

Intermediate Algebra Word Problems

Examples and Practice Questions

Example: A pot contains 6 quarts of brine at a concentration of 150 grams/quart.
How much water should boil off to increase the concentration to 200 grams/quart?

The focus is concentration (of salt)

	amount	%	Total
Original	6	150 g/q	900 g
burn off	-X	0	0
Mixture/ concentration	6 - X	200 g/q	900 g

$$(6 - X)(200) = 900$$

$$1200 - 200X = 900$$

$$300 = 200X$$

$X = 1.5 \text{ quarts}$

Example: A party has 6 liters in the punch bowl, consisting of 40% seltzer and 60% juice.
If the host adds 1 liter of seltzer, what is the % of seltzer in the punch bowl?

before: 40% of 6 = 1.6 liters seltzer
after: 1.6 + 1 = 2.6 liters of seltzer
2.6 out of 7 liters of punch...

37.1%

Example: A chemist has 2 beakers of chemicals with acid.
If he blends 300 mL from beaker 1 with 600 mL from beaker 2, he ends up with a 15% solution of acid.
If he blends 100 mL from beaker 1 with 500 mL from beaker 2, he ends up with a 12.5% solution of acid.

What are the acid concentrations in each beaker?

$$300X + 600Y = (.15)900$$

$$100X + 500Y = (.125)600$$

$$300X + 600Y = 135$$

$$100X + 500Y = 75$$

$$-300X + -1500Y = -225$$

$$-900Y = -90$$

$$Y = .10$$

$$X = .25$$

$10\% \text{ in beaker 2}$

 $25\% \text{ in beaker 1}$

Example: A 1.5 liter bottle of fruit punch contains 50% pure fruit juice.
Kathy drinks 100 mL. Then, she refills the bottle with a cheaper brand that contains less pure fruit juice.

If the refilled 1.5 liter bottle contains 48% pure fruit, then what is the concentrate of pure fruit in the cheaper brand?

The focus of the problem is "pure fruit juice"...

	amount	%	Total
Original bottle	1500 mL	.50	750
"Kathy drinks"	-100 mL	.50	-50
the "refill"	+100 mL	X	100X
Mixed bottle	1500 mL	.48	720

$$1500(.50) - 100(.50) + 100(X) = 1500(.48)$$

$$750 - 50 + 100X = 720$$

$$100X = 20$$

$X = .20$

The cheaper brand contains 20% pure fruit juice...

- 4) Theophylline, an asthma medicine, is to be prepared from an elixir with a drug concentration of 5 mg/mL and a cherry-flavored syrup that is added to hide the taste of the drug... How much of each must be used to prepare 100 mL of solution with a drug concentration of 2 mg/mL?

- 5) Tom is currently twice Jerry's age.
Five years ago, Tom was three times Jerry's age.
How old is each person now?

- 6) Working together, Charlie and Lucy can paint a room in 4 hours.
Separately, it takes Charlie 3 hours longer to paint a room than it does Lucy.

How long does it take for each person to paint a room alone?

- 7) Scrooge has 50 coins in his pocket.
They are nickels, dimes, and quarters.
There are twice as many dimes as quarters, and the
total amount is \$6.10.

How many of each coin does Scrooge have?

- 8) A guy will surround a rectangular garden with 150 feet of fencing.
If he wants the area to be greater than 1250 square feet, what are the possible lengths?

- 9) A company has 2 water pumps for draining their tanks..
The large pump can drain the tank in 5 hours. And, the
small pump can drain the tank in 8 hours.
At Noon, the foreman turns on the large pump. If he
wants to finish draining the tank and leave at 4:00, what is the
latest time he must turn on the small pump?

10) Your uncle has a bunch of money in his pocket.

He gives $\frac{1}{2}$ of the money in his pocket to your sister....

Then, he gives $\frac{1}{4}$ of what's left in his pocket to your brother....

Then, he gives $\frac{1}{3}$ of what's left in his pocket to your friend...

Then, he gives $\frac{1}{2}$ of what's left in his pocket to you....

If he has \$3 left, how much was in his pocket in the beginning?

11) Pump A can fill a tank in 7 hours...

And, Pump B could fill a tank in 4 hours....

When the full tank is drained, it takes 22 hours to empty through the drainage hole...

Philip decides to fill the empty tank by turning both pumps on...

Unfortunately, he forgot to close the drainage hole!

How long will it take to fill the tank?

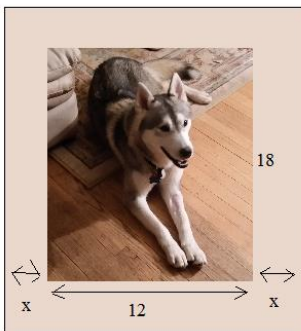
12) In a medical test, an adult drinks 7 ounces of a 30% glucose solution.

When the test is administered to a child, the glucose concentration must be decreased to 20%. How much 30% glucose solution and how much water should be used to prepare 7 ounces of 20% glucose solution?

- 13) You're looking to construct an outdoor enclosed area.
There are 2 choices: a square area or a circular area.
The material needed for the enclosure costs \$3 per foot for straight borders and \$4 per foot for bendable curved borders.
- If you plan to use 40 feet of material, which area will be larger?
 - If you plan to spend \$180, which area would be larger?

- 14) A square plot of land has a building 80 feet by 30 feet located in the northeast corner.
The rest of the land is a 12,000 square foot parking lot.
What are the dimensions of the plot of land?

- 15) In my house, I have an 18" x 12" photo of my dog that is placed on a mat with a uniform border.
If the perimeter of the framed piece is 88 inches, then how wide is the strip?




Berger Middle School
Since 1948
Over 1 billion taught...

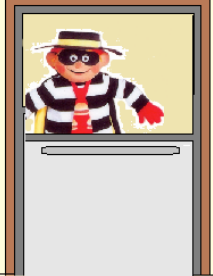
"...Everyone has to take the exam again, because someone stole the answer key..."

$y = mx + b$

Mr. McDonald
Algebra 1



(slope)



I'll bet it was the hamburger.

"Ray, this is a crock! I don't wanna take the test again.."

Geez, our teacher is a clown, and this class is a joke..



Algebra

McAlgebra Class

SOLUTIONS-→

- 1) A school play sells 133 tickets, raising \$490...
 The theater group charged \$7 for adults, \$3 for children, and \$2 for students...
 If they sold twice as many children tickets as adult tickets,
 how many student tickets, did they sell?

Linear System of
3 equations and 3 unknowns

Step 1: Identify variables

Let A = # of adult tickets
 C = # of children tickets
 S = # of student tickets

Step 3: Solve (and, answer question)

Using substitution: $A + (2A) + S = 133$
 $7A + 3(2A) + 2S = 490$

$3A + S = 133$
 $13A + 2S = 490$

$13A + 2(133 - 3A) = 490$
 $7A = 224$
 $A = 32$

Step 2: Determine equations

$A + C + S = 133$ "sells 133 tickets"
 $7A + 3C + 2S = 490$ "raising \$490"
 $2A = C$ "sold twice as many..."

$2A = C$
 $64 = C$

$A + C + S = 133$
 $32 + 64 + S = 133$ $S = 37$

3 equations, 3 unknowns

Step 4: Check

32 adult	224
64 children	192
37 students	74
133	\$490 ✓

- 2) At home, you have a 10 ounce glass of orange juice, which contains
 40% of pure juice and 60% of water and other juices...
 If you pour out some of the glass and replace it with pure orange juice,
 how much should you pour out to end up with a mixture of 70%
 orange juice and 30% water and juices?

Mixture question
(involving replacement)

Equation representing "pure oj content"

	amount	concentration	total OJ
start	10	40%	4
pour out	(10 - x)	40%	.4(10 - x)
add	x	100%	1x
finish	10	70%	7

starting cup pour out replace finishing cup

$10(.4) - x(.4) + x(1.0) = 10(.7)$
 $.4(10 - x) + x = 7$
 $4 - .4x + x = 7$
 $.6x = 3$

$x = 5$
 pour out 1/2 and
 replace with 5 ounces
 of pure OJ...

- 3) The product of two consecutive numbers is 14 less than 10 times the smaller number.
 What is the smaller number?

Algebra and Integers

Establish Variables Let x = smaller number...

Since they are consecutive numbers, let x + 1 = larger number...

"product of two consecutive numbers": $x(x + 1)$

"10 times the smaller number": $10x$

"is": =

Set up Equations

$x(x + 1) = 10x$

Then, since the product is less than, we'll add 14 to that side to create an equality...

$x(x + 1) + 14 = 10x$

Solve

$x^2 + x + 14 = 10x$

$x^2 - 9x + 14 = 0$

$(x - 2)(x - 7) = 0$

$x = 2$ or 7

Check Answers

If x = 2, then x + 1 = 3

$(2 \times 3) + 14 = 10(2)$

$20 = 20$ ✓

If x = 7, then x + 1 = 8

$(7 \times 8) + 14 = 10(7)$

$70 = 70$ ✓

Algebra 2 Word Problems

- 4) Theophylline, an asthma medicine, is to be prepared from an elixir with a drug concentration of 5 mg/mL and a cherry-flavored syrup that is added to hide the taste of the drug... How much of each must be used to prepare 100 mL of solution with a drug concentration of 2 mg/mL?

mixture problem

	Theo	=	Elixir	+	Syrup	
Amount	100 mL		X mL		Y mL	X = amount of Elixir Y = amount of Syrup
drug Concentration	2 mg/1 mL		5 mg/1 mL		0 mg/1 mL	X + Y = 100 mL (quantity of inputs) 5X mg + 0Y mg = 200 mg (amount of drug)
Total drug	200 mg		5X mg		0Y mg	(Now, solve the system of linear equations)

(If there is 100mL of medicine, there must be 200 mg of the drug..)

X = 40 mL of elixir
Y = 60 mL of elixir

Quick check: 60 mL of syrup contains 0 medicine...
40 mL of elixir will contain 200 mg of medicine
100 mL of 200 mg of medicine ✓

- 5) Tom is currently twice Jerry's age. Five years ago, Tom was three times Jerry's age. How old is each person now?

Age algebra question

Let T = Tom's age T = 2J "currently twice..."
J = Jerry's age (T - 5) = 3 · (J - 5) "Five years ago,..."

T - 5 = 3J - 15
2J - 5 = 3J - 15

J = 10
so, T = 20

- 6) Working together, Charlie and Lucy can paint a room in 4 hours. Separately, it takes Charlie 3 hours longer to paint a room than it does Lucy.

Work Problem

How long does it take for each person to paint a room alone?

"distance = rate x time"

Charlie: 1 room = rate · (t + 3) $r_c = \frac{1 \text{ room}}{(t + 3) \text{ hours}}$

Lucy: 1 room = rate · (t) $r_L = \frac{1 \text{ room}}{t \text{ hours}}$

Working together:

Charlie Lucy
(rate_c)(time) + (rate_L)(time) = 1 room

$\frac{1 \text{ room}}{(t + 3) \text{ hours}} (4 \text{ hours}) + \frac{1 \text{ room}}{t \text{ hours}} (4 \text{ hours}) = 1 \text{ room}$

$\frac{4}{t + 3} + \frac{4}{t} = 1$

$\frac{4t + 4(t + 3)}{(t + 3)(t)} = 1$

$8t + 12 = t^2 + 3t$

$t^2 - 5t - 12 = 0$ $t \approx -1.77$ or 6.77

since t cannot be negative,

Charlie takes 9.77 hours 9 hr 46 min
Lucy takes 6.77 hours 6 hr 46 min

quick check:

In 4 hours: lucy paints $4 \cdot \frac{1}{6.77} = .59$ room

In 4 hours: charlie paints $4 \cdot \frac{1}{9.77} = .41$ room ✓

- 7) Scrooge has 50 coins in his pocket. They are nickels, dimes, and quarters. There are twice as many dimes as quarters, and the total amount is \$6.10.

System of Linear Equations

How many of each coin does Scrooge have?

Let $N = \#$ of nickels $N + D + Q = 50$ (number of coins)
 $D = \#$ of dimes $.05N + .10D + .25Q = 6.10$ (amount of coins)
 $Q = \#$ of quarters $D = 2Q$ (twice as many dimes as quarters)

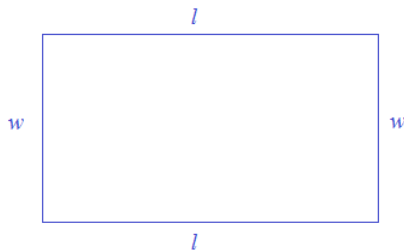
$$\begin{aligned} N + (2Q) + Q &= 50 \\ 5N + 10(2Q) + 25Q &= 610 \\ 5N + 15Q &= 250 \\ - 5N + 45Q &= 610 \\ \hline -30Q &= -360 \\ Q &= 12 \end{aligned}$$

Then, $D = 2(12) = 24$

And, $N + (24) + (12) = 50$
 $N = 14$

- 8) A guy will surround a rectangular garden with 150 feet of fencing. If he wants the area to be greater than 1250 square feet, what are the possible lengths?

Area Inequalities (restrictions)



$$\begin{aligned} 2l + 2w &= 150 \text{ feet} \\ lw &> 1250 \text{ square feet} \\ l + w &= 75 \text{ feet} \\ w &= 75 \text{ feet} - l \end{aligned}$$

$$\begin{aligned} l(75 \text{ feet} - l) &> 1250 \text{ sq feet} \\ 75l - l^2 &> 1250 \\ l^2 - 75l + 1250 &< 0 \\ (l - 25)(l - 50) &< 0 \end{aligned}$$



$25 < l < 50$

- 9) A company has 2 water pumps for draining their tanks.. The large pump can drain the tank in 5 hours. And, the small pump can drain the tank in 8 hours. At Noon, the foreman turns on the large pump. If he wants to finish draining the tank and leave at 4:00, what is the latest time he must turn on the small pump?

Work Problem

$$\begin{aligned} \frac{1}{5}t + \frac{1}{8}t &= 1 \\ \text{large pump runs for full 4 hours} \\ \frac{4}{5} + \frac{1}{8}t &= 1 \\ \frac{1}{8}t &= \frac{1}{5} \end{aligned}$$

$t = 8/5$ hours

small pump must run for 1 hour 36 minutes

2:24 is the latest time he must turn on the small pump!

10) Your uncle has a bunch of money in his pocket.

SOLUTIONS

He gives 1/2 of the money in his pocket to your sister....
 Then, he gives 1/4 of what's left in his pocket to your brother....
 Then, he gives 1/3 of what's left in his pocket to your friend...
 Then, he gives 1/2 of what's left in his pocket to you....
 If he has \$3 left, how much was in his pocket in the beginning?

Let X = initial amount of money...

$$(1/2) \left((2/3) \left((3/4) \left((1/2)X \right) \right) \right) = \$3$$

$$\frac{1}{8} x = 3 \quad \boxed{X = \$24}$$

working backward:
 \$3 at the end...
 So, he gave you \$3... (\$6)
 So, he gave \$3 to your friend (\$9)
 So, he gave \$3 to your brother (\$12)
 So, he gave \$12 to your sister (\$24)

11) Pump A can fill a tank in 7 hours...
 And, Pump B could fill a tank in 4 hours....
 When the full tank is drained, it takes 22 hours to empty through the drainage hole...

Work Problem

Philip decides to fill the empty tank by turning both pumps on...
 Unfortunately, he forgot to close the drainage hole!

How long will it take to fill the tank?

pumpA pumpB drainage

$$\frac{1 \text{ tank}}{7 \text{ hours}} t + \frac{1 \text{ tank}}{4 \text{ hours}} t - \frac{1 \text{ tank}}{22 \text{ hours}} t = 1 \text{ tank}$$

multiply by $\frac{308 \text{ hours}}{\text{tank}}$

$$44t + 77t - 14t = 308 \text{ hours}$$

$$t = 2.88$$

2.88 x (1/7)	.41
2.88 x (1/4)	.72
2.88 x (1/22)	-.13
= 1.00 ✓	

approx. 2 hours 53 minutes

12) In a medical test, an adult drinks 7 ounces of a 30% glucose solution.
 When the test is administered to a child, the glucose concentration must be decreased to 20%. How much 30% glucose solution and how much water should be used to prepare 7 ounces of 20% glucose solution?

Mixture Problem

Using a table to express the mixtures

	Solution amt.	% of glucose	amount of glucose
Adult	A ounces	30%	.30A
+			
Water	W ounces	0%	0 x W = 0 ounces
Child	A + W = 7	20%	.20 x 7 = 1.4 ounces

Write a system of equations...

We need 7 ounces of the child's solution:

A + W = 7 ounces (We have an equation with 2 variables.. So, we need one more equation.)

the amount of glucose:

.30A + 0W = 1.4 ounces 7 x .20 = 1.4 ounces...

A + W = 7

.3A = 1.4

A = 4 2/3

W = 2 1/3

quick check:

20% of 7 ounces is 1.4 ounces

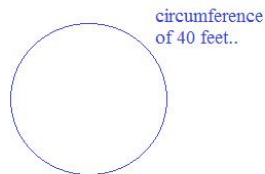
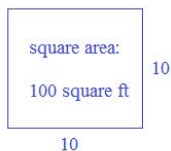
and, 30% of 4.667 is 1.4 ounces

- 13) You're looking to construct an outdoor enclosed area. There are 2 choices: a square area or a circular area.

The material needed for the enclosure costs \$3 per foot for straight borders and \$4 per foot for bendable curved borders.

- a) If you plan to use 40 feet of material, which area will be larger?
 b) If you plan to spend \$180, which area would be larger?

a) If you use 40 feet of material...



$$C = 2\pi(\text{radius})$$

$$40 = 2\pi(\text{radius})$$

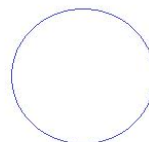
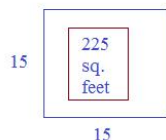
$$\text{radius} = 6.37$$

$$\text{area} = \pi(\text{radius})^2$$

$$\text{area} = 127 \text{ feet (approx)}$$

b) if you plan to spend \$180.....

at \$3 per square foot,
60 feet of border...



at \$4 per square foot,

can afford 45 feet of bendable material...

$$C = 2\pi(\text{radius})$$

$$45 = 2\pi(\text{radius})$$

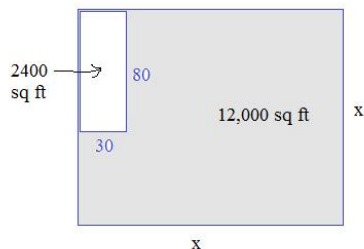
$$\text{radius} = 7.16$$

$$\text{area} = \pi(\text{radius})^2$$

$$\text{area} = 161 \text{ feet (approx)}$$

If length of material is constraint, then round area is greater...
 But, if cost of material is constraint, then square area is greater!

- 14) A square plot of land has a building 80 feet by 30 feet located in the northeast corner. The rest of the land is a 12,000 square foot parking lot. What are the dimensions of the plot of land?



The area of the parking lot : 12,000

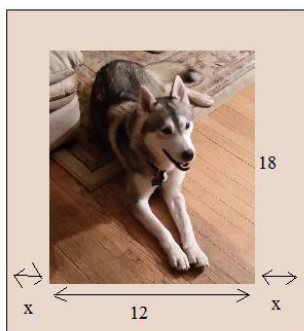
The area of the building: 2,400

So, the total area: 14,400

So, the dimensions of the square lot are

120 x 120

- 15) In my house, I have an 18" x 12" photo of my dog that is placed on a mat with a uniform border. If the perimeter of the framed piece is 88 inches, then how wide is the strip?



Perimeter of rectangle = 2(length) + 2(width)

$$88 = 2(2x + 12) + 2(2x + 18)$$

$$88 = 8x + 60$$

$$28 = 8x$$

$$x = 3.5 \text{ inches}$$