Solving Rational Equations

Notes, Examples, and practice (with solutions)

Topics include cross multiplying, word problems, factoring, inequalities, extraneous answers, and more.

Solving Rational Equalities/Equations

Method 1: Combine terms (using least common denominator)

Example: $\dfrac{x}{2} + \dfrac{x}{5} = 14$ (Combine terms on left)

$\dfrac{5x}{10} + \dfrac{2x}{10} = 14$ Find least common multiple of 2 and 5

$\dfrac{7x}{10} = 14$ (Solve)

$7x = 140$

$x = 20$ Check solution : $\dfrac{20}{2} + \dfrac{20}{5} = 14$

$10 + 4 = 14$ ✓

Method 2: Multiply _entire_ equation by least common denominator

Example: $\dfrac{5n}{2} + \dfrac{1}{4} = 4$ (multiply entire equation by LCD to get rid of the fractions)

$4 \cdot \left(\dfrac{5n}{2} + \dfrac{1}{4} = 4 \right)$ Denominators are 2, 4, and 1... LCM of 1, 2 and 4 is 4...

$10n + 1 = 16$ Solve

$10n = 15$ Check solution: $\dfrac{5 \cdot \frac{3}{2}}{2} + \dfrac{1}{4} = 4$

$n = \dfrac{3}{2}$

$\dfrac{\frac{15}{2}}{2} + \dfrac{1}{4} = 4$

$\dfrac{15}{4} + \dfrac{1}{4} = 4$ ✓

2 rational expressions in an equality? Try cross multiplying...

Example:

$\dfrac{1}{x^2 + 1} = \dfrac{3}{2x + 4}$

$(2x + 4)(1) = (3)(x^2 + 1)$ Check solutions: $\dfrac{1}{(1)^2 + 1} = \dfrac{3}{2(1) + 4}$

$2x + 4 = 3x^2 + 3$

$0 = 3x^2 - 2x - 1$ $\dfrac{1}{2} = \dfrac{3}{6}$ ✓

$(3x + 1)(x - 1) = 0$ $\dfrac{1}{(-1/3)^2 + 1} = \dfrac{3}{2(-1/3) + 4}$

$x = -1/3 \ \text{or} \ 1$ $\dfrac{1}{\frac{10}{9}} = \dfrac{3}{\frac{-2}{3} + \frac{12}{3}} = \dfrac{9}{10}$ ✓

Example: $\dfrac{x}{5} = \dfrac{x+3}{8}$ Cross Multiply

$8x = 5(x + 3)$

$8x = 5x + 15$ Check: $\dfrac{5}{5} = \dfrac{5+3}{8}$

$3x = 15$ $1 = 1$ ✓

$x = 5$

Important: Check your answers! Sometimes, math techniques produce *extraneous* solutions

Example: $\dfrac{1}{x+1} = \dfrac{3}{x^2 - 1}$ Cross Multiply

$1(x^2 - 1) = 3(x + 1)$

$x^2 - 1 = 3x + 3$ Check solutions: 4: $\dfrac{1}{4+1} = \dfrac{3}{4^2 - 1}$

$x^2 - 3x - 4 = 0$ (substitute in the original equation) $\dfrac{1}{5} = \dfrac{3}{15}$ ✓

$(x - 4)(x + 1) = 0$

$\boxed{x = 4, \; \cancel{-1}}$ $x = -1$ is an extraneous solution... $\dfrac{1}{-1+1} = \dfrac{3}{(-1)^2 - 1}$

$\dfrac{1}{0} = \dfrac{3}{0}$ ✗

Pick the approach that you prefer...

Example: $\dfrac{3x+2}{12} - \dfrac{1}{6} = \dfrac{1}{3}$

method 1: combine terms and cross multiply

$$\dfrac{3x+2}{12} - \dfrac{2}{12} = \dfrac{1}{3}$$

$$\dfrac{3x+2-2}{12} = \dfrac{1}{3}$$

$$\dfrac{3x}{12} = \dfrac{1}{3}$$

$$3(3x) = 12(1)$$

$$9x = 12$$

$$x = \dfrac{4}{3}$$

method 2: get rid of denominators and solve

Least common multiple of 12, 6, and 3 is 12

--> multiply entire equation by 12

$$12\left(\dfrac{3x+2}{12} - \dfrac{1}{6} = \dfrac{1}{3}\right)$$

$$3x + 2 - 2 = 4$$

$$3x = 4$$

$$x = \dfrac{4}{3}$$

Word Problems that use Rational Expressions

Example: Underground pipes can fill a swimming pool in 4 hours.
A regular garden hose can fill the pool in 15 hours.
If both are used at the same time, how long will it take to fill the pool?

