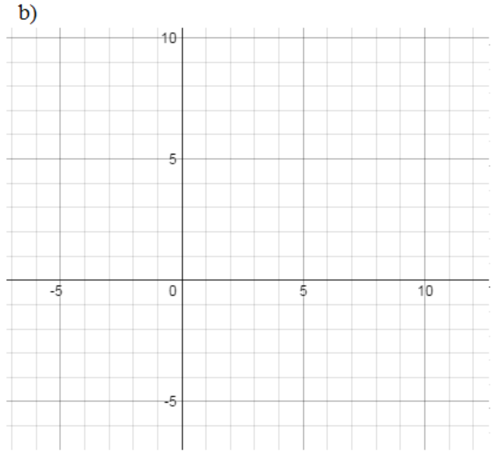
c) $u - 2v$

Answer both graphically and using the components...

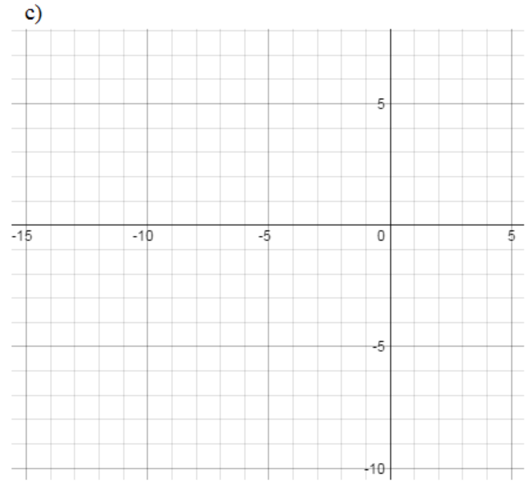
a)



b)



c)



7) A plane flying N52W at an *air speed* of 340 miles per hour has a *ground speed* of 325 miles per hour going N47W. What is the bearing and speed of the wind?

AIR SPEED + WIND SPEED = GROUND SPEED

Vector Questions

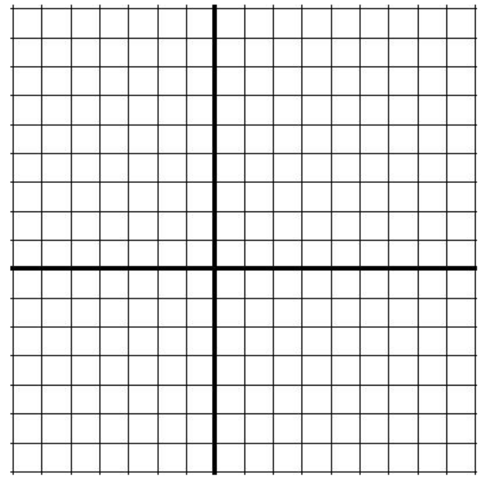
8) $u = \langle -3, 2 \rangle$ $v = \langle 4, 8 \rangle$

find $(u \cdot v)v$

9) Graph the vectors and find the angle between them...

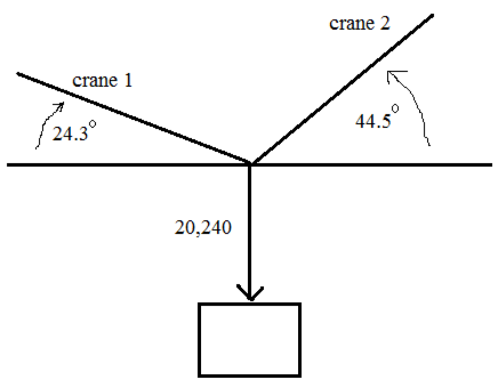
$u = 2i + 4j$

$v = -3i + 5j$



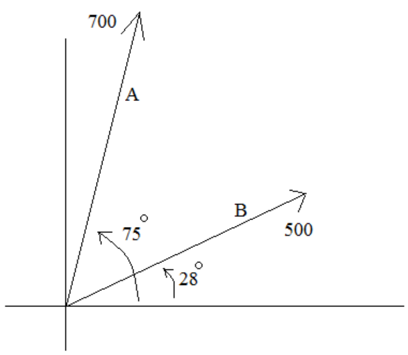
10) Two cranes lift an object. The diagram shows the known angles and weight.

Find the force of each crane:



Vectors Questions

11) Find the magnitude and direction of the resultant vector of vectors A and B.

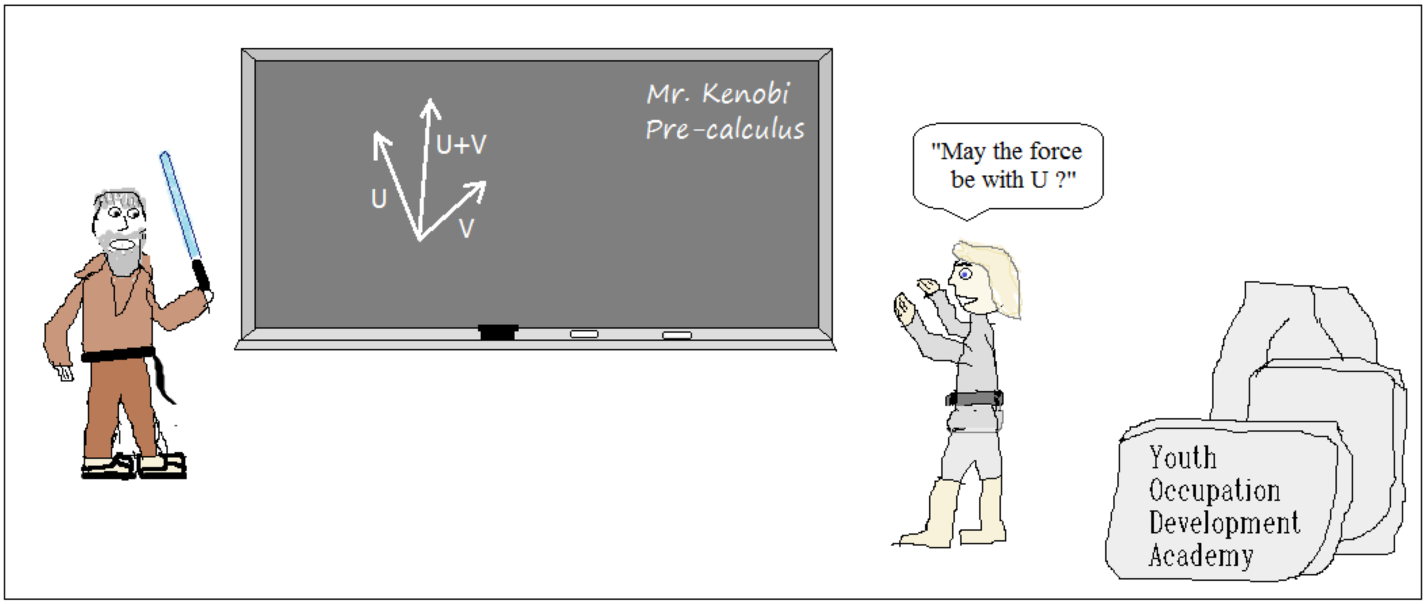


12) Given the following forces: Force 1: 55 lbs Force 2: 80 lbs The resultant force: 125 lbs

Find the angle between the forces...

A long time ago,
in a classroom
far, far away...

Math Lessons from the Jedi



LanceAF #72 2-17-13
www.mathplane.com

Obi-Wan teaches Luke about resultant vectors and (the) force

Solutions -→

SOLUTION

1) Find the unit vector of $\vec{v} = 2i - 5j$

unit vector $\hat{v} = \dfrac{v}{\| v \|}$ $\| v \| = \sqrt{2^2 + (-5)^2} = \sqrt{29}$

$$\boxed{\hat{v} = \frac{2}{\sqrt{29}}\, i - \frac{5}{\sqrt{29}}\, j}$$

2) For the given vector $\vec{u} = \langle -4, 9 \rangle$

a) What is the length of the vector?

$\| v \| = \sqrt{(-4)^2 + (9)^2} = \boxed{\sqrt{97}}$

b) Find the unit vector of $\vec{u}$

unit vector $= \dfrac{u}{\| u \|}$

unit vector $\vec{u}$

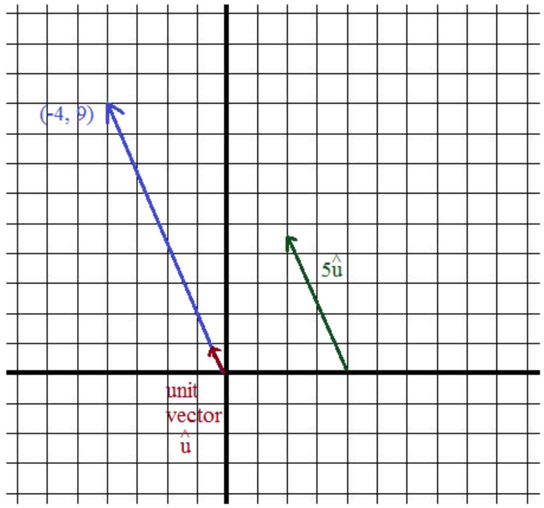
$$\hat{u} = \boxed{\left\langle \frac{-4}{\sqrt{97}}, \frac{9}{\sqrt{97}} \right\rangle}$$

c) Determine a vector in the same direction with length 5...

Since unit vector has length 1,
a vector in the same direction with length 5:

$$5 \cdot \hat{u} \quad \boxed{\left\langle \frac{-20}{\sqrt{97}}, \frac{45}{\sqrt{97}} \right\rangle}$$

d) Graph the vectors...



3) For the given vectors $v = \langle 3, -4 \rangle$ $w = \langle 1, K \rangle$

a) If v and w are parallel, what is K?

If the vectors are parallel, they are going in the same direction..
therefore, there 'slopes' are the same...

direction (slope) of v $= \dfrac{-4}{3}$ direction of w $= \dfrac{K}{1}$ $\boxed{K = -4/3}$

b) If v and w are orthogonal (or perpendicular), what is K?

If the vectors are orthogonal, then the dot product equals 0.

$v \cdot w = (3)(1) + (-4)(K) = 0$

$3 - 4K = 0$ $\boxed{K = 3/4}$

4) Find the vector with the given magnitude and direction:

$\| v \| = 8$ in the direction $(5, 6)$

first, find the unit vector in the direction (5, 6): $\left\langle \dfrac{5}{\sqrt{61}}, \dfrac{6}{\sqrt{61}} \right\rangle$ (magnitude = 1)

then, adjust it to the correct magnitude:

$$8 \left\langle \frac{5}{\sqrt{61}}, \frac{6}{\sqrt{61}} \right\rangle = \boxed{\left\langle \frac{40}{\sqrt{61}}, \frac{48}{\sqrt{61}} \right\rangle \text{ or } \frac{40}{\sqrt{61}}\, i + \frac{48}{\sqrt{61}}\, j}$$

5) Given the following magnitudes:

$\| F_1 \| = 72$ $\| F_2 \| = 38$

$\| F_1 + F_2 \| = 93$

what is the angle between the vectors?

Use law of cosines...

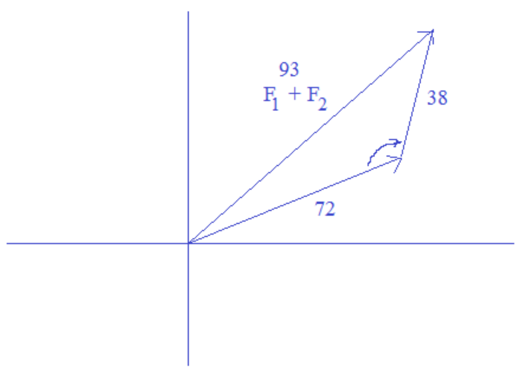
$93^2 = 72^2 + 38^2 - 2(72)(38)\cos\theta$

$8649 = 5184 + 1444 - 5472\cos\theta$

$2021 = -5472\cos\theta$ $\boxed{\theta = 111.7^\circ}$

$-.369 = \cos\theta$

(possible sketch)



Vectors Questions

SOLUTIONS

6) Given the vectors in component form

u = < -4, -4 >   v = < 4, 2 >

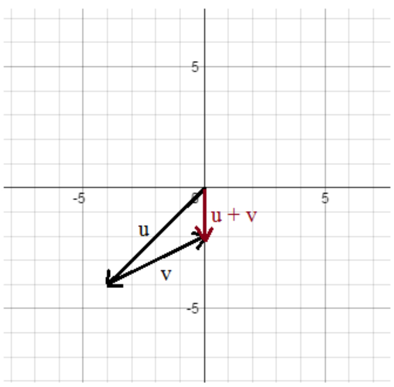
Find a) u + v   < -4, -4 > + < 4, 2 > = < 0, -2 >

b) u + 3v   < -4, -4 > + < 12, 6 > = < 8, 2 >

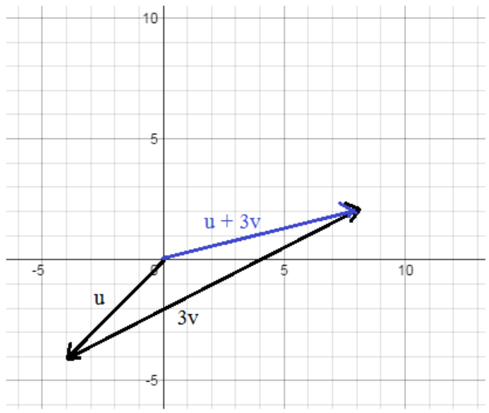
c) u − 2v   < -4, -4 > − < 8, 4 > = < -12, -8 >

Answer both graphically and using the components...

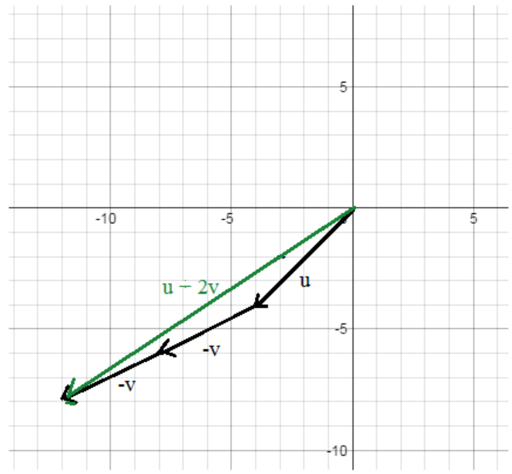
a)



b)



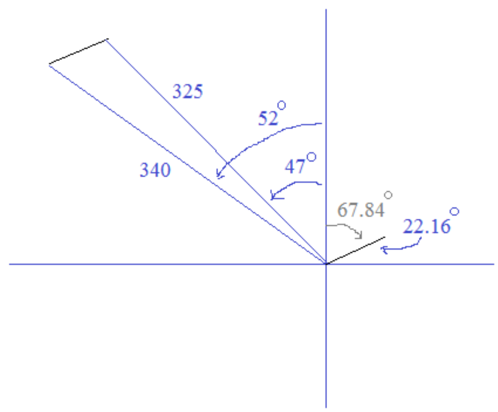
c)



7) A plane flying N52W at an *air speed* of 340 miles per hour has a *ground speed* of 325 miles per hour going N47W. What is the bearing and speed of the wind?

AIR SPEED + WIND SPEED = GROUND SPEED

Step 1: Draw a diagram



Step 2: Determine the component vectors

$< r\cos\theta \,,\, r\sin\theta >$

$< 340\cos(142^\circ), 340\sin(142^\circ) >$

$< -267.9, 209.3 >$

$< 325\cos(137^\circ), 325\sin(137^\circ) >$

$< -237.7, 221.6 >$

NOTE: we changed the orientation from navigation bearings to geometry plane

EX: N52W becomes 142°

Step 3: Find resultant vector and magnitude...

Air speed + Wind speed = Ground Speed

$< -267.9, 209.3 > + < W_i \,, W_j > = < -237.7, 221.6 >$

Wind vector = < 30.2, 12.3 >

$\| W \| = \sqrt{30.2^2 + 12.3^2} = \boxed{32.6 \text{ mph}}$

Step 4: Find the wind direction

$\tan^{-1} \dfrac{12.3}{30.2} = \theta$

$= 22.16^\circ$

since the angle is 22.16, the <u>bearing</u> is

90 - 22.16 ----> $\boxed{\text{N67.84E}}$

8) $u = \langle -3, 2 \rangle$ $\quad v = \langle 4, 8 \rangle$

find $(u \cdot v)v$

(order of operations -- parenthesis first)

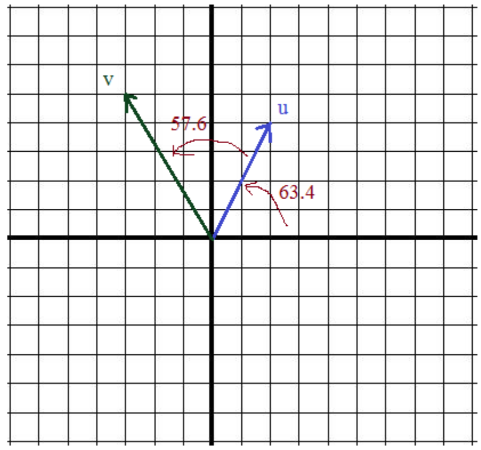
$u \cdot v = (-3 \times 4) + (2 \times 8) = 4$

$4v = \boxed{\langle 16, 32 \rangle}$

9) Graph the vectors and find the angle between them...

$u = 2i + 4j$

$v = -3i + 5j$



$$\boxed{\cos\theta = \frac{u \cdot v}{|u|\,|v|}} = \frac{(2 \times (-3)) + (4 \times 5)}{\sqrt{20} \times \sqrt{34}} = \frac{14}{26.08}$$

$$\theta = \cos^{-1}\left(\frac{14}{26.08}\right) = \boxed{57.5 \text{ degrees}}$$

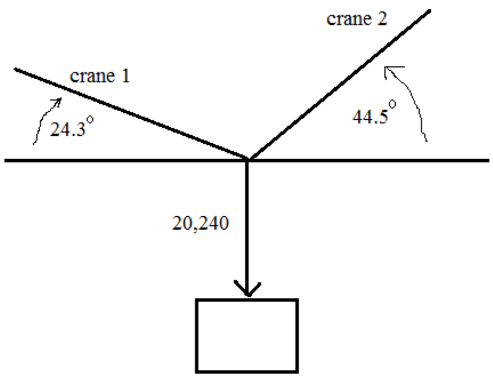
Alternate method:

$u = \langle 2, 4 \rangle$ $\quad \tan^{-1}\left(\frac{4}{2}\right) = 63.4$ degrees $\quad$ $121 - 63.4 = 57.6$ ✓

$v = \langle -3, 5 \rangle$ $\quad \tan^{-1}\left(\frac{5}{-3}\right) = -59.0 + 180 = 121$ degrees

10) Two cranes lift an object. The diagram shows the known angles and weight.

Find the force of each crane:



c_1 = force of crane 1

c_2 = force of crane 2

Using component vectors:

$$\langle c_2 \cos(44.5),\ c_2 \sin(44.5) \rangle$$
$$+$$
$$\langle c_1 \cos(155.7),\ c_1 \sin(155.7) \rangle$$
$$=$$
$$\langle 20{,}240\cos(270),\ 20{,}240\sin(270) \rangle$$

$$\langle .7133c_2,\ .7009c_2 \rangle$$
$$+$$
$$\langle -.9114c_1,\ .4115c_1 \rangle$$
$$=$$
$$\langle 0,\ -20{,}240 \rangle$$

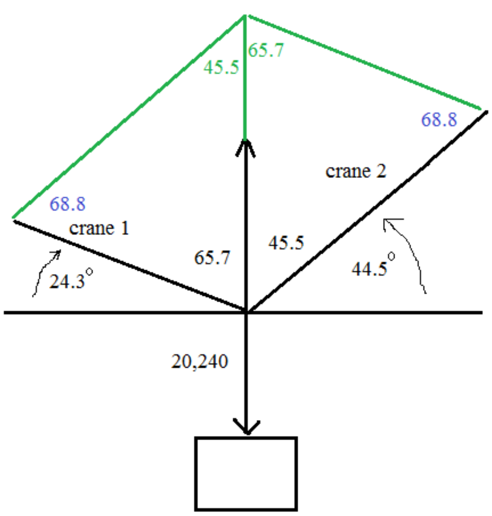
$.7133c_2 + -.9114c_1 = 0$

$.7009c_2 + .4115c_1 = -20{,}240$

solve system of equations

$$\boxed{\begin{array}{l} c_2 = -19{,}786 \\ c_1 = -15{,}485 \end{array}}$$

Using trigonometry, geometry, and law of sines:



Note: consecutive angles of parallelogram must be supplementary...

$$\frac{\sin(65.7)}{c_2} = \frac{\sin(68.8)}{20{,}240}$$

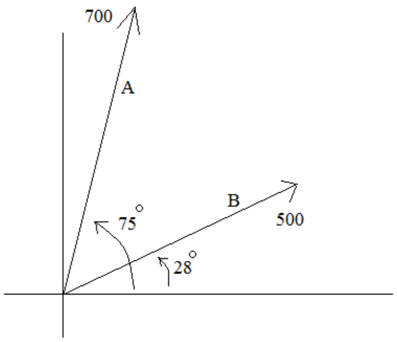
$$\frac{\sin(45.5)}{c_1} = \frac{\sin(68.8)}{20{,}240}$$

$$\boxed{\begin{array}{l} \text{crane 2} = 19{,}785 \\ \text{crane 1} = 15{,}484 \end{array}}$$

Vectors Questions

SOLUTIONS

11) Find the magnitude and direction of the resultant vector of vectors A and B.



Method 1: Convert to component vectors

28 degrees and magnitude 500 $\Rightarrow$ $\langle 500\cos(28), 500\sin(28) \rangle$ $\langle 441.47, 234.74 \rangle$

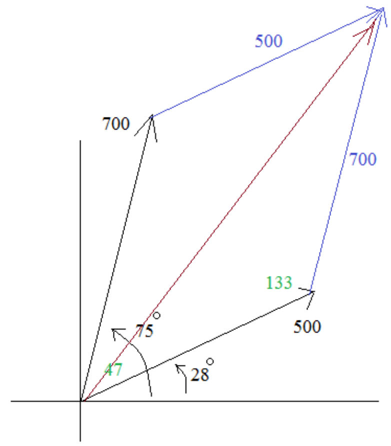
75 degrees and magnifude 700 $\Rightarrow$ $\langle 700\cos(75), 700\sin(75) \rangle$ $\langle 181.17, 676.15 \rangle$

add the vectors.....

$\langle 622.65, 910.89 \rangle$

$$\boxed{\begin{array}{l} \tan^{-1}\left(\dfrac{910.89}{622.65}\right) = 55.64^\circ \\ \text{magnitude: } \sqrt{622.65^2 + 910.89^2} = 1103.36 \text{ (approximately)} \end{array}}$$

Method 2: Using law of sines and cosines and parallelogram



Consecutive angles of a parallelogram are supplementary...

Using law of cosines where sides are 500 and 700,
and the included angle is 133 degrees

$$\text{magnitude}^2 = 500^2 + 700^2 - 2(500)(700)\cos(133^\circ)$$

$$\text{magnitude} = 1103.36$$

Then, use law of sines to get the angle direction...

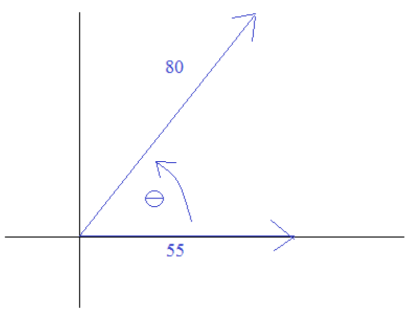
$$\frac{\sin A}{700} = \frac{\sin(133)}{1103.36} \qquad A = 27.64^\circ$$

Therefore, direction of vector is $27.64 + 28 = 55.64^\circ$

12) Given the following forces: Force 1: 55 lbs Force 2: 80 lbs The resultant force: 125 lbs

Find the angle between the forces...

Step 1: Draw a diagram



Note: we intentionally placed one vector on the x-axis...

Step 2: Convert the forces into vectors

Force 1: $\langle 55, 0 \rangle$

Force 2: $\langle 80\cos(\theta), 80\sin(\theta) \rangle$

Step 3: Find the resultant

First, add the forces: $\langle 55 + 80\cos(\theta), 80\sin(\theta) \rangle$

We know the magnitude of the resultant is $\sqrt{(55 + 80\cos(\theta))^2 + (80\sin(\theta))^2} = 125$

$$(55 + 80\cos(\theta))^2 + (80\sin(\theta))^2 = 15625$$

$$3025 + 8800\cos(\theta) + \underbrace{6400\cos^2\theta + 6400\sin^2\theta}_{6400} = 15625$$

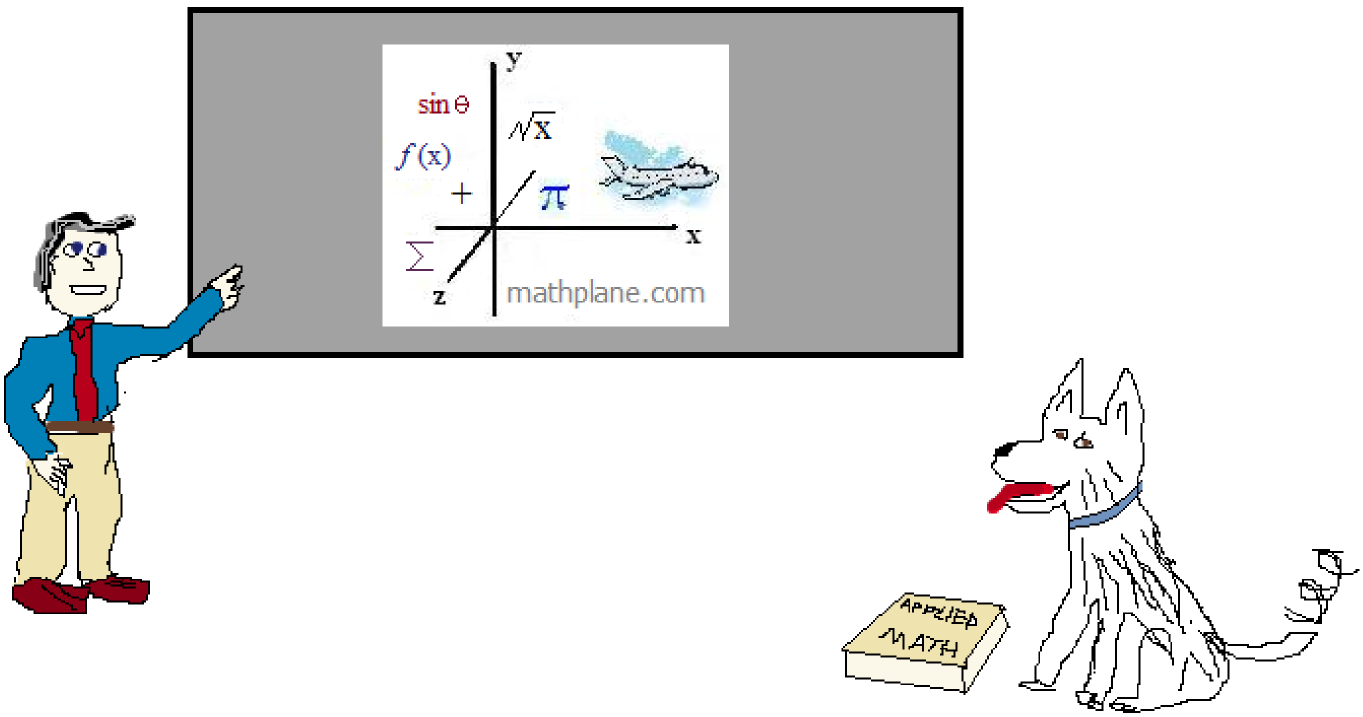
$$8800\cos(\theta) = 6200$$

$$\boxed{45.2 \text{ degrees}}$$

Thanks for visiting. (Hope it helps!)

If you have questions, suggestions, or requests, let us know.

Cheers



Also, at TeachersPayTeachers…

And, Mathplane.ORG for mobile and tablets

One more question…

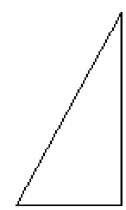
Find the component form of the vector V if $\|V\| = 8$, and the angle it makes with the *x-axis* is 60 degrees…

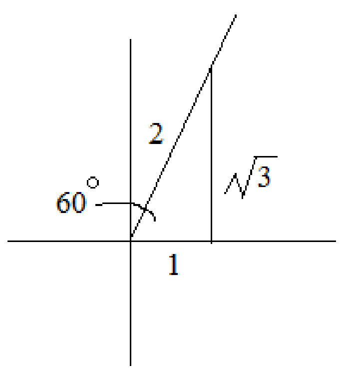
Answer on the next page→

Find the component form of the vector V if $\|V\| = 8$

and the angle it makes with the *x-axis* is 60 degrees.

30-60-90 right triangle





$i + \sqrt{3}\ j = V$

$\|V\| = 2$

Unit vector: $\frac{i}{2} + \frac{\sqrt{3}\ j}{2}$

Unit vector x 8 = $4i + 4\sqrt{3}\,j$

60 degree angle and length 8!!