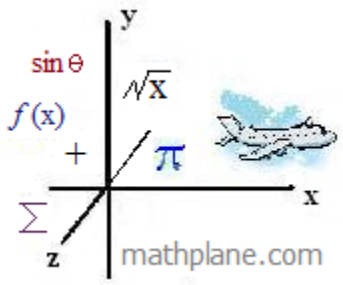
Simplifying Negative Exponents and Variables

Brief notes, examples, and Worksheet (& solutions)



Exponent Rules: Notes and Examples

Exponent *definition*:

$$X^A = X_1 \cdot X_2 \cdot X_3 \cdot \ \dots \ \cdot X_{A-2} \cdot X_{A-1} \cdot X_A$$

Examples: $4^5 = 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 = 1024$

$$\left(\frac{2}{3}\right)^3 = \frac{2}{3} \cdot \frac{2}{3} \cdot \frac{2}{3} = \frac{8}{27}$$

$$(-2)^7 = (-2) \cdot (-2) \cdot (-2) \cdot (-2) \cdot (-2) \cdot (-2) \cdot (-2) = -128$$

$$(-2)^6 = (-2) \cdot (-2) \cdot (-2) \cdot (-2) \cdot (-2) \cdot (-2) = 64$$

Rule #1 ('Addition Rule')

$$X^A \cdot X^B = X^{A+B}$$

Examples:

$$X^3 \cdot X^5 = X^8$$

$$5^3 \cdot 5^2 = 125 \cdot 25 = 3125 = 5^5$$

Note:

$$Y^2 \cdot Y^4 = Y^6$$

$$\underset{2}{(Y \times Y)} \cdot \underset{4}{(Y \times Y \times Y \times Y)} = \underset{6 \text{ total Y's}}{Y \times Y \times Y \times Y \times Y \times Y}$$

Rule #2: ('Multiplication Rule')

$$(X^A)^B = X^{AB}$$

Examples: $(X^4)^3 = X^{12}$

$$(4^2)^4 = 4^8 = 16^4 = 65536$$

Note:

$$\underset{5}{(Y^5)^3} = \underset{5}{(Y \times Y \times Y \times Y \times Y)} \cdot \underset{5}{(Y \times Y \times Y \times Y \times Y)} \cdot \underset{5}{(Y \times Y \times Y \times Y \times Y)} = Y^{15}$$

3 groups of 5 Y's
total: 3 x 5 = 15 Y's

Rule #3: ('zero exponent')

$$X^0 = 1$$

Examples: $Y^0 = 1$

$8^0 = 1$

$(3cd)^0 = 1$

What is 0^0 ? $0^A = 0$ because $0 \cdot 0 \cdot 0 \cdot 0 \dots = 0$ (if $A \neq 0$)

$X^0 = 1$ (zero exponent rule)

$0^0 = 1$

Note: $Z^5 \cdot Z^{-5} = Z^0 = 1$

addition rule --- then, zero exponent rule

$$\frac{\overbrace{Z \times Z \times Z \times Z \times Z}^{Z^5}}{\underbrace{Z \times Z \times Z \times Z \times Z}_{Z^{-5}}} = 1$$

Exponent Rules: Notes and Examples

Rule #4: ('negatives' or 'reciprocal rule')

$$X^{(-A)} = \frac{1}{X^A}$$

Examples:

$$X^{-3} = \frac{1}{X^3}$$

$$5^{-2} = \frac{1}{25}$$ It is not equal to -25!!!

$$\left(\frac{1}{3}\right)^{-4} = 81$$

Note: $$Y^{(-A)} = Y^{(-A)} \cdot \frac{Y^A}{Y^A} = \frac{Y^{(-A)} \cdot Y^A}{Y^A} = \frac{Y^{(-A+A)}}{Y^A} = \frac{Y^0}{Y^A} = \frac{1}{Y^A}$$

multiply by one; exponent addition rule; zero exponent

Rule #5: ('base rule')

$$X^A \cdot Y^A = (XY)^A$$

Examples: $$5^3 \cdot 7^3 = 125 \times 343 = 42875 = 35^3$$

$$= (5 \times 5 \times 5) \times (7 \times 7 \times 7) = (5 \times 7) \times (5 \times 7) \times (5 \times 7)$$

$$4^{\frac{1}{2}} \cdot 16^{\frac{1}{2}} = 64^{(1/2)} = 8$$

$$\sqrt{4} \times \sqrt{16} = \sqrt{4 \times 16} = \sqrt{64}$$

Rule #6: ('rational exponents')

$$X^{(1/2)} = \sqrt{X} \qquad X^{\left(\frac{A}{B}\right)} = \sqrt[B]{X^A}$$

Examples:

$$25^{(1/2)} = \sqrt{25} = 5$$

$$8^{(1/3)} = \sqrt[3]{8} = 2$$ ('cubed root of 8')

$$121^{(.5)} = 11$$

Note: $$Y^{(1/2)} \cdot Y^{(1/2)} = Y^1 \qquad \sqrt{Y} \cdot \sqrt{Y} = Y$$

(addition exponent rule)

$$8^{(1/3)} \cdot 8^{(1/3)} \cdot 8^{(1/3)} = 8^1 = 8$$

Example: Simplify $\frac{x^5}{x^8}$

Simplifying variables with exponents

Method 1: Use Exponent Rules

$$\frac{x^5}{x^8} \Rightarrow x^{5-8} = x^{-3} = \frac{1}{x^3}$$

"Subtraction Rule" "Negatives/Reciprocal Rule"

Method 2: Count the variables

$$\frac{x^5}{x^8} \Rightarrow \frac{\cancel{x\,x\,x\,x\,x}}{\cancel{x\,x\,x\,x\,x}\,x\,x\,x} = \frac{1}{x\,x\,x} = \frac{1}{x^3}$$

5 cancel out

Example: Simplify $\frac{a^2b^3c^4}{a^5b^3c^8}$

This is a 3-part problem: 1) simplify the a's 2) simplify the b's 3) simplify the c's

$\frac{a^2b^3c^4}{a^5b^3c^8}$ — 2 a's, 5 a's — so, two a's cancel out

$\frac{b^3c^4}{a^3b^3c^8}$ — 3 b's, 3 b's — so, three b's cancelled

$\frac{(1)\,c^4}{a^3\,c^8}$ — 4 c's, 8 c's — so, four c's cancel out

$$\frac{(1)}{a^3\,c^4} = \frac{1}{a^3\,c^4}$$

Exponents and Variables Examples: $x = -2 \quad y = 4 \quad z = \frac{1}{3}$

a) x^{-2} b) xy^0z^3 c) $\frac{y^{-3}zx^5}{x^2z^2}$ d) $\frac{5x^3y^3}{25xy^{-2}z^{-1}}$ e) $\frac{(x^2y^{-3}z^4)^2}{x^{-1}y^{-3}z^6}$

a) $x^{-2} = \frac{1}{x^2}$ simplify the variable

$\frac{1}{(-2)^2}$ substitute

$\boxed{\frac{1}{4}}$

b) $xy^0z^3 = (-2)(4)^0(\frac{1}{3})^3$ substitute

$-2 \cdot 1 \cdot \frac{1}{27}$ simplify

$\boxed{\frac{-2}{27}}$

c) $\frac{y^{-3}zx^5}{x^2z^2} = \frac{x^5z}{x^2y^3z^2}$ rearrange variables (reciprocal rule)

$\frac{x^3}{y^3z}$ simplify

$\frac{(-2)^3}{(4)^3(\frac{1}{3})}$ substitute

$\frac{-8}{\frac{64}{3}} = \boxed{\frac{-3}{8}}$

d) $\frac{5x^3y^3}{25xy^{-2}z^{-1}} = \frac{1\,x^2\,y^5\,z^1}{5}$

simplify each variable and the numbers

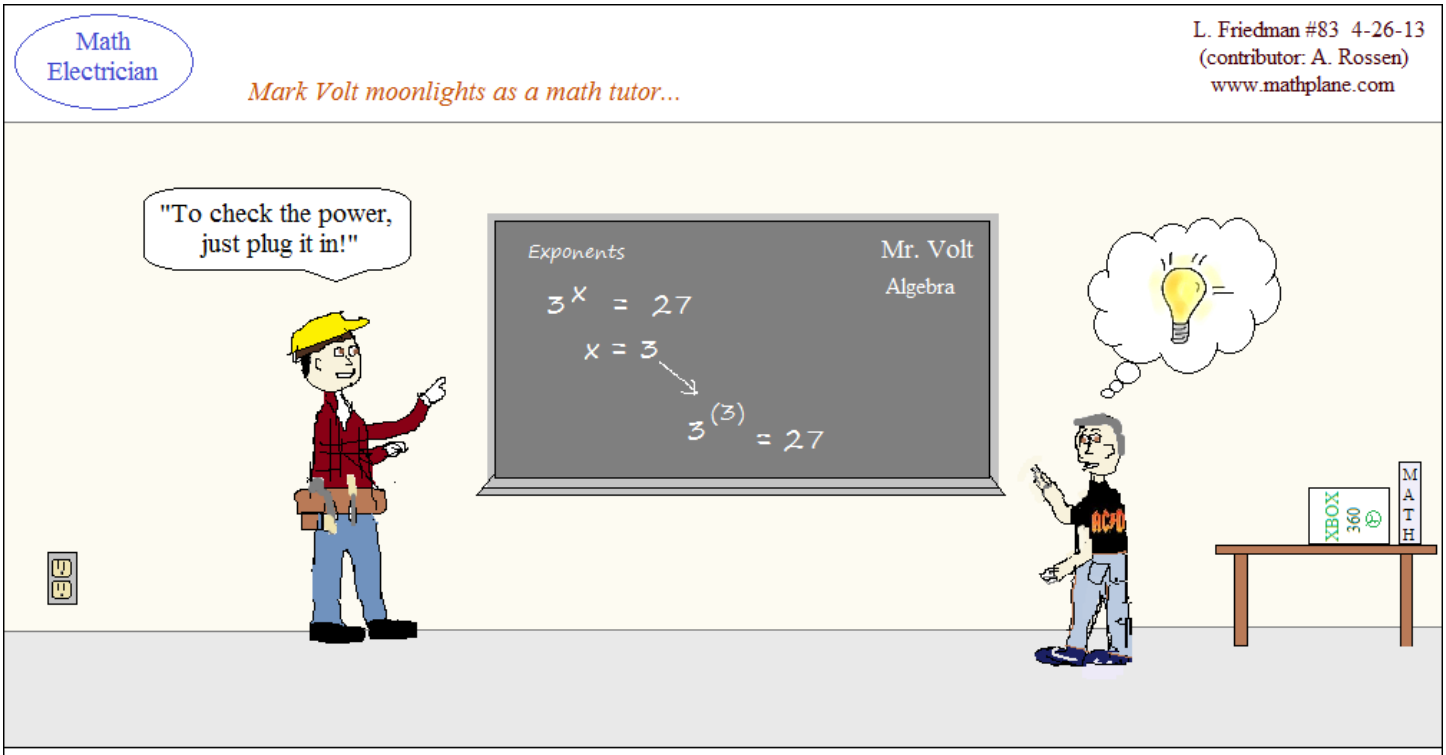
then, substitute $\frac{1 \cdot (-2)^2 \cdot (4)^5 \cdot (\frac{1}{3})}{5}$

$\boxed{\frac{1024}{15}}$

e) $\frac{(x^2y^{-3}z^4)^2}{x^{-1}y^{-3}z^6} = \frac{x^4\,y^{-6}\,z^8}{x^{-1}y^{-3}z^6}$ (exponent rule)

$\frac{x^5\,z^2}{y^3}$ (addition/subtraction rule)

then, substitute $\frac{(-2)^5 \cdot (\frac{1}{3})^2}{(4)^3} = \boxed{\frac{-1}{18}}$



Alex gets the connection!

Worksheets and Solutions-→

Simplify the expressions (leaving no negative exponents).

Simplifying (Negative) Exponents Drill

$a^{3} \cdot a^{8}$

$a^{4} \cdot a^{-7}$

$a^{-5} \cdot a^{-2}$

$a^{0} \cdot a^{-5}$

$\frac{x^{5}}{x^{2}}$

$\frac{x^{2}}{x^{5}}$

$\frac{x^{-6}}{x^{7}}$

$\frac{x^{7}}{x^{-4}}$

$b^{4} \cdot b^{-8}$

$b^{4} \div b^{-8}$

$b^{8} \cdot b^{-4}$

$b^{8} \div b^{-4}$

$\frac{m^{6} \cdot m^{-2}}{m^{3}}$

$m^{5} \cdot m^{-3} \cdot m^{0}$

$\frac{m^{7} \cdot m^{5}}{m^{-4}}$

$m^{-6} \cdot m^{-3} \cdot m$

Positive/Negative Exponents & Variables

Simplify the following:

1) $(3a^2)^3$

2) $(2b^3)^2$

3) $(-2c^3)^{-2}$

4) $(-4d^2)(-5^{-3})$

5) $(3e^2)(-2e^0)^2$

6) $(-8f^2)(fg^{-2})$

7) $h^{-2}(h^2j^3)^4(-jk)^5$

8) $(3k^7lm)^{-2}(l^2m)^0$

9) $\dfrac{6p^3q^2}{4pq^{-2}}$

10) $\dfrac{a^3b^2c^{-1}}{b^2cd^{-3}}$

11) $\dfrac{-2tu^5v^{10}}{(4t^2v)^2}$

12) $\dfrac{5r(s^0t)^{-3}}{(5st^2)^2}$

12) $\dfrac{(x^2y^{-3})(3xy)}{(xy^2)^2(6x^2y)}$

13) $\dfrac{(7x^9y^8)^0(3xy^2)^{-2}}{-2(6x^2)(xy^{-2})}$

14) $\dfrac{(a^2b^5)^3}{(ab)^{-2}} \div \dfrac{(ab^{-1})}{(a^0b)^2}$

15) $\dfrac{def}{(d^2e^3f)^{-1}} \div fed$

Simplify the expressions (leaving no negative exponents).

SOLUTIONS

Simplifying (Negative) Exponents Drill

$a^3 \cdot a^8$

add exponents

a^{11}

$a^4 \cdot a^{-7}$

add exponents

a^{-3}

"flip" $\frac{1}{a^3}$

"flip" $\frac{a^4}{a^7}$

cancel

$\frac{aaaa}{aaaaaaa}$

$\frac{1}{a^3}$

$a^{-5} \cdot a^{-2}$

add exponents

a^{-7}

"flip" $\frac{1}{a^7}$

$a^0 \cdot a^{-5}$

$1 \cdot a^{-5}$

"flip" $\frac{1}{a^5}$

$\frac{x^5}{x^2}$

subtract

x^3

cancel

$\frac{xx\,xxx}{xx}$

x^3

$\frac{x^2}{x^5}$

subtract

x^{-3}

flip

$\frac{1}{x^3}$

cancel

$\frac{xx}{xx\,xxx}$

$\frac{1}{x^3}$

$\frac{x^{-6}}{x^7}$

subtract

x^{-13}

flip

$\frac{1}{x^{13}}$

flip first

$\frac{1}{x^6 \cdot x^7}$

add

$\frac{1}{x^{13}}$

$\frac{x^7}{x^{-4}}$

subtract

x^{11}

flip

x^{11}

flip first

$\frac{x^4 \cdot x^7}{1}$

add

x^{11}

$b^4 \cdot b^{-8}$

add

b^{-4}

flip

$\frac{1}{b^4}$

flip

$\frac{b^4}{b^8}$

cancel

$\frac{bbbb}{bbbbbbbb}$

$\frac{1}{b^4}$

$b^4 \div b^{-8}$

rewrite

$\frac{b^4}{b^{-8}}$

subtract

b^{12}

rewrite

$\frac{b^4}{b^{-8}}$

flip

$b^4 b^8$

add

b^{12}

$b^8 \cdot b^{-4}$

add

b^4

flip

$\frac{b^8}{b^4}$

cancel

$\frac{bbbbbbbb}{bbbb}$

b^4

$b^8 \div b^{-4}$

rewrite

$\frac{b^8}{b^{-4}}$

subtract

b^{12}

rewrite

$\frac{b^8}{b^{-4}}$

flip

$b^4 b^8$

add

b^{12}

$\frac{m^6 \cdot m^{-2}}{m^3}$

add (numerator)

$\frac{m^4}{m^3}$

subtract

m

flip

$\frac{m^6}{m^3\ m^2}$

cancel

$\frac{mmmmmmm}{mmm\ mm}$

$m^5 \cdot m^{-3} \cdot m^0$

$m^5\ m^{-3}\ (1)$

add

m^2

flip

$\frac{m^5}{m^3}$

cancel

$\frac{mmmmm}{mmm}$

$\frac{m^{-7} \cdot m^5}{m^{-4}}$

add (numerator)

$\frac{m^{12}}{m^{-4}}$

subtract

m^{16}

$m^{-6} \cdot m^{-3} \cdot m^1$

add terms

m^{-8}

flip

$\frac{1}{m^8}$

flip

$\frac{m}{m^3\ m^6}$

cancel

$\frac{m}{mmm\ mmmmmm}$

Positive/Negative Exponents & Variables

SOLUTIONS

Simplify the following:

1) $(3a^2)^3$

$27a^6$

2) $(2b^3)^2$

$4b^6$

3) $(-2c^3)^{-2}$

$\frac{1}{(-2c^3)^2} = \frac{1}{(-2c^3)(-2c^3)}$

$\frac{1}{4c^6}$

4) $(-4d^2)(-5^{-3})$

$(-4d^2)\ \frac{-1}{125}$

$\frac{4d^2}{125}$

5) $(3e^2)(-2e^0)^2$

$(3e^2)(-2)^2$

$12e^2$

6) $(-8f^2)(fg^{-2})$

$\frac{-8f^2}{1} \cdot \frac{f}{g^2}$

$\frac{-8f^3}{g^2}$

7) $h^{-2}(h^2j^3)^4(-jk)^5$

$\frac{h^8 j^{12}}{h^2} \cdot (-1)^5 j^5 k^5$

$-h^6 j^{17} k^5$

8) $(3k^7lm)^{-2}(l^2m)^0$

$\frac{1}{(3k^7lm)^2}$

$\frac{1}{9k^{14}l^2m^2}$

9) $\frac{6p^3q^2}{4pq^{-2}}$

$\frac{3p^2q^4}{2}$

10) $\frac{a^3b^2c^{-1}}{b^2cd^{-3}}$

$\frac{a^3d^3}{c^2}$

11) $\frac{-2tu^5v^{10}}{(4t^2v)^2}$

$\frac{-2tu^5v^{10}}{16t^4v^2}$

$\frac{-u^5v^8}{8t^3}$

12) $\frac{5r(s^0t)^{-3}}{(5st^2)^2}$

$\frac{5r(t)^{-3}}{25s^2t^4}$

$\frac{r}{5s^2t^7}$

12) $\frac{(x^2y^{-3})(3xy)}{(xy^2)^2(6x^2y)}$

$\frac{\frac{x^2}{y^3} \cdot 3xy}{x^2y^4 \cdot 6x^2y}$

$\frac{\frac{3x^3}{y^2}}{6x^4y^5}$

$\frac{1}{2xy^7}$

13) $\frac{(7x^9y^8)^0(3xy^2)^{-2}}{-2(6x^2)(xy^{-2})}$

$\frac{1 \cdot 3^{-2}x^{-2}y^{-4}}{-12x^3y^{-2}}$

$\frac{\frac{1}{9}}{-12x^5 \cdot y^2}$

$\frac{-1}{108x^5y^2}$

14) $\frac{(a^2b^5)^3}{(ab)^{-2}} \div \frac{(ab^{-1})}{(a^0b)^2}$

$\frac{a^6b^{15}}{a^{-2}b^{-2}} \div \frac{ab^{-1}}{b^2}$

$a^8b^{17} \div \frac{a}{b^3}$

$a^8b^{17} \cdot \frac{b^3}{a}$

a^7b^{20}

15) $\frac{def}{(d^2e^3f)^{-1}} \div fed$

$\frac{\cancel{def}}{(d^2e^3f)^{-1}} \cdot \frac{1}{\cancel{fed}}$

d^2e^3f