Step 1: Write Formulas and Variables

distance = rate x time

In this case, filling pool = (rate)(time)

therefore, rate = $\dfrac{\text{filling one pool}}{\text{time}}$

$\text{rate}_p = \dfrac{1 \text{ pool}}{4 \text{ hours}}$ (filling rate of pipes)

$\text{rate}_h = \dfrac{1 \text{ pool}}{15 \text{ hours}}$ (filling rate of hose)

Step 2: Establish equation and solve

We want to fill a pool with BOTH hose and pipes...

hose water + pipes water = total water

$$r_h(t) \quad + \quad r_p(t) \quad = \text{total}$$

$$\dfrac{1 \text{ pool}}{15 \text{ hours}} t + \dfrac{1 \text{ pool}}{4 \text{ hours}} t = 1 \text{ pool}$$

(multiply by 60 hours; divide out the pool)

$$4t \quad + \quad 15t \quad = \quad 60 \text{ hours}$$

$$t = \dfrac{60}{19} \text{ hours or } 3.158 \text{ hours}$$
(approx)

Step 3: Check Answer!

If time is 3.158 hours,

the pipes will add

$3.158 \text{hrs x } \dfrac{1 \text{ pool}}{4 \text{ hours}}$

.79 pools...

the hose will add

$3.158 \text{ x } \dfrac{1 \text{ pool}}{15 \text{ hours}}$

.21 pools...

Example: Tom can paint a fence in 5 hours. Huck can paint a fence in 4 hours.
On Saturday, they have to paint 3 houses. Huck begins at 9:00am, and Tom
shows up late (and begins at noon).. Working together, how long did it take to paint
the 3 houses?

Step 1: Write formulas and variables

'task' = rate x time

In this case, painting the house = (rate)(time)

therefore, rate = $\dfrac{\text{painting 1 house}}{\text{time}}$

$\text{rate}_H = \dfrac{1 \text{ house}}{4 \text{ hours}}$ (Huck's painting rate)

$\text{rate}_T = \dfrac{1 \text{ house}}{5 \text{ hours}}$ (Tom's painting rate)

Step 2: Establish equation and solve

We want to paint THREE houses, and we have to adjust the time each spends painting...

Tom paint + Huck paint = 3 houses

$$\text{rate}_T \ (t_T) + \text{rate}_H \ (t_H) = 3 \text{ houses}$$

$$\dfrac{1 \text{ house}}{5 \text{ hours}}(t - 3 \text{hrs}) + \dfrac{1 \text{ house}}{4 \text{ hours}}(t) = 3 \text{ houses}$$

(note: Tom's painting time is 3 less than Huck's)

(multiply by 20 hours; divide out the 'houses')

$$4(t - 3 \text{hrs}) + 5(t) = 60 \text{ hours}$$

$$4t - 12 \text{hrs} + 5t = 60 \text{ hours}$$

$$t = 8 \text{ hours}$$

Step 3: Check your answer!

Since t = 8 hours, Huck painted for 8 hours and Tom painted for 5 hours..

Huck paint = $(8 \text{ hours}) \dfrac{1 \text{ house}}{4 \text{ hours}} = $ 2 houses

Tom paint = $(5 \text{ hours}) \dfrac{1 \text{ house}}{5 \text{ hours}} = $ 1 house

Here are a few more examples to examine....

Example: $9x^{-1} + 4(6x + 3)^{-1} = 2(6x + 3)^{-1}$

$$\frac{9}{x} + \frac{4}{(6x + 3)} = \frac{2}{(6x + 3)}$$ change to positive exponents

$$\frac{9}{x} = \frac{2}{(6x + 3)} - \frac{4}{(6x + 3)}$$ collect 'like' terms

$$\frac{9}{x} = \frac{-2}{(6x + 3)}$$ cross multiply

$$54x + 27 = -2x$$

$$56x = -27$$

$$\boxed{x = \frac{-27}{56}}$$

Example: $\dfrac{1}{x + 2} + \dfrac{3}{x + 7} = \dfrac{5}{x^2 + 9x + 14}$

$$\frac{(x + 7)\,1}{(x + 7)(x + 2)} + \frac{3\,(x + 2)}{(x + 7)(x + 2)} = \frac{5}{(x + 7)(x + 2)}$$ Common denominators

$$x + 7 + 3x + 6 = 5$$ Simplify

$$4x = -8$$

$$x = -2$$ since $\dfrac{1}{x + 2}$ is undefined at $x = 0$, there is $\boxed{\text{NO SOLUTION!}}$

Example: $\dfrac{1}{x + 6} - \dfrac{3}{x^2 - 36} = \dfrac{x}{x - 6}$

Since the least common multiple of the denominators is $x^2 - 36$

$$(x^2 - 36)\left(\frac{1}{x + 6} - \frac{3}{x^2 - 36} = \frac{x}{x - 6}\right) = \frac{1(x^2 - 36)}{x + 6} - \frac{3(x^2 - 36)}{x^2 - 36} = \frac{x(x^2 - 36)}{x - 6}$$

$$\frac{1(x + 6)(x - 6)}{x + 6} - \frac{3(x^2 - 36)}{x^2 - 36} = \frac{x(x + 6)(x - 6)}{x - 6}$$

$$1(x - 6) - 3 = x(x + 6)$$

$$x - 9 = x^2 + 6x$$

$$0 = x^2 + 5x + 9$$

No real solutions

$$\boxed{x = \frac{-5 \pm i\sqrt{11}}{2}}$$

Example: Solve $\dfrac{2}{x+3} - \dfrac{3}{4-x} = \dfrac{2x-2}{x^2-x-12}$

Factor the terms to reveal factors...

$$\frac{2}{x+3} - \frac{3}{4-x} = \frac{2(x-1)}{(x+3)(x-4)}$$

"Reverse the middle term" for convenience...

$$(x+3)(x-4)\left(\frac{2}{x+3} - \frac{-3}{x-4} = \frac{2(x-1)}{(x+3)(x-4)} \right)$$

Multiply by least common denominator... (this removes the denominators!)

$$2(x-4) - -3(x+3) = 2(x-1)$$

$$2x - 8 + 3x + 9 = 2x - 2$$

$$3x = -3$$

$$\boxed{x = -1}$$

Quick check: $\dfrac{2}{(-1)+3} - \dfrac{3}{4-(-1)} = \dfrac{2(-1)-2}{(-1)^2-(-1)-12}$

$$\frac{2}{2} - \frac{3}{5} = \frac{-4}{-10}$$

$$\frac{2}{5} = \frac{2}{5} \checkmark$$

Example: Solve $\dfrac{4}{3x+3} = \dfrac{12}{x^2-1}$ Cross-multiply

$$36x + 36 = 4x^2 - 4 \qquad \text{Divide by 4}$$

$$9x + 9 = x^2 - 1 \qquad \text{Collect terms to one side}$$

$$x^2 - 9x - 10 = 0$$

$$(x-10)(x+1) = 0$$

$$x = 10, -1$$

Quick check: If x = -1, the right equation is undefined! ✗

If x = 10, $\dfrac{4}{33} = \dfrac{12}{99}$ ✓

$$\boxed{x = 10}$$

Example: $\dfrac{x+2}{6} = \dfrac{x+2}{x-1}$

Obviously, x = 7 is a solution (because the numerators are the same and $6 = (7) - 1$)

Let's go through the steps and cross-multiply...

$$(x+2)(x-1) = (x+2)(6)$$

$$x^2 + x - 2 = 6x + 12$$

$$x^2 - 5x - 14 = 0$$

$$(x-7)(x+2) = 0$$

$\boxed{x = 7 \text{ and } -2}$

$0 = 0$ ✓

and

$3/2 = 3/2$ ✓

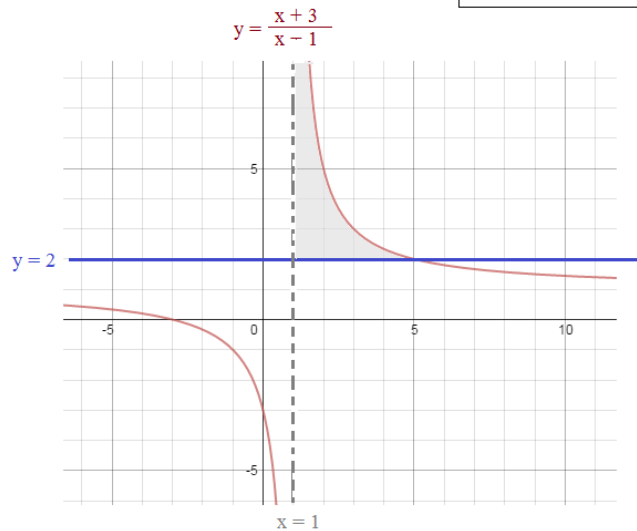*Example:* Solve and graph $\dfrac{x+3}{x-1} \geq 2$

Step 1: Look for "critical values"

$$\frac{x+3}{x-1} = \frac{2}{1}$$

$$x+3 = 2x-2$$

$$x = 5$$

AND, there is a vertical asymptote
at
$$x = 1$$

Step 2: Set up number line (with open and closed intervals)

Step 3; Test regions

If x = 0: $\dfrac{0+3}{0-1} = -3 \ngeq 2$

If x = 3: $\dfrac{3+3}{3-1} = 3 \geq 2$

If x = 7: $\dfrac{7+3}{7-1} = 5/3 \ngeq 2$

$(1, 5]$ or $1 < x \leq 5$

$$y = \frac{x+3}{x-1}$$

$$y = 2$$

$$x = 1$$

Example: Solve $\dfrac{x+2}{x+3} \leq \dfrac{x-1}{x-2}$

Method 1: Cross-Multiply, Solve, and Check Regions

$$(x+2)(x-2) = (x+3)(x-1)$$

$$x^2 - 4 = x^2 + 2x - 3$$

$$-1 = 2x$$

$$x = -1/2$$

Also, the equations are undefined at

$$x = -3 \quad \text{or} \quad x = 2$$

asymptotes/undefined points are always open circles.
since inequality is less than or equal, it's a closed circle

test -5: $\dfrac{-5+2}{-5+3} \leq \dfrac{-5-1}{-5-2}$ $3/2 \leq 6/7$ NO

test -2: $\dfrac{-2+2}{-2+3} \leq \dfrac{-2-1}{-2-2}$ $0 \leq 3/4$ YES

test 0: $\dfrac{0+2}{0+3} \leq \dfrac{0-1}{0-2}$ $2/3 \leq 1/2$ NO

test 5: $\dfrac{5+2}{5+3} \leq \dfrac{5-1}{5-2}$ $7/8 \leq 4/3$ YES

Method 2: Combine Terms on one-side and Check Regions

$$\frac{x+2}{x+3} - \frac{x-1}{x-2} \leq 0$$

$$\frac{x+2}{x+3} \cdot \frac{(x-2)}{(x-2)} - \frac{x-1}{x-2} \cdot \frac{(x+3)}{(x+3)} \leq 0$$

$$\frac{x^2 - 4 - (x^2 + 2x - 3)}{(x+3)(x-2)} \leq 0$$

$$\frac{-2x - 1}{(x+3)(x-2)} \leq 0$$

	-3	-1/2	2
-2x -1	+	+	−	−
x + 3	−	+	+	+
x - 2	−	−	−	+
All multiplied together	+	−	+	−

Less than zero

$$(-3, -1/2] \cup (2, \infty)$$

mathplane.com

Despite a slight stutter and bizarre math examples,
many enjoyed his Saturday morning Algebra lessons.

Practice Questions -→

1) $\dfrac{x}{2} + \dfrac{x}{10} = 6$

2) $\dfrac{3y}{2} + \dfrac{7}{4} = 1$

3) $\dfrac{3}{x+1} = \dfrac{1}{x^2 - 1}$

4) $\dfrac{11}{3x} - \dfrac{1}{3} = \dfrac{-4}{x^2}$

5) If Jim can paint a fence in 6 hours, and Tim can paint a fence in 4 hours. How long will it take them to paint a fence *working together*?

6) $$\dfrac{\dfrac{4}{x^2 - 25} + \dfrac{2}{x - 5}}{\dfrac{1}{x + 5} + \dfrac{1}{x - 5}} = 1$$

7) $$\dfrac{2}{x + 1} + \dfrac{x}{x - 1} = \dfrac{2}{x^2 - 1}$$

8) $$\dfrac{6x}{x + 4} + 4 = \dfrac{2x + 2}{x - 1}$$

9) $$\dfrac{3}{x + 1} = \dfrac{27}{x^2 - 1}$$

10) $\dfrac{x}{5} + \dfrac{3x}{x+1} = \dfrac{36}{5x+5}$

11) $\dfrac{7}{n-3} = \dfrac{2n+1}{21}$

12) $\dfrac{5}{z+1} + \dfrac{z}{z+3} = \dfrac{10z+15}{z^2+4z+3}$

13) $\dfrac{x}{x+3} - \dfrac{2}{x+6} = \dfrac{-18}{2(x^2+9x+18)}$

Rational Expressions Quiz

Simplify:

1) $\dfrac{x}{3} + \dfrac{x}{5}$

2) $\dfrac{1}{a+4} + \dfrac{3}{a+4}$

3) $\dfrac{4}{x^2+4x+3} - \dfrac{1}{x+3}$

4) $\dfrac{3}{2x+6} + \dfrac{4}{6x+18}$

5) $\dfrac{7}{2d} - \dfrac{3}{2d}$

6) $\dfrac{2x}{x^2-1} - \dfrac{3}{x+1}$

7) $\dfrac{k-10}{20-2k}$

Solve:

1) $\dfrac{x}{x+2} + \dfrac{4}{x-2} = 1$

2) $\dfrac{1}{x} + \dfrac{1}{2x} = \dfrac{1}{6}$

3) $\dfrac{1}{3s} = \dfrac{s}{2} - \dfrac{1}{6s}$

4) $\dfrac{x+2}{x+8} = \dfrac{x-2}{x+4}$

5) $1 - \dfrac{3}{z} = \dfrac{4}{z^2}$

6) $\dfrac{d}{3} + \dfrac{1}{2} = \dfrac{1}{3d}$

7) $\dfrac{1}{c-3} = \dfrac{c}{4}$

8) $\dfrac{5}{x-2} = \dfrac{5x+10}{x^2}$

1) A girl can wash the windows in 45 minutes.
Working with her brother, they can wash the windows in 20 minutes.

Working by himself, how fast can the brother wash the windows?

2) Mac wants to fill a fuel tank.
Pump 1 can fill a tank in 5 hours.
Pump 2 can fill a tank in 8 hours.

At 1:00 pm, Mac starts filling the tank with Pump 1...
Later in the afternoon, Pump 2 becomes available. So, Mac starts using it to
fill the fuel tank -- as he continues to use Pump 1. (So, both are filling the tank.)

At 5:00, the fuel tank is full.

What time did Mac begin using Pump 2?

3) My garden hose can fill the swimming pool in 3 hours.
Or, a high-powered tank can fill the pool in 40 minutes.

At 1:00, I began filling the pool, using the garden hose.
An hour later, the tank arrived. So, at 2:00,
we continued filling the pool with both the hose and the tank.

What time did we fill the pool?

4) At 9:00 am, Danny goes out for his daily jog.
    He runs due north at 7 miles per hour. Then, turns around and runs due south at a pace of 8 miles per hour.
    If he returns to his starting spot at 9:50 am, how far did he run?

5) Xavier runs 14 miles and walks 18 miles in 5 hours.
    If he is able to run twice as fast as he can walk, how fast does he walk?

6) On an oval track, a rider can go around in 30 seconds...
    Another rider can go around one lap in 40 seconds...

    If the first rider passes the second rider, how long will it take to pass the rider again?!?!

1) $x(x-3)^{-1} + x(x+3)^{-1} = 18(x^2-9)^{-1}$

2) $\dfrac{4}{x} - \dfrac{4}{x-6} = \dfrac{24}{6x-x^2}$

3) Given the rational equation $\dfrac{x}{x-2} + \dfrac{x+3}{x+2} = \dfrac{a}{x^2-4}$

When this equation is solved for x, one of the solutions of the quadratic equation is extraneous.... List all possible values for a....
Also, identify the solutions of x.

4) Simplify and give restrictions...

$$\dfrac{x^2+6x+5}{x^2-4} \cdot \dfrac{x^2-1}{x^2+3x+2}$$

5) Find the missing expression:

$$\frac{x^2 + 6x - 7}{x^2 + 1} \div \frac{????}{(x + 10)(x + 3)} = \frac{x^2 + 17x + 70}{(x + 1)^2}$$

6) $\dfrac{1}{x + 1} + \dfrac{x}{x - 6} - \dfrac{5x - 2}{x^2 - 5x - 6}$

7) $\dfrac{x + 1}{3} - \dfrac{x + 1}{x + 2} = 0$

8) $2 + \dfrac{5}{x - 4} = \dfrac{x + 1}{x - 4}$

Solving Rational Equations Quiz	SOLUTIONS

1) $\dfrac{x}{2} + \dfrac{x}{10} = 6$

$\dfrac{10x}{20} + \dfrac{2x}{20} = 6$

$\dfrac{12x}{20} = 6$

$12x = 120$

$\boxed{x = 10}$

Quick Check:

$\dfrac{(10)}{2} + \dfrac{(10)}{10} = 6$

$5 + 1 = 6$ ✓

2) $\dfrac{3y}{2} + \dfrac{7}{4} = 1$ (multiply entire equation by 4)

$\dfrac{12y}{2} + \dfrac{28}{4} = 4$

$6y = -3$

$\boxed{y = \dfrac{-1}{2}}$

Quick Check:

$\dfrac{3(-1/2)}{2} + \dfrac{7}{4} = 1$

$\dfrac{-3}{4} + \dfrac{7}{4} = 1$ ✓

3) $\dfrac{3}{x+1} = \dfrac{1}{x^2 - 1}$ Cross Multiply

$3x^2 - 3 = x + 1$

$3x^2 - x - 4 = 0$

$(3x - 4)(x + 1) = 0$

$\boxed{x = \dfrac{4}{3}}$ or $\cancel{-1}$

Check solutions:

-1: EXTRANEOUS

$\dfrac{3}{(-1)+1} = \dfrac{3}{0}$

4/3:

$\dfrac{3}{(4/3)+1} = \dfrac{1}{(4/3)^2 - 1}$

$\dfrac{9}{7} = \dfrac{9}{7}$ ✓

4) $\dfrac{11}{3x} - \dfrac{1}{3} = \dfrac{-4}{x^2}$ combine terms on left side of equation

$\dfrac{11}{3x} - \dfrac{x}{3x} = \dfrac{-4}{x^2}$

$\dfrac{11 - x}{3x} = \dfrac{-4}{x^2}$ cross multiply

$-12x = -x^3 + 11x^2$

$x^3 - 11x^2 - 12x = 0$ factor/solve

$x(x - 12)(x + 1) = 0$

$\boxed{x = \cancel{0}, 12, -1}$ check solutions!!

0: extraneous (because undefined) ✗

12: $\dfrac{11}{36} - \dfrac{1}{3} = \dfrac{-4}{144}$

$\dfrac{-1}{36} = \dfrac{-1}{36}$ ✓

-1: $\dfrac{11}{-3} - \dfrac{1}{3} = \dfrac{-4}{1}$ ✓

5) If Jim can paint a fence in 6 hours, and Tim can paint a fence in 4 hours. How long will it take them to paint a fence *working together*?

$\dfrac{\text{amount of fence}}{} = (\text{rate})(\text{time})$

therefore, $\boxed{\text{rate} = \dfrac{\text{fence}}{\text{time}}}$

Jim's rate: $\text{rate}_J = \dfrac{1 \text{ fence}}{6 \text{ hours}}$

Tim's rate: $\text{rate}_T = \dfrac{1 \text{ fence}}{4 \text{ hours}}$

Working together:

Jim's amount + Tim's amount = 1 fence

$\text{rate}_J (t) + \text{rate}_T (t) = 1$ fence

$\dfrac{1 \text{ fence}}{6 \text{ hours}}(t) + \dfrac{1 \text{ fence}}{4 \text{ hours}}(t) = 1$ fence

Solve (multiply by 24 hours; divide by fence)

$4t + 6t = 24$ hours

$\boxed{t = 12/5 \text{ hours} \quad (2 \text{ hours}, 24 \text{ minutes})}$

Check:

Jim: $(1/6)(12/5) = 2/5$

Tim: $(1/4)(12/5) = 3/5$

total: $5/5$ ✓

SOLUTIONS

6)
$$\cfrac{\dfrac{4}{x^2-25}+\dfrac{2}{x-5}}{\dfrac{1}{x+5}+\dfrac{1}{x-5}}=1$$

numerator $\longrightarrow$ $\dfrac{4}{(x-5)(x+5)}+\dfrac{2}{(x-5)(x+5)}\dfrac{(x+5)}{}$ $\qquad$ $\dfrac{2x+14}{(x-5)(x+5)}$

denominator $\longrightarrow$ $\dfrac{1\cdot(x-5)}{(x+5)(x-5)}+\dfrac{1\cdot(x+5)}{(x-5)(x+5)}$ $\qquad$ $\dfrac{2x}{(x-5)(x+5)}$

Strategy: Simplify numerator and denominator separately...

Then, divide!

To divide fractions, we can 'invert and multiply':

$$\dfrac{2x+14}{(x-5)(x+5)}\cdot\dfrac{(x-5)(x+5)}{2x}=\dfrac{2x+14}{2x}=\dfrac{x+7}{x}$$

$$\dfrac{x+7}{x}=1 \qquad \boxed{\text{NO SOLUTIONS!}}$$

7) $\dfrac{2}{x+1}+\dfrac{x}{x-1}=\dfrac{2}{x^2-1}$

multiply entire equation by LCD

$$2(x-1)+x(x+1)=2$$
$$2x-2+x^2+x=2$$
$$x^2+3x-4=0$$
$$(x+4)(x-1)=0$$
$$x=-4,\ \cancel{1}$$

$\boxed{x=-4}$

Least Common Denominator of the 3 rational expressions is

$$x^2-1$$

Check Solutions:

If $x=1$, then there are 0's in the denominators of 2 of the expressions... therefore, it is extraneous.... $\times$

If $x=-4$, then $\dfrac{2}{(-4)+1}+\dfrac{(-4)}{(-4)-1}=\dfrac{2}{(-4)^2-1}$

$$\dfrac{2}{-3}+\dfrac{-4}{-5}=\dfrac{2}{15}$$

$$\dfrac{-10}{15}+\dfrac{12}{15}=\dfrac{2}{15}\ \checkmark$$

8) $\dfrac{6x}{x+4}+4=\dfrac{2x+2}{x-1}$

$$\dfrac{6x+4(x+4)}{(x+4)}=\dfrac{2x+2}{x-1}$$

$$\dfrac{10x+16}{(x+4)}=\dfrac{2x+2}{x-1}$$

cross multiply: FOIL

$$10x^2+16x-10x-16=2x^2+8x+2x+8$$
$$8x^2-4x-24=0$$
$$4(2x^2-x-6)=0$$
$$4(2x+3)(x-2)=0$$
$$\boxed{x=-3/2,\ 2}$$

Approach: combine terms on left side.. then, cross-multiply..

Test answers:

If $x=2$, $\qquad \dfrac{6(2)}{(2)+4}+4=\dfrac{2(2)+2}{(2)-1}$

$$2+4=6\ \checkmark$$

If $x=-3/2$,

$$\dfrac{6(-3/2)}{(-3/2)+4}+4=\dfrac{2(-3/2)+2}{(-3/2)-1}$$

$$\dfrac{-9}{(5/2)}+4=\dfrac{-1}{(-5/2)}$$

$$\dfrac{-18}{5}+\dfrac{20}{5}=\dfrac{2}{5}\ \checkmark$$

9) $\dfrac{3}{x+1}=\dfrac{27}{x^2-1}$

Cross multiply: $\qquad 3x^2-3=27x+27$

Collect terms, factor, and solve: $\quad 3x^2-27x-30=0$

$$3(x^2-9x-10=0$$
$$3(x+1)(x-10)=0$$
$$x=\cancel{-1},\ 10$$

Check solutions: $\qquad x=-1$:

$$\dfrac{3}{(-1)+1}=\dfrac{27}{(-1)^2-1}$$

$$\dfrac{3}{0}\text{ is undefined;}$$

$$-1\text{ is extraneous!}$$

$\boxed{x=10}$ $\qquad \dfrac{3}{(10)+1}=\dfrac{27}{(10)^2-1}$

$$\checkmark\ \dfrac{3}{11}=\dfrac{27}{99}$$

10) $\dfrac{x}{5} + \dfrac{3x}{x+1} = \dfrac{36}{5x+5}$ Since least common denominator / least common multiple
is $5x + 5$,

$$5x+5\left(\dfrac{x}{5} + \dfrac{3x}{x+1} = \dfrac{36}{5x+5}\right) = x(x+1) + 3x(5) = 36$$

$$= x^2 + x + 15x - 36 = 0$$

$$\boxed{x = 2, -18}$$

$$= (x-2)(x+18) = 0$$

Check: $\dfrac{(2)}{5} + \dfrac{3(2)}{(2)+1} = \dfrac{36}{5(2)+5}$ $\dfrac{(-18)}{5} + \dfrac{3(-18)}{(-18)+1} = \dfrac{36}{5(-18)+5}$

$2\dfrac{2}{5} = \dfrac{36}{15}$ ✓ $\dfrac{-18}{5} + \dfrac{-54}{-17} = \dfrac{36}{-85}$ $\dfrac{306}{-85} + \dfrac{-270}{-85} = \dfrac{36}{-85}$ ✓

11) $\dfrac{7}{n-3} = \dfrac{2n+1}{21}$ Cross multiply...

$(n-3)(2n+1) = (7)(21)$ Check: $\dfrac{7}{(-7.5)-3} = \dfrac{2(-7.5)+1}{21}$

$2n^2 - 5n - 3 = 147$ $\dfrac{7}{-10.5} = \dfrac{-14}{21}$ ✓

$2n^2 - 5n - 150 = 0$

$(2n+15)(n-10) = 0$ $\dfrac{7}{(10)-3} = \dfrac{2(10)+1}{21}$

$\boxed{n = -15/2,\ 10}$ $\dfrac{7}{7} = \dfrac{21}{21}$ ✓

12) $\dfrac{5}{z+1} + \dfrac{z}{z+3} = \dfrac{10z+15}{z^2+4z+3}$

Since $(z+3)(z+1)$ is common denominator,

$(z+3)(z+1)$

$$\dfrac{5(z+3)}{(z+1)(z+3)} + \dfrac{z(z+1)}{(z+3)(z+1)} = \dfrac{10z+15}{(z+3)(z+1)}$$

Eliminate all the denominators and collect terms...

$5z + 15 + z^2 + z = 10z + 15$ Check: $\dfrac{5}{(0)+1} + \dfrac{(0)}{(0)+3} = \dfrac{10(0)+15}{(0)+4(0)+3}$

$z^2 - 4z = 0$ $5 + 0 = 5$ ✓

$z(z-4) = 0$ $\dfrac{5}{(4)+1} + \dfrac{(4)}{(4)+3} = \dfrac{10(4)+15}{(4)^2+4(4)+3}$

$\boxed{z = 0,\ 4}$ $1\dfrac{4}{7} = \dfrac{55}{35} = \dfrac{11}{7}$ ✓

13) $\dfrac{x}{x+3} - \dfrac{2}{x+6} = \dfrac{-18}{2(x^2+9x+18)}$

$$\dfrac{2x(x+6)}{2(x+3)(x+6)} - \dfrac{2(2)(x+3)}{2(x+6)(x+3)} = \dfrac{-18}{2(x+3)(x+6)}$$

$2x^2 + 12x - (4x + 12) = -18$ Check: if $x = -3$, then $\dfrac{x}{x+3}$ is undefined.. ✗

$2x^2 + 8x + 6 = 0$ if $x = -1$, then

$2(x+1)(x+3) = 0$ $\dfrac{(-1)}{(-1)+3} - \dfrac{2}{(-1)+6} = \dfrac{-18}{2(10)}$

$\boxed{x = -1,\ -3}$ $-1/2 - (2/5) = -9/10$ ✓

-3 is extraneous...

Simplify:

1) $\dfrac{x}{3} + \dfrac{x}{5}$

$\dfrac{5x}{15} + \dfrac{3x}{15} =$

$\boxed{\dfrac{8x}{15}}$

2) $\dfrac{1}{a+4} + \dfrac{3}{a+4}$

$\boxed{\dfrac{4}{a+4}}$

3) $\dfrac{4}{x^2+4x+3} - \dfrac{1}{x+3}$

$\dfrac{4}{(x+1)(x+3)} - \dfrac{1}{(x+3)} =$

$\dfrac{4}{(x+1)(x+3)} - \dfrac{(x+1)}{(x+1)(x+3)} =$

$\boxed{\dfrac{3-x}{(x+1)(x+3)}}$

4) $\dfrac{3}{2x+6} + \dfrac{4}{6x+18}$

$\dfrac{3}{2(x+3)} + \dfrac{4}{6(x+3)} =$

$\dfrac{9}{6(x+3)} + \dfrac{4}{6(x+3)} =$

$\boxed{\dfrac{13}{6(x+3)}}$

5) $\dfrac{7}{2d} - \dfrac{3}{2d}$

$\dfrac{4}{2d} =$

$\boxed{\dfrac{2}{d}}$

6) $\dfrac{2x}{x^2-1} - \dfrac{3}{x+1}$

$\dfrac{2x}{(x+1)(x-1)} - \dfrac{3}{(x+1)} =$

$\dfrac{2x}{(x+1)(x-1)} - \dfrac{3(x-1)}{(x+1)(x-1)} =$

$\dfrac{-x+3}{(x+1)(x-1)} = \boxed{\dfrac{3-x}{x^2-1}}$

7) $\dfrac{k-10}{20-2k}$

$\dfrac{k-10}{-2(k-10)} =$

$\boxed{-\dfrac{1}{2}}$

Solve: (Plug in to check solutions)

1) $\dfrac{x}{x+2} + \dfrac{4}{x-2} = 1$

$\dfrac{x(x-2)}{(x+2)(x-2)} + \dfrac{4(x+2)}{(x-2)(x+2)} = 1$

$\dfrac{x^2-2x+4x+8}{x^2-4} = \dfrac{1}{1}$

$x^2-4 = x^2+2x+8$

$\boxed{x = -6}$

$\dfrac{(-6)}{(-6)+2} + \dfrac{4}{(-6)-2}$

$\dfrac{3}{2} + \dfrac{4}{-8} = 1$ ✓

2) $\dfrac{1}{x} + \dfrac{1}{2x} = \dfrac{1}{6}$

Use common denominator

$\dfrac{6}{6x} + \dfrac{3}{6x} = \dfrac{x}{6x}$

$\dfrac{9}{6x} = \dfrac{x}{6x}$

$\boxed{x = 9}$

$\dfrac{1}{(9)} + \dfrac{1}{2(9)} = \dfrac{1}{6}$

$\dfrac{2}{18} + \dfrac{1}{18} = \dfrac{3}{18}$ ✓

3) $\dfrac{1}{3s} = \dfrac{s}{2} - \dfrac{1}{6s}$

multiply by 6s

$2 = 3s^2 - 1$

$3s^2 = 3$

$\boxed{s = 1, -1}$

$\dfrac{1}{3(1)} = \dfrac{(1)}{2} - \dfrac{1}{6(1)}$

$\dfrac{1}{3} = \dfrac{1}{2} - \dfrac{1}{6}$ ✓

$\dfrac{1}{3(-1)} = \dfrac{(-1)}{2} - \dfrac{1}{6(-1)}$

$\dfrac{-1}{3} = \dfrac{-1}{2} + \dfrac{1}{6}$ ✓

4) $\dfrac{x+2}{x+8} = \dfrac{x-2}{x+4}$

cross multiply

$x^2+2x+4x+8 = x^2+8x-2x-16$

$6x+8 = 6x-16$

$\boxed{\text{No solution!}}$

5) $1 - \dfrac{3}{z} = \dfrac{4}{z^2}$

multiply by z^2

$z^2-3z = 4$

$z^2-3z-4 = 0$

$(z-4)(z+1) = 0$

$\boxed{z = 4, -1}$

$1 - \dfrac{3}{(4)} = \dfrac{4}{(4)^2}$

$1 - 3/4 = 1/4$ ✓

$1 - \dfrac{3}{(-1)} = \dfrac{4}{(-1)^2}$

$1 - (-3) = 4/1$ ✓

6) $\dfrac{d}{3} + \dfrac{1}{2} = \dfrac{1}{3d}$

multiply by 6d

$2d^2 + 3d = 2$

$2d^2 + 3d - 2 = 0$

$(2d-1)(d+2) = 0$

$\boxed{d = 1/2, -2}$

$\dfrac{(-2)}{3} + \dfrac{1}{2} = \dfrac{1}{3(-2)}$

$\dfrac{-4}{6} + \dfrac{3}{6} = \dfