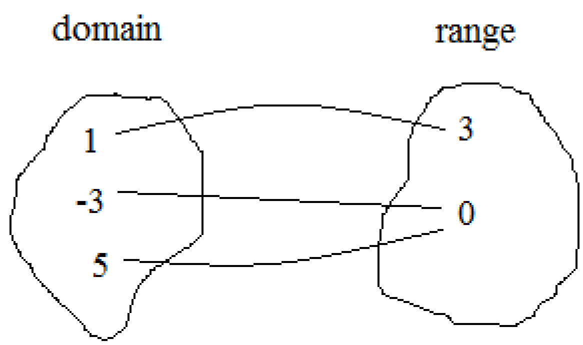
Domain/Range & Functions/Relations

Notes, examples, and quiz (with answers)



Topics include absolute value, graphs, radicals, vertical line test, piecewise functions, and more.

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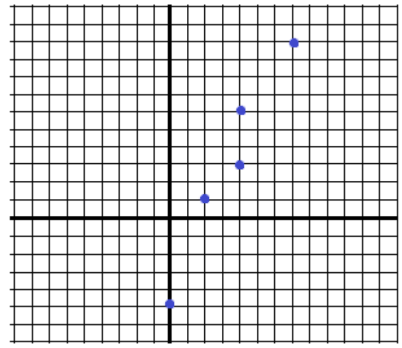
Domain & Range: Notes and Examples

What is the domain and range? Domain --- "*all the X values*"
Range --- "*all the Y values*"

Example: For the relation, (4, 3) (2, 1) (0, -5) (4, 6) (7, 10)

Domain = {0, 2, 4, 7}
Range = {-5, 1, 3, 6, 10}

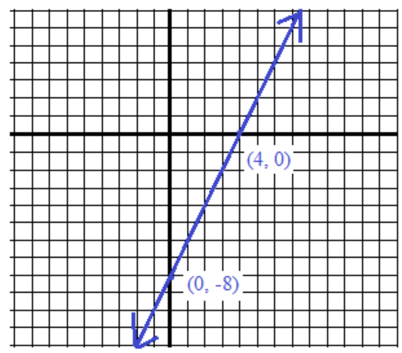
(numbers that are represented by points in the graph)



Example: For the function, $f(x) = 2x - 8$

Domain = {all real numbers}
Range = {all real numbers}

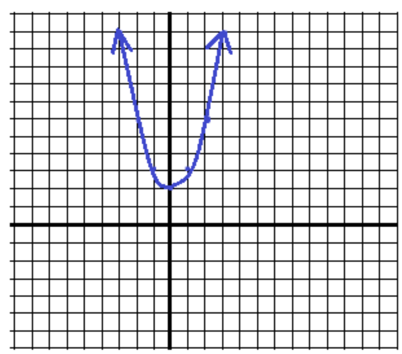
(any real number can go into x; and this will produce all possible real numbers)



Example: For the function, $y = x^2 + 2$

Domain = {all real numbers}
Range = $\{y \geq 2\}$

(any number can go into x; but, it will only produce real numbers that are ≥ 2)



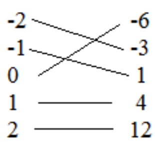
x	y
-3	11
-2	6
-1	3
0	2
1	3
2	6
3	11

Domain & Range: Notes and Examples

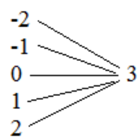
What is a relation? A function?

Relation --- a set containing pairs of related numbers (elements)
Function --- a relation where *for each input value (X), there is* *<u>only one</u> output value (Y)*

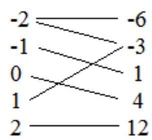
Note: All functions are relations... But, not all relations are functions.



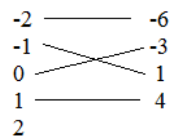
function



function



relation
(-2 has two range elements)

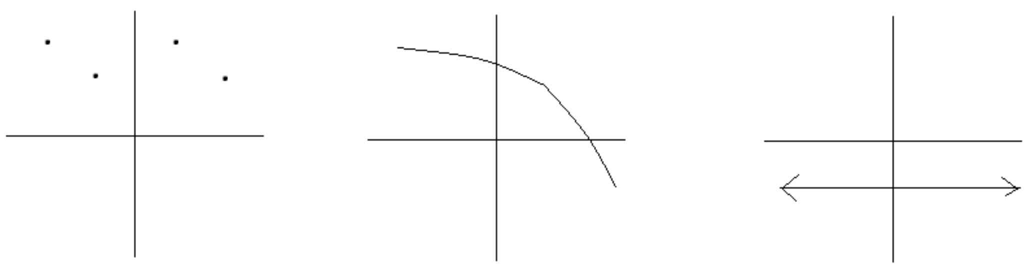


neither
(the 2 is in the domain; but, it has no range element!)

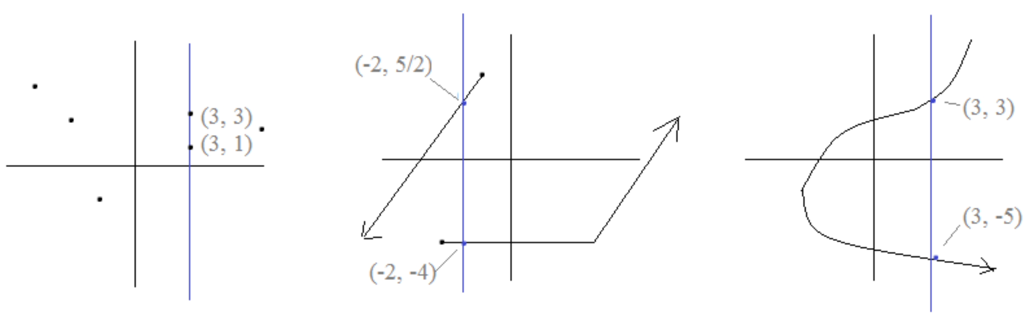
Vertical Line Test --- To determine if a set of pairs is a function, try the vertical line test.

Graph the set.
If you can draw a vertical line through *2 or more points*, then it is <u>not</u> a function.

(Note: "if a set of ordered pairs has 2 or more X values, then it is not a function")



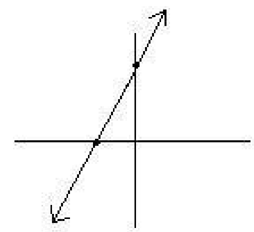
Functions: Every x has only one y value; passes vertical line test



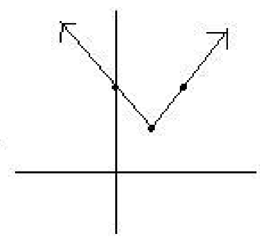
Relations that are not functions: sets of number pairs; but, each fails the vertical line test

Domain and Range examples and "exceptions"

1) $y = 3x + 5$ linear equation; domain: all real numbers
range: all real numbers
It is a function...



2) $f(x) = |x - 2| + 3$ absolute value equation;
domain: all real numbers
(any number can go into x)
range: all real numbers ≥ 3
(there is no possible way to get an output less than 3)
It is a function



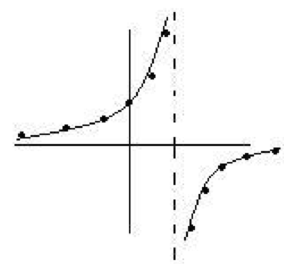
3) $f(x) = \dfrac{3}{2 - x}$

X	Y
-7	1/3
-4	1/2
-1	1
2	undefined
5	-1
8	-1/2
11	-1/3

domain: all real numbers $\neq 2$
(can't have 0 in denominator)
range: all real numbers $\neq 0$

It is a function

(passes vertical line test;
It has an asymptote at x=2,
so it is not continuous.
Nevertheless, it is a
function)

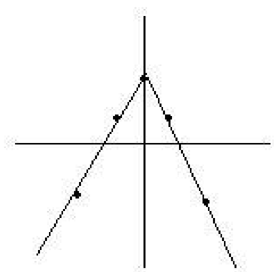


"Set Notation or Interval Notation"

4) $y = -2|x| + 3$

domain: all real numbers
D: {all real numbers}
domain: $(-\infty, \infty)$

range: all real numbers ≤ 3
set notation — R: {all real such that $y \leq 3$}
interval notation — range: $(-\infty, 3]$



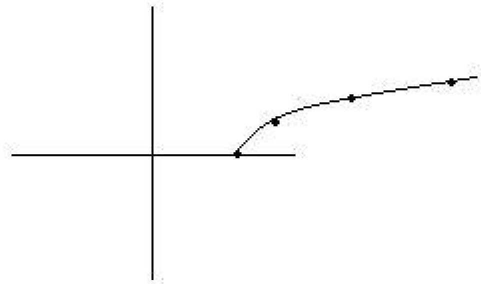
5) $f(x) = \sqrt{x - 3}$

domain: all numbers ≥ 3

range: all numbers ≥ 0

x	f (x)
2	$\sqrt{-1}$ — not real
3	0
4	1
7	2
12	3

This is a square root function.



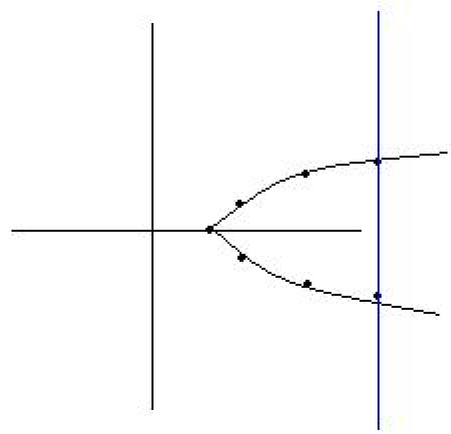
5a) $y = \sqrt{x - 3}$

domain: $[3, \infty)$

range: $(-\infty, \infty)$

x	y
3	0
4	± 1
7	± 2
12	± 3
19	± 4

This is a square root relation.



fails "vertical line test"
Not a function

Mapping: Domain/Range, Functions/Relations

Try this exercise: for the following sets of ordered pairs, determine

a) domain
b) range
c) whether the set is a function..

then,

d) map the relationships

1) (1, 1) (1, 2) (1, 3) (1, 4)

2) (3, 3) (4, 3) (5, 3) (6, 3)

3) (-3, 7) (0, 0) (1, 6) (6, 1)

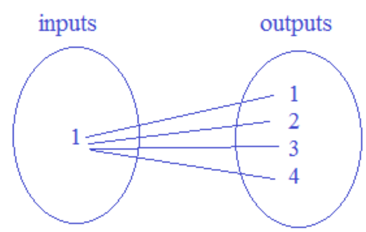
4) (1, 0) (0, 1) (2, 0) (0, 2)

SOLUTIONS....

1) (1, 1) (1, 2) (1, 3) (1, 4)

domain: { 1 }
range: {1, 2, 3, 4}

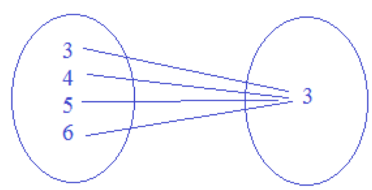
not a function



2) (3, 3) (4, 3) (5, 3) (6, 3)

domain: {3, 4, 5, 6}
range: { 3 }

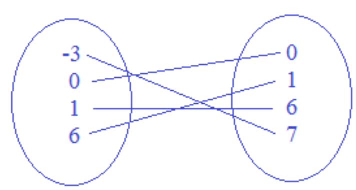
function
(each input has only one output..)



3) (-3, 7) (0, 0) (1, 6) (6, 1)

domain: { -3, 0, 1, 6}
range: { 0, 1, 6, 7}

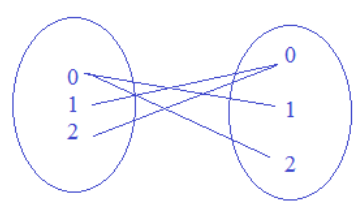
function



4) (1, 0) (0, 1) (2, 0) (0, 2)

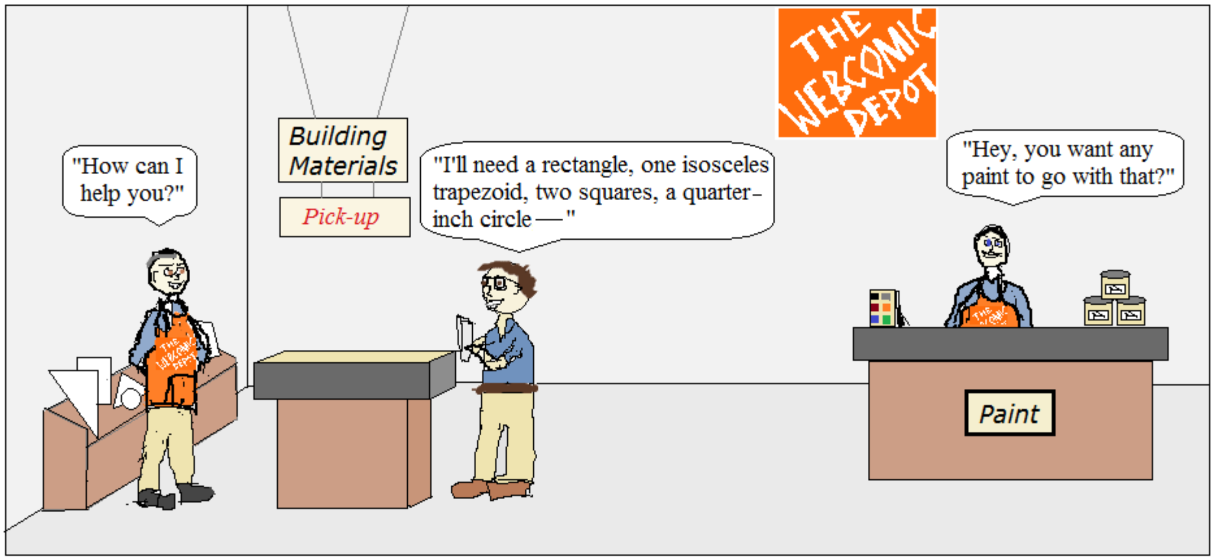
domain: {0, 1, 2}
range: {0, 1, 2}

not a function
(because input 0
has 2 different outputs!)

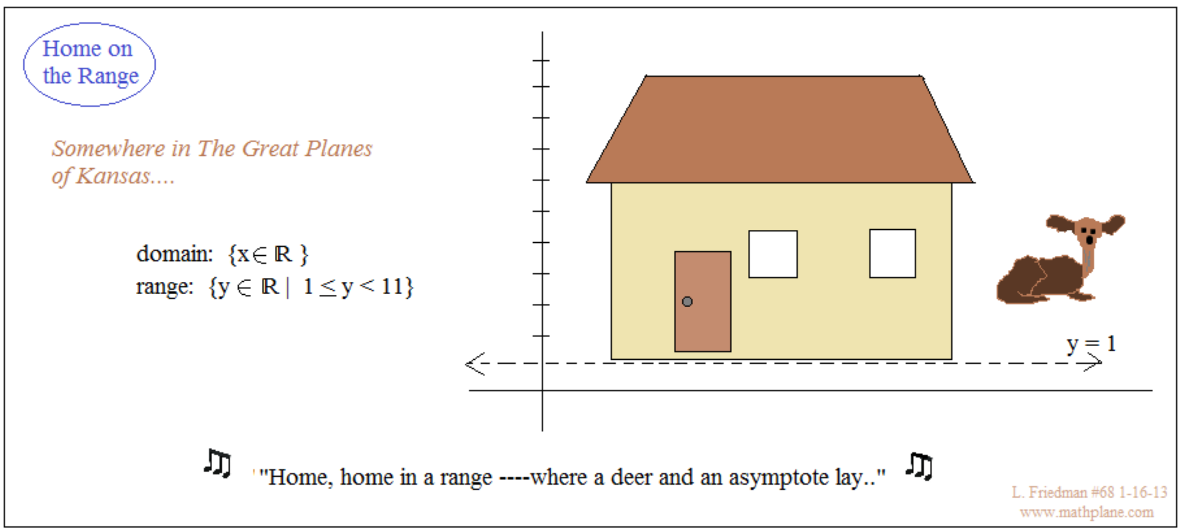


Math Homebuilders

Last Weekend...



Today...



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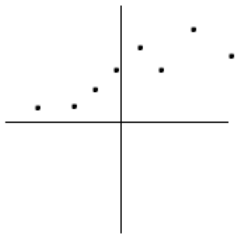
Practice Quiz-→

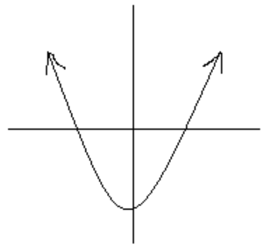
Relations and Functions

1) Which of the following relations are *functions*?

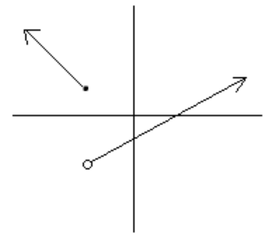
a)



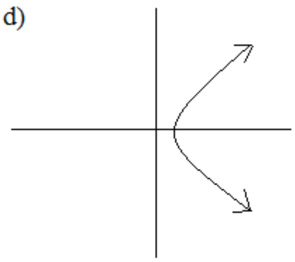
b)



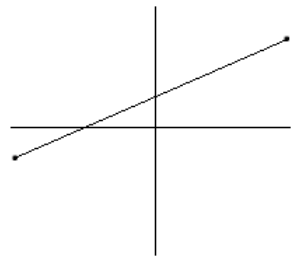
c)



d)



e)



2) The following points represent *relation* y: (-1, 4) (-1, 8) (1, -4) (6, -4) (0, 0)

Remove one point that would change the set to a *function*.

a) (-1, 4)
b) (-1, 8)
c) (1, -4)
d) (6, -4)
e) (0, 0)

3) Determine the domain of $f(x) = \dfrac{5}{\sqrt{x-3}}$

a) $x > 3$

b) $x \geq 3$

c) $x = 5$

d) all real numbers

e) all positive real numbers

What is the range of $y = -|x+4| + 2$?

a) $y \geq -4$

b) $y \leq -4$

c) $y \geq 2$

d) $y \leq 2$

e) all positive real numbers

4) Which of the following are functions?

a) $y = -6$

b) $x = 3$

c) $y = 3x^2$

d) $x = y^2 + 2$

e) $y = \frac{4}{x}$

5) Consider the 26 letters of the alphabet.

a) Identify 3 letters that when mapped on a coordinate plane, the result could be a function.

b) Identify 3 letters that when mapped on a coordinate plane, the result would not be a function.

6) What is the domain and range of $g(x) = x^2 - 6x + 5$?

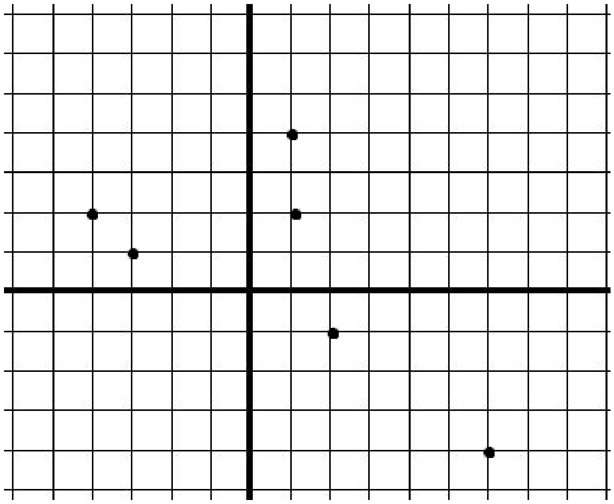
7) What is the domain and range of the following *piecewise* function?

$$f(x) = \begin{cases} 5 - x & \text{if} \quad x < 0 \\ 2 & \text{if} \quad 0 \leq x < 5 \\ x - 9 & \text{if} \quad x \geq 5 \end{cases}$$

8) Write the domain and range of each relation/function (in set notation or interval notation).

Relations and Functions

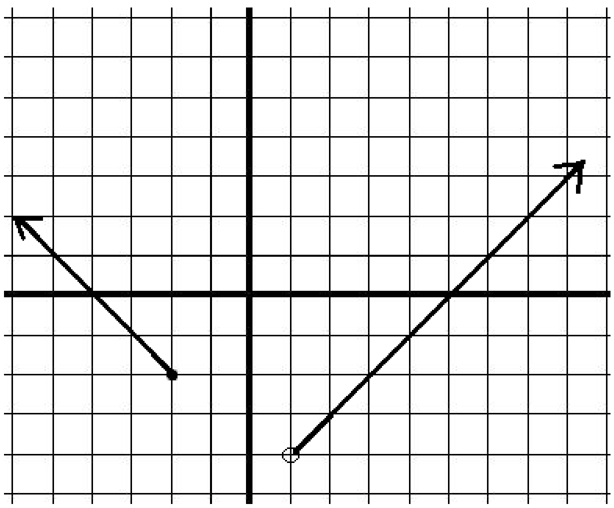
a)



Domain:

Range:

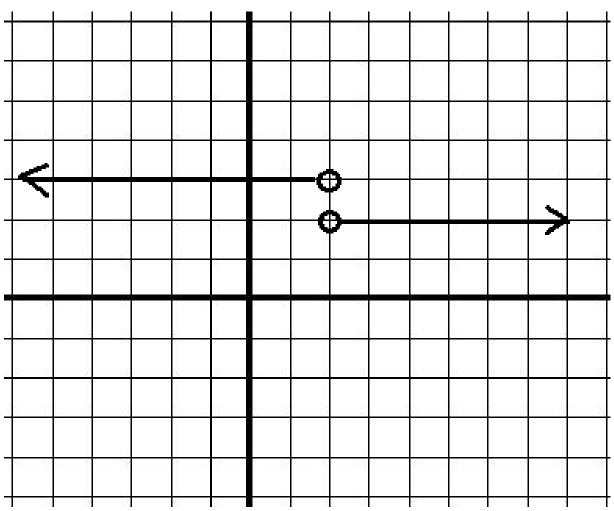
b)



Domain:

Range:

c)



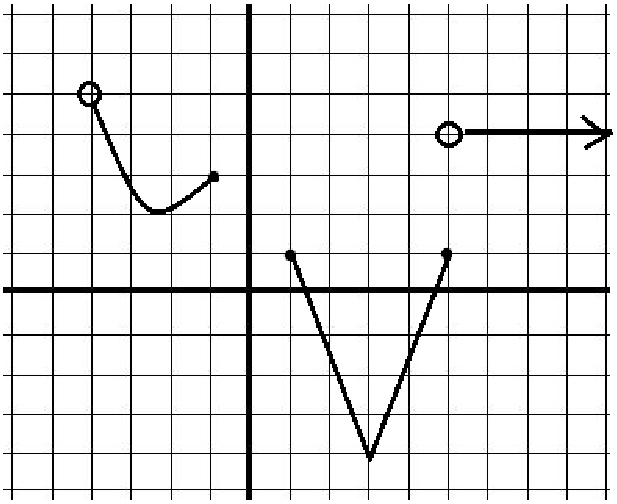
Domain:

Range:

Relations and Functions

Write the domain and range of each relation/function (in set notation or interval notation).

d)



Domain:

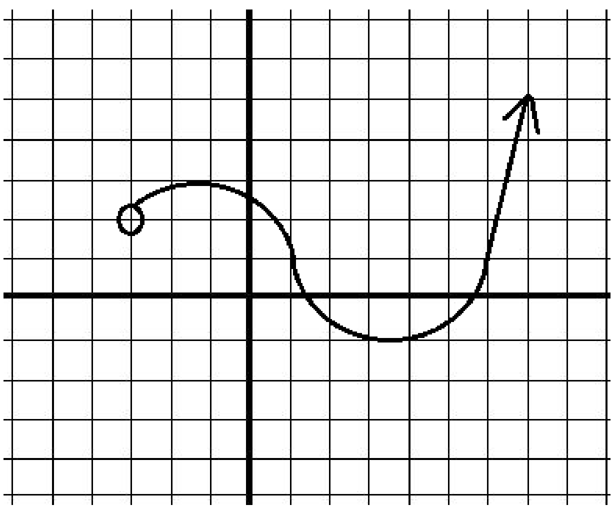
Range:

Increasing Invervals:

Decreasing Intervals:

Constant Intervals:

e)



Domain:

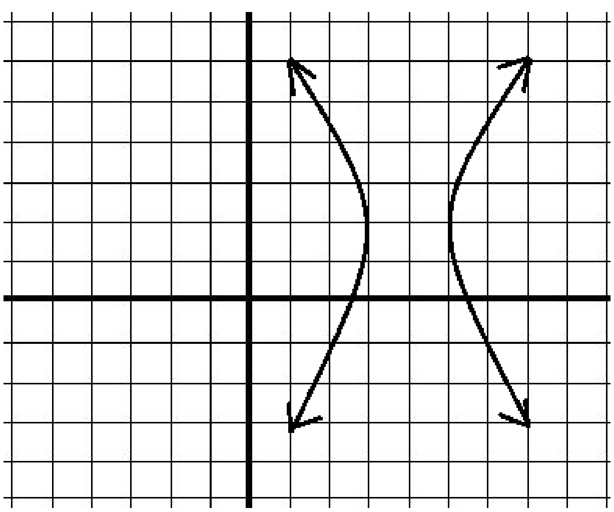
Range:

Increasing Invervals:

Decreasing Intervals:

Constant Intervals:

f)



Domain:

Range:

Increasing Invervals:

Decreasing Intervals:

Constant Intervals:

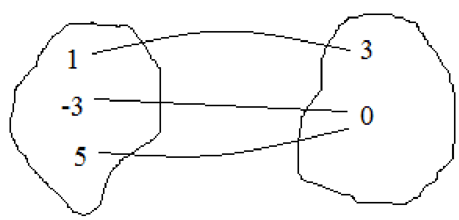
9) Determine if the relation given by the set of ordered pairs shown is a function. Justify your answer.

a) { (2, 4) (3, 4) (5, -4) (0, 0) (1, 1) }

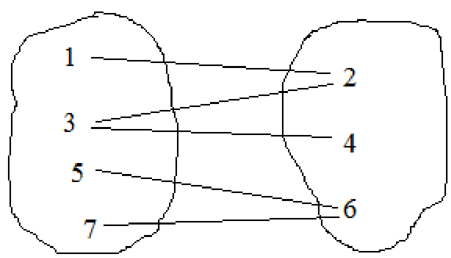
b) { (1, 2) (2, 3) (3, 4) (4, 5) (5, 6) }

c) { (10, 5) (10, -5) (5, 0) (-5, 0) (0, 0) }

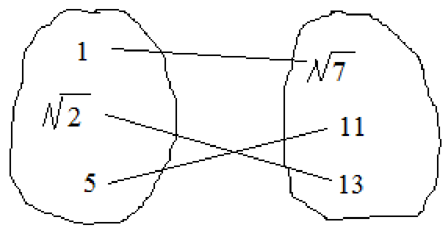
d) domain range



e) domain range



f) domain range



Domain Worksheet

Determine the domain of the following functions.

1) $y = \frac{x + 2}{3x - 2}$

2) $y = x + \sqrt{6}$

3) $f(x) = \frac{\sqrt{x + 4}}{x}$

4) $g(x) = \frac{3}{x^2 + 5}$

5) $y = \sqrt{3 - x} + 5$

6) $h(x) = \sqrt[3]{x + 1}$

7) $y = \frac{3}{x^2 + 12x}$

8) $f(x) = \frac{1}{\sqrt{x^2 - 10x + 21}}$

9) $y = \frac{x + 7}{|x - 3| - 5}$

10) $y = ln(x + 5)$

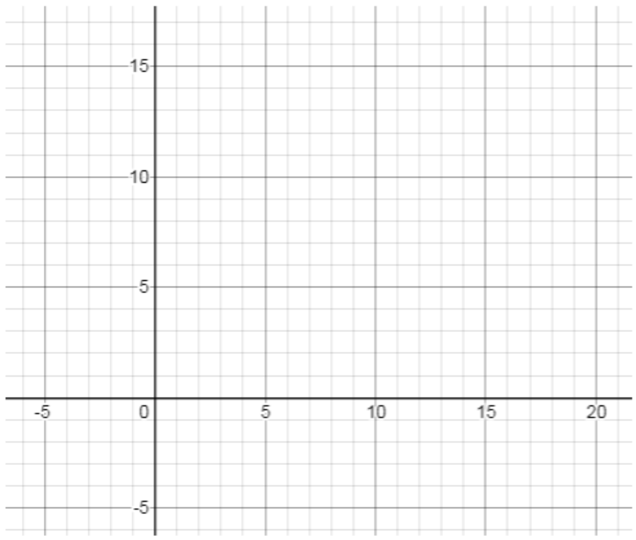
11) $y = \frac{1}{\log_2 (x)}$

12) $f(x) = \sin^{-1}(\frac{x}{3})$

Sketch a function with the given domain and range.

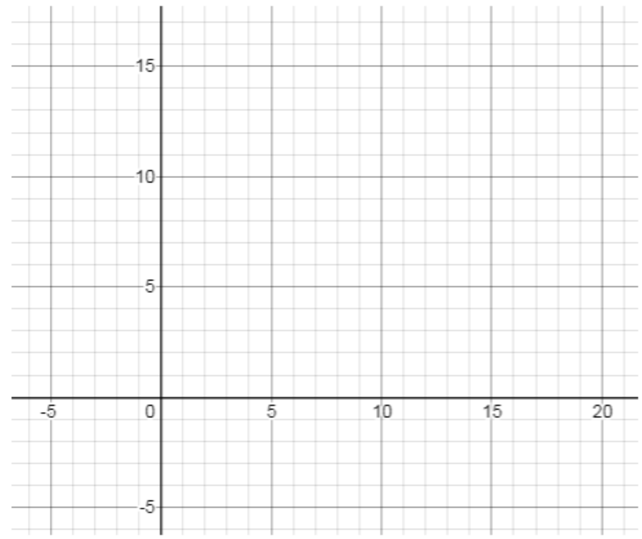
1) Domain: $3 \le x \le 17$

Range: $1 \le y \le 14$



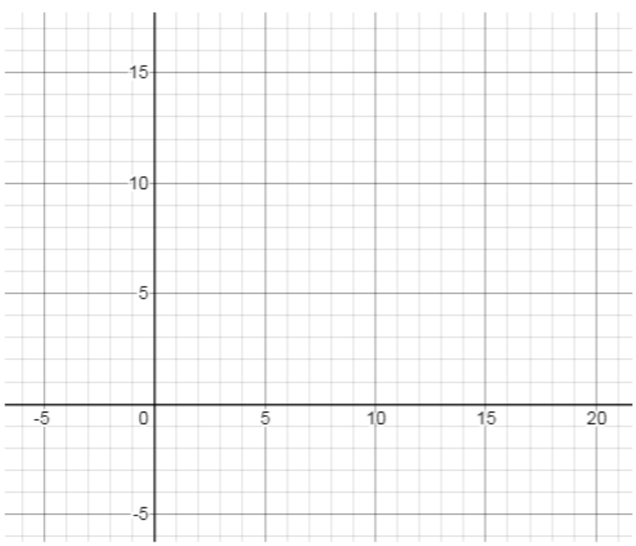
2) Domain: $1 < x < 16$

Range: $3 < y \le 10$



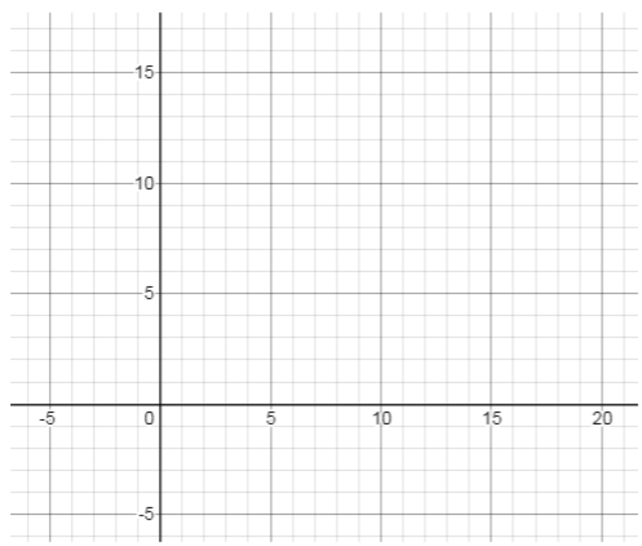
3) Domain: (-3, 11]

Range: {1, 3, 5}



4) Domain: [-2, 12)

Range: [-6, 5) U (8, 14]



Calculate and identify the domain

I. $f(x) = \sqrt{x+4}$ $g(x) = 2x^2$

$f(x)$ domain :

$g(x)$ domain :

$f(5) =$

$g(-3) =$

$(f+g)(5) =$

$(fg)(12) =$

$\left(\frac{f}{g}\right)(-3) =$

function	domain
$f+g =$	
$f \cdot g =$	
$\frac{f}{g} =$	
$\frac{g}{f} =$	

II. $f(x) = \sqrt{x+6}$ $g(x) = \sqrt{3-x}$

$f(x)$ domain :

$g(x)$ domain :

$f(g(3)) =$

$g(f(3)) =$

$(f+g)(2) =$

$(fg)(1) =$

function	domain
$f-g =$	
$f \cdot g =$	
$\frac{f}{g} =$	
$\frac{g}{f} =$	

III. $f(x) = \frac{2}{x-1}$ $g(x) = \frac{7}{x-1}$

$f(g(2)) =$

$f(g(1)) =$

$f(g(8)) =$

function	domain
$g-f =$	
$f \cdot g =$	
$\frac{f}{g} =$	
$f \circ g =$	

IV. Find the domain of these <u>identities</u>.

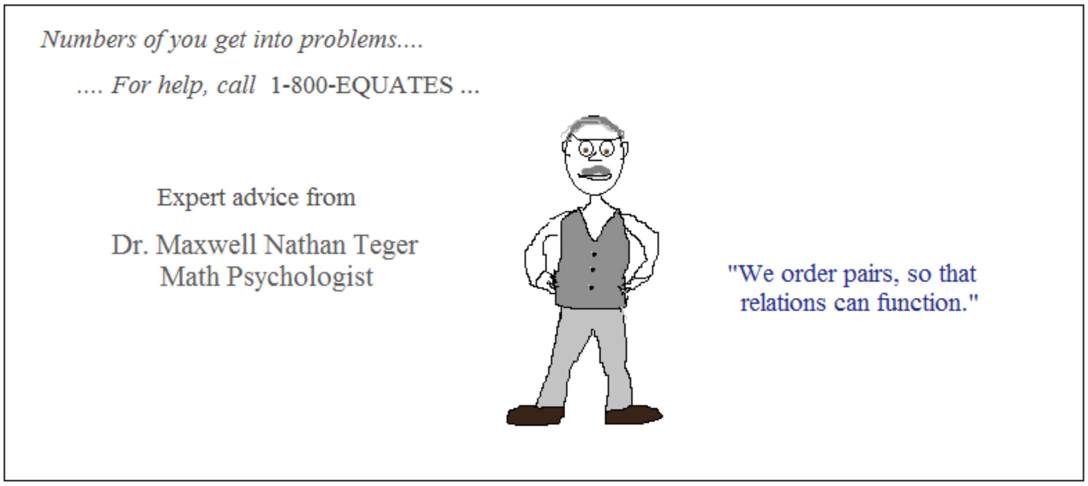
$g(x) = \frac{1}{x^2}$ $h(x) = x^2$

$g(x) \cdot f(x) = (g \cdot f)(x)$

$f(x) = \frac{2}{x-1}$ $g(x) = \frac{7}{x-1}$

$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$

Math Mediator



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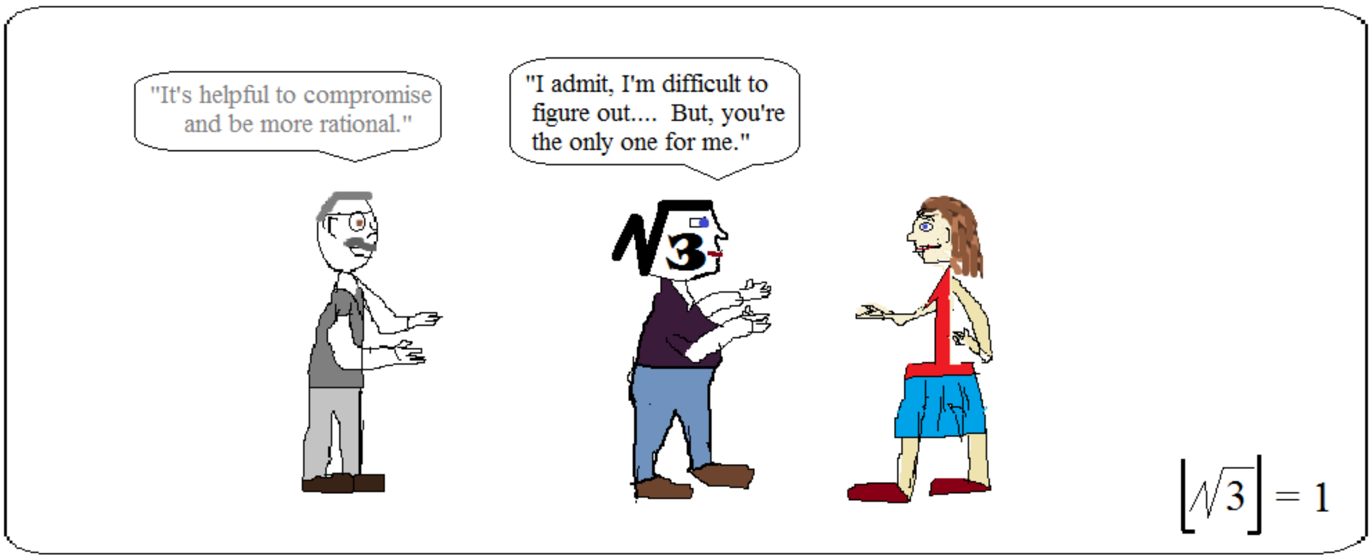


Thanks to Max N. Teger, countless pairs have ended up together.

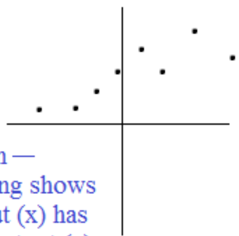
Solutions-→

Relations and Functions

SOLUTIONS

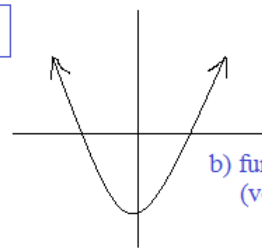
1) Which of the following relations are *functions*?

a)



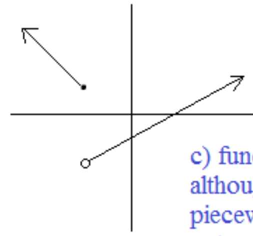
a) function — the mapping shows every input (x) has a unique output (y)

b)



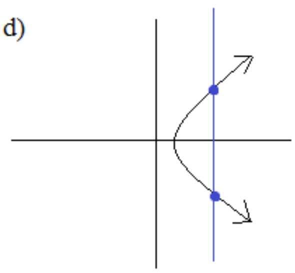
b) function — (vertical parabola)

c)



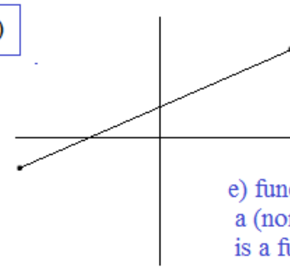
c) function — although this piecewise function is not continuous, it does satisfy the vertical line test.

d)



d) NOT a function! violates the "vertical line test"

e)



e) function — a (non-vertical) line segment is a function

2) The following points represent *relation* y: (-1, 4) (-1, 8) (1, -4) (6, -4) (0, 0)

Remove a point that would change the set to a *function*.

a) (-1, 4)
b) (-1, 8)
c) (1, -4)
d) (6, -4)
e) (0, 0)

a) or b)

In the relation, -1 has TWO outputs. Therefore, removing either point would change the set ot a function.

3) Determine the domain of $f(x) = \dfrac{5}{\sqrt{x-3}}$

a) $x > 3$
b) $x \geq 3$
c) $x = 5$
d) all real numbers
e) all positive real numbers

a) correct: any real number > 3 can go into the function
b) wrong: at x = 3, the function is undefined! $f(0) = 5/0$
c) wrong: x = 5 is only one element in the infinite domain
d) wrong: x = 3, $f(x)$ is undefined; x < 3, there is a negative number under the radical
e) wrong: counter-example: let x = 2

What is the range of $y = -|x+4| + 2$?

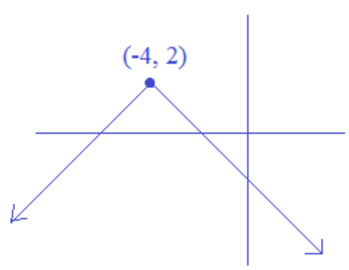
a) $y \geq -4$
b) $y \leq -4$
c) $y \geq 2$
d) $y \leq 2$
e) all positive real numbers

The range represents all the possible "y values/outputs"

2 is the maximum value; and, there in no minimum.

$(-\infty, 2]$

Note: $|x + 4| \geq 0$; therefore, $-|x + 4| \leq 0$



Relations and Functions

Solutions

4) Which of the following are functions?

a) $y = -6$ function (horizontal line)

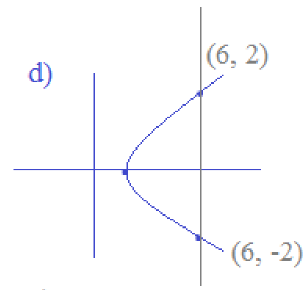
b) $x = 3$ NOT a function (vertical line)

c) $y = 3x^2$ function (parabola -- satisfies the "vertical line test"

d) $x = y^2 + 2$ NOT a function ('horizontal parabola' -- fails "vertical line test")

e) $y = \frac{4}{x}$ function (inverse function with asymptotes)
every x input has a unique y input!

d)

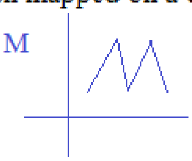


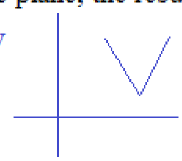
6 has TWO outputs; not a function

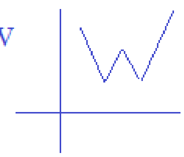
5) Consider the 26 letters of the alphabet.

a) Identify 3 letters that when mapped on a coordinate plane, the result could be a function.

Examples may include M, V, and W

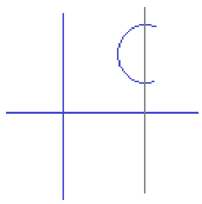
M


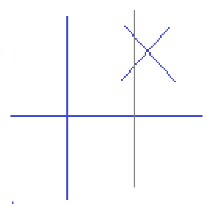
V


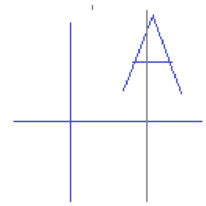
W


b) Identify 3 letters that when mapped on a coordinate plane, the result would not be a function.

Examples include C, A, and X...
All fail the vertical line test...

C


X


A


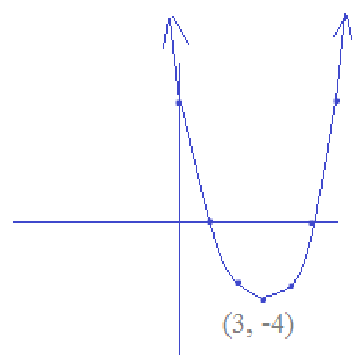
6) What is the domain and range of $g(x) = x^2 - 6x + 5$?

Domain is all real numbers... (Any real number can be put into the function)

Range is $[-4, \infty)$

$y \geq -4$

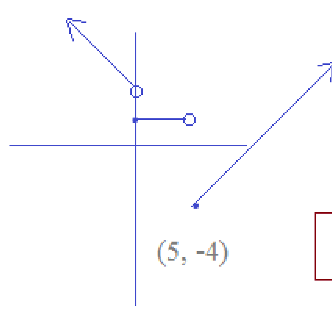
3 is the minimum value (the vertex of the parabola/quadratic)



7) What is the domain and range of the following *piecewise* function?

$$f(x) = \begin{cases} 5 - x & \text{if } x < 0 \\ 2 & \text{if } 0 \leq x < 5 \\ x - 9 & \text{if } x \geq 5 \end{cases}$$

domain: all real numbers

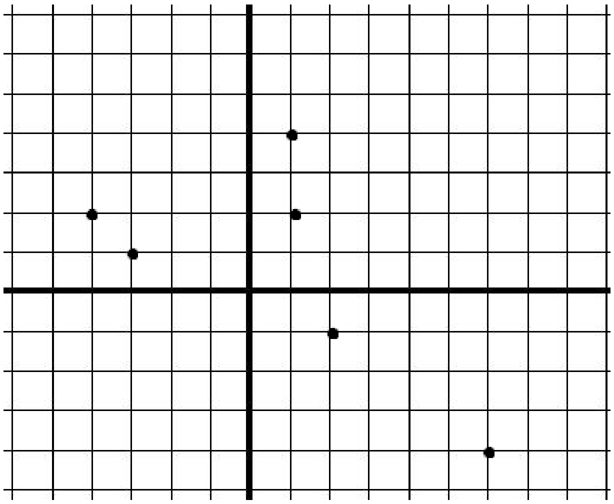


range: $[-4, \infty)$

Relations and Functions

8) Write the domain and range of each relation/function (in set notation or interval notation).

a)



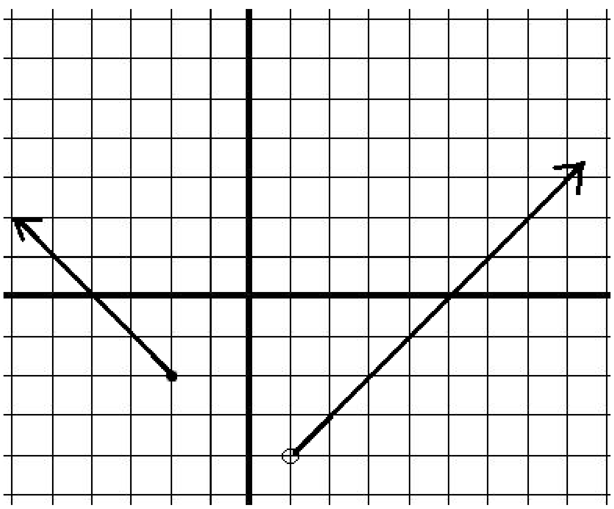
Domain: (all the x values)
{-4, -3, 1, 2, 6}

Range: (all the y values)
{-4, -1, 1, 2, 4}

(this is a relation because x = 1 has TWO possible outputs!)

Solutions

b)

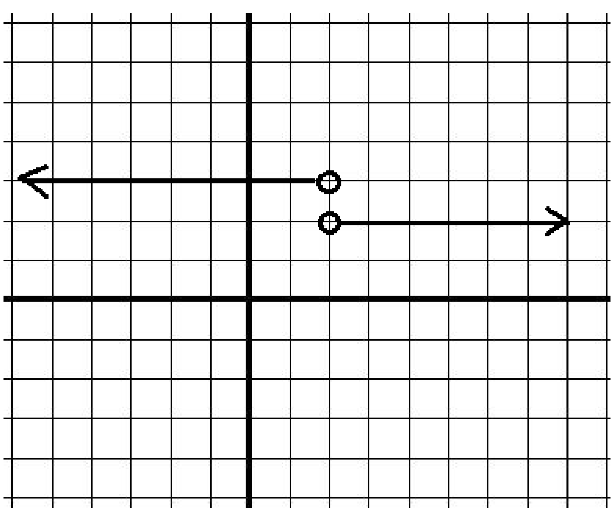


Domain: $(-\infty, -2] \cup (1, \infty)$
D = {x ≤ 2 or x > 1}

Range: $(-4, \infty)$
R = {y > -4}

(outputs/y values will be every number 'above' -4)

c)



Domain: $(-\infty, 2) \cup (2, \infty)$
D = {all real numbers, where x ≠ 2}

Range: R = {2, 3}

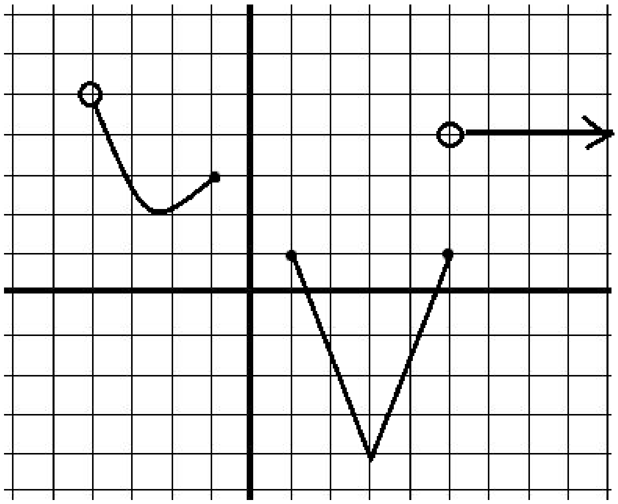
(this is a function with an infinite domain and a range of just 2 values)

Relations and Functions

Write the domain and range of each relation/function (in set notation or interval notation).

Solutions

d)



Domain: (-4, -1] U [1, 5] U (5, ∞)

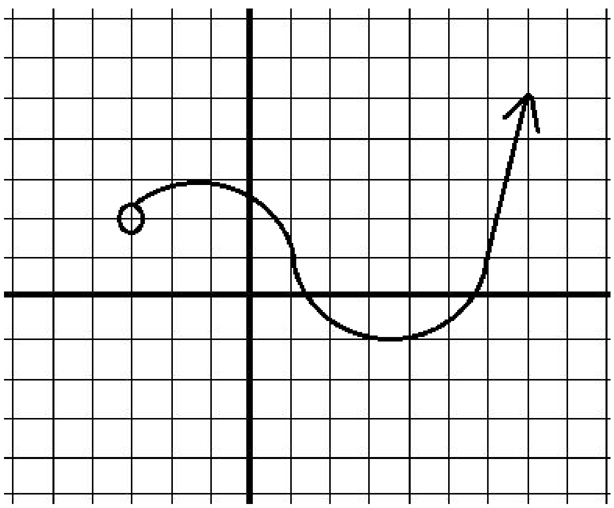
Range: [-4, 1] U [2, 5)

Increasing Invervals: (-2.5, 1) U (3, 5)

Decreasing Intervals: (-4, -2.5) U (1, 3)

Constant Intervals: (5, ∞)

e)



Domain: (-3, ∞)

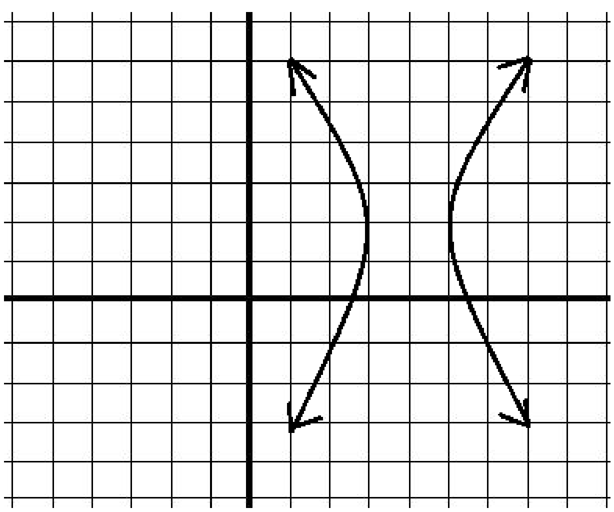
Range: (-1, ∞)

Increasing Invervals: (-3, -1.5) U (3.5, ∞)

Decreasing Intervals: (-1.5, 3.5)

Constant Intervals: None

f)



Domain: (- ∞ , 3) U (5, ∞)

Range: (- ∞ , ∞)

Increasing Invervals:

Decreasing Intervals:

Ambiguous
(It's not a function!)

Constant Intervals: None

Relations and Functions

9) Determine if the relation given by the set of ordered pairs shown is a function. Justify your answer.

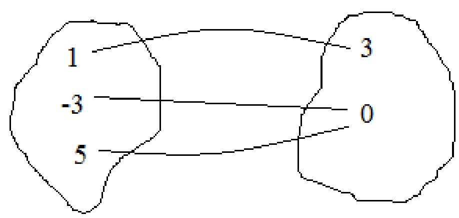
Solutions

a) { (2, 4) (3, 4) (5, -4) (0, 0) (1, 1) } — Function: every input -- 2, 3, 5, 0, and 1 -- has a unique output

b) { (1, 2) (2, 3) (3, 4) (4, 5) (5, 6) } — Function: every input -- x value in each ordered pair -- has a unique output y

c) { (10, 5) (10, -5) (5, 0) (-5, 0) (0, 0) } — Relation: The input 10 has more than one possible output --- 5 or -5 (Not a function)

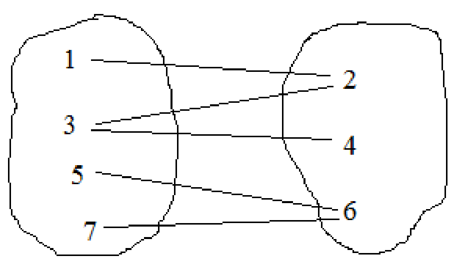
d) domain range



mapping: (1, 3)
(-3, 0)
(5, 0)

Function: each input has a specific output...

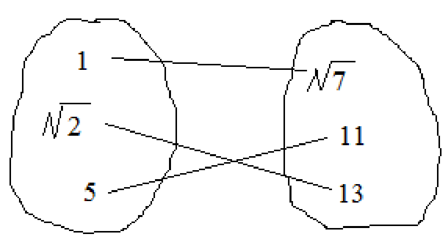
e) domain range



NOT a function:

If x = 3, then y can be 2 OR 4

f) domain range



one-to-one Function

Domain Worksheet

Determine the domain of the following functions.

What to look for:
a) denominator cannot equal 0
b) no negatives under square root
c) logarithms > 0
d) inverse cosine/sine: $-1 \leq x \leq 1$

SOLUTIONS

1) $y = \frac{x+2}{3x-2}$

$3x - 2 \neq 0$

all reals where $x \neq 2/3$

$(-\infty, 2/3) \cup (2/3, \infty)$

2) $y = x + \sqrt{6}$

it's a line...

all real numbers

$(-\infty, \infty)$

3) $f(x) = \frac{\sqrt{x+4}}{x}$

$x \neq 0$

$x + 4 \geq 0 \quad x \geq -4$

$[-4, 0) \cup (0, \infty)$

4) $g(x) = \frac{3}{x^2+5}$

since x^2 is positive, the denominator will never equal 0

all real numbers

$(-\infty, \infty)$

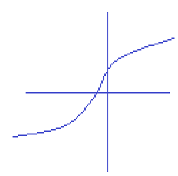
5) $y = \sqrt{3-x} + 5$

find where $3 - x \geq 0$

$x \leq 3$

$(-\infty, 3]$

6) $h(x) = \sqrt[3]{x+1}$



cube root of negative is acceptable..

so, domain is all real numbers

7) $y = \frac{3}{x^2 + 12x}$

$x^2 + 12x \neq 0$

$x(x+12) \neq 0$

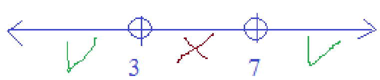
$x \neq 0, -12$

all reals, except -12 or 0

8) $f(x) = \frac{1}{\sqrt{x^2 - 10x + 21}}$

$x^2 - 10x + 21 > 0$

$(x-3)(x-7) > 0$ (test points on number line)



$(-\infty, 3) \cup (7, \infty)$

9) $y = \frac{x+7}{|x-3| - 5}$

$|x-3| - 5 \neq 0$

$|x-3| \neq 5$

$x \neq 8$ or -2

all reals, except -2 or 8

10) $y = \ln(x+5)$

$x + 5 > 0$

$x > -5$

$(-5, \infty)$

11) $y = \frac{1}{\log_2(x)}$

$x > 0$ (log must be > 0)

then, $\log_2(x)$ cannot equal zero in the denominator

$\log_2(1) = 0$

$x > 0$ and $x \neq 1$

$(0, 1) \cup (1, \infty)$

12) $f(x) = \sin^{-1}\left(\frac{x}{3}\right)$

$-1 \leq \frac{x}{3} \leq 1$

$-3 \leq x \leq 3$

$[-3, 3]$

Sketch a function with the given domain and range.

SOLUTIONS

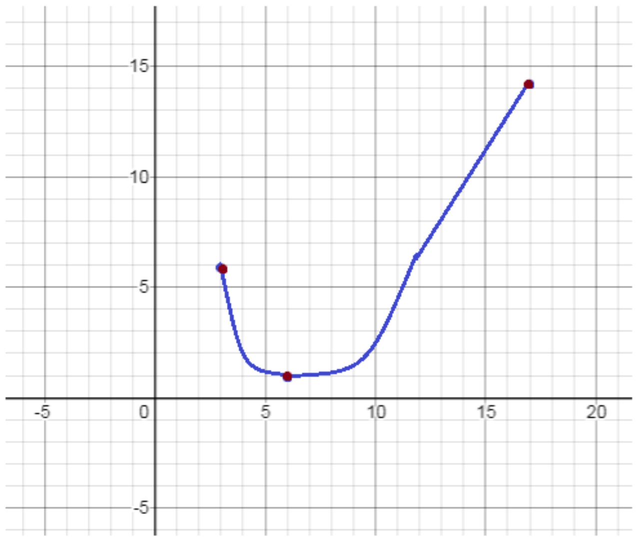
NOTE: There are other possible solutions

1) Domain: $3 \le x \le 17$

Range: $1 \le y \le 14$

Every x value is covered from 3 to 17

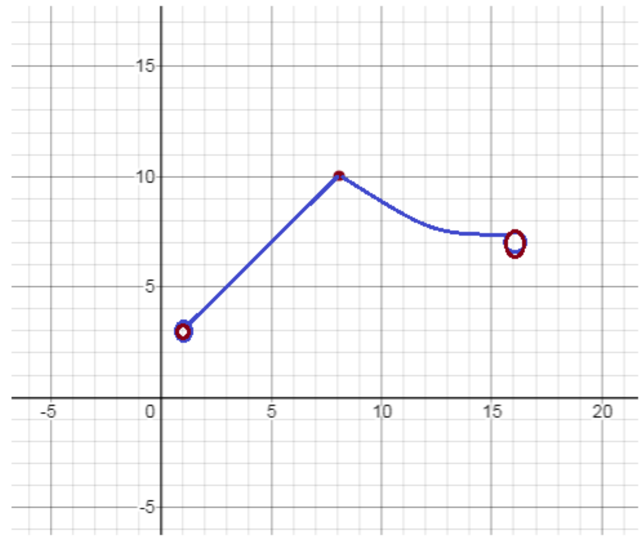
Every y value is covered from 1 to 14



2) Domain: $1 < x < 16$

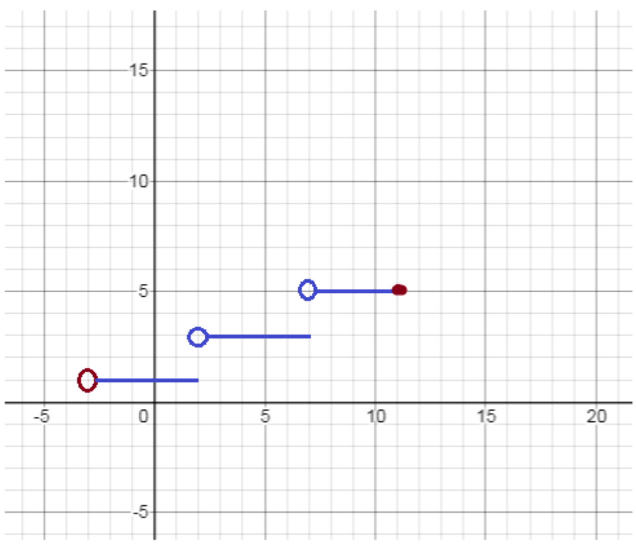
Range: $3 < y \le 10$

Open circles are placed at boundaries for $<$ signs...



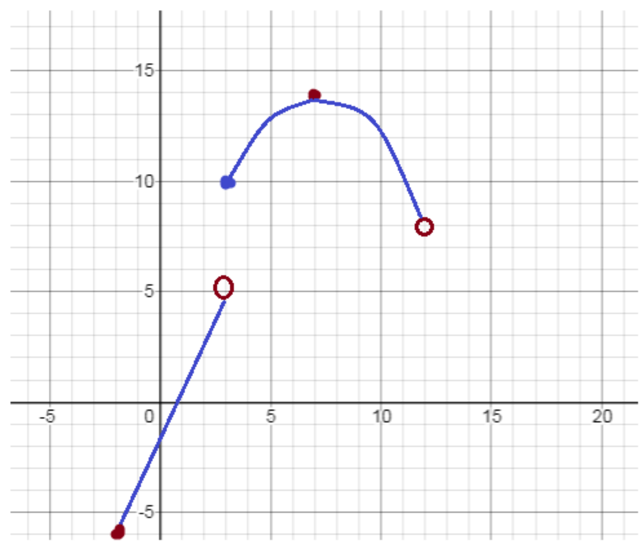
3) Domain: (-3, 11]

Range: {1, 3, 5}



4) Domain: [-2, 12)

Range: [-6, 5) U (8, 14] ,



Calculate and identify the domain

SOLUTIONS

I. $f(x) = \sqrt{x+4}$ $g(x) = 2x^2$

$f(x)$ domain : $x+4 \geq 0 \Rightarrow [-4, \infty)$

$g(x)$ domain : all real numbers

$f(5) = 3$

$g(-3) = 18$

$(f+g)(5) = \sqrt{(5)+4} + 2(5)^2 = 53$

$(fg)(12) = \sqrt{12+4} \cdot 2(12)^2 = 4 \times 288 = 1152$

$\left(\frac{f}{g}\right)(-3) = \frac{\sqrt{(-3)+4}}{2(-3)^2} = \frac{1}{18}$

function	domain
$f+g = \sqrt{x+4} + 2x^2$	$[-4, \infty)$
$f \cdot g = 2x^2\sqrt{x+4}$	$[-4, \infty)$
$\frac{f}{g} = \frac{\sqrt{x+4}}{2x^2}$	$[-4, 0) \cup (0, \infty)$
$\frac{g}{f} = \frac{2x^2}{\sqrt{x+4}}$	$(-4, \infty)$

II. $f(x) = \sqrt{x+6}$ $g(x) = \sqrt{3-x}$

$f(x)$ domain $x+6 \geq 0 \Rightarrow x \geq -6$

$g(x)$ domain $3-x \geq 0 \Rightarrow x \leq 3$

$f(g(3)) = f(0) = \sqrt{6}$

$g(f(3)) = g(3) = 0$

$(f+g)(2) = \sqrt{8} + 1$

$(fg)(1) = \sqrt{7}\sqrt{2} = \sqrt{14}$

function	domain
$f-g = \sqrt{x+6} - \sqrt{3-x}$	$[-6, 3]$
$f \cdot g = \sqrt{(x+6)(3-x)}$	$[-6, 3]$
$\frac{f}{g} = \frac{\sqrt{x+6}}{\sqrt{3-x}}$	$-6 \leq x < 3$
$\frac{g}{f} = \frac{\sqrt{3-x}}{\sqrt{x+6}}$	$-6 < x \leq 3$

III. $f(x) = \frac{2}{x-1}$ $g(x) = \frac{7}{x-1}$

$f(g(2)) = f(7) = 1/3$

$f(g(1)) =$ since $g(1)$ is $\frac{7}{0}$, this is undefined

$f(g(8)) = g(8) = 1$

then, $f(1)$ is undefined

function	domain
$g-f = \frac{5}{x-1}$	all real numbers $\neq 1$
$f \cdot g = \frac{14}{(x-1)^2}$	all real numbers $\neq 1$
$\frac{f}{g} = \frac{2}{7}$	all real numbers
$f \circ g = \frac{2(x-1)}{8-x}$	all real numbers except 1 and 8

$$\frac{2}{\frac{7}{x-1} - 1} \Rightarrow \frac{2}{\frac{7}{x-1} - \frac{x-1}{x-1}} \Rightarrow \frac{2}{\frac{8-x}{x-1}}$$

IV. Find the domain of these <u>identities</u>.

$g(x) = \frac{1}{x^2}$ $h(x) = x^2$

$g(x) \cdot f(x) = (g \cdot f)(x)$

$g(7) \cdot h(7) = (gh)(7) = 1$

but

$g(0) \cdot h(0) = (gh)(0)$

undefined 1

domain of $g(x) \cdot f(x)$ excludes 0

domain of $(gf)(x)$ includes 0

$f(x) = \frac{2}{x-1}$ $g(x) = \frac{7}{x-1}$

$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$

Individually, the domains exclude 1

Together, the domain is all real numbers...

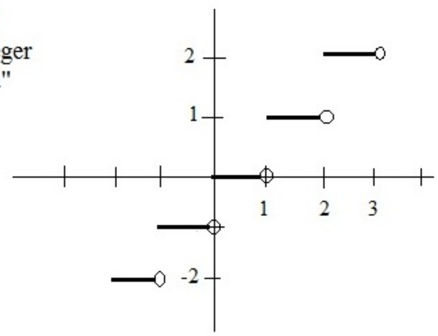
The domain of the <u>identity</u> for all numbers is all reals except 1

One more topic….

Greatest Integer Function

A function that maps a real number to the largest *previous* integer.
It shows the greatest integer that is not greater than the input.
The greatest integer function is commonly called the *floor function*.

$g(x) = \lfloor x \rfloor$
"greatest integer function"



$g(1.5) = 1$

$g(-2.3) = -3$

$g(2) = 2$

$g(1.00034) = 1$

Notation and symbols: There are several ways the greatest (or least) integer function may be expressed.

	Greatest integer value	Least integer value
Bracket with top (or bottom) cut off:	$\lfloor X \rfloor$	$\lceil X \rceil$
Alternate names:	floor function	ceiling function
Word form:	floor(x)	ceil(x)

Note: Instead of an open bracket (with the top cut off), the greatest integer function may use brackets in various forms:

open bracket	bracket	bold bracket	double bracket
$\lfloor x \rfloor$	[x]	**[x]**	[[x]]
$\lfloor 1.2 \rfloor = 1$	[2.4] = 2	**[** -3.3 **]** = -4	[[6.77]] = 6

The least integer function or "ceiling function" maps a rational number to the next smallest integer.

ceil(x) or ceiling(x) = $\lceil x \rceil$ is the smallest integer not less than x.

$\lceil 4.33 \rceil = 5$ $\lceil 3 \rceil = 3$ $\lceil -7.89 \rceil = -7$

Application of greatest integer function:

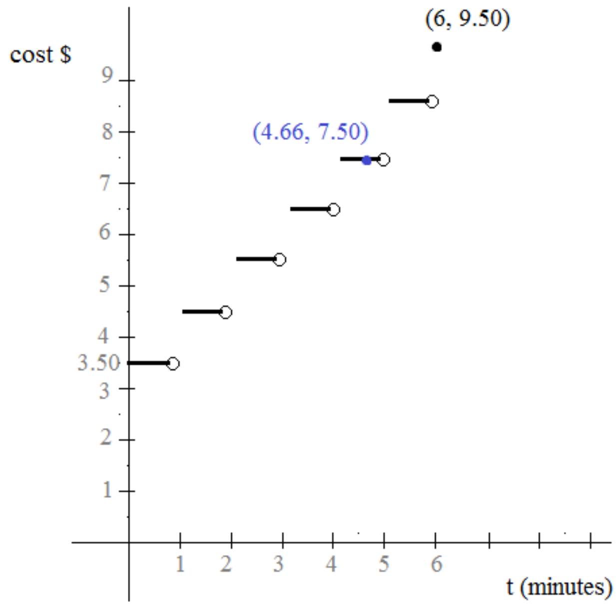
Example: The telephone company charges \$3.50 to connect and \$1 per minute for an international call.

Write an equation that shows the cost of making an international call. (Use the greatest integer, or floor, function). Graph the function, representing calls of 0 to 6 minutes. How much would a 4 minute and 40 second call cost?

$$\text{Cost} = \$3.50 + \lfloor t \rfloor \times (\$1)$$

$$\text{Cost of 4 minute 40 second call} = \$3.50 + \left\lfloor 4\tfrac{2}{3} \right\rfloor \times (\$1)$$

$$= \$3.50 + 4(\$1) = \$7.50$$



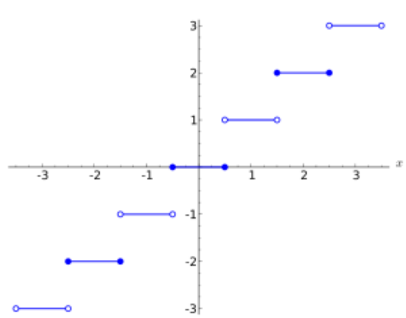
Nearest integer function: $[x]$

The nearest integer function is used in computer science.

Round the 1/2 numbers to the nearest even integer.

(or, round up, round down, round away from zero, or randomly round up or down)

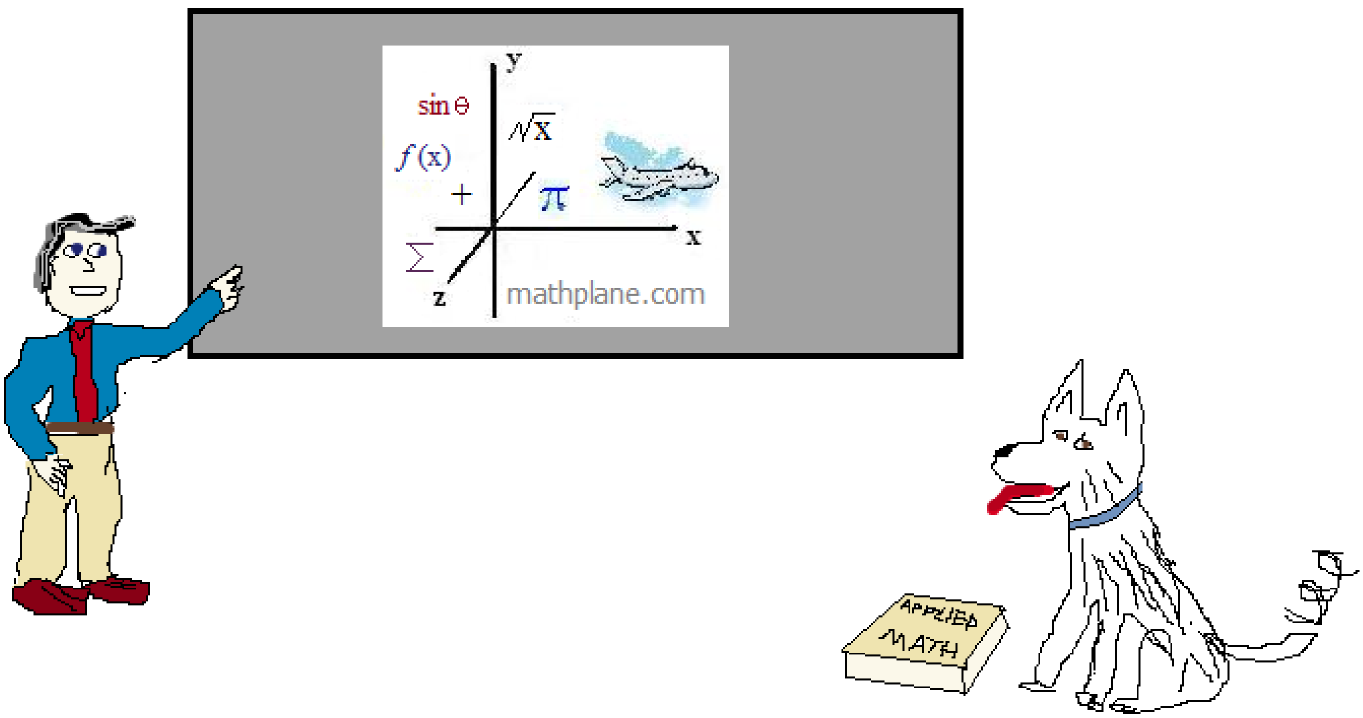
nint(x) or Round(x) $\lfloor x \rceil$



Thanks for visiting. (Hope it helped!)

If you have questions, suggestions, or requests, let us know.

Cheers



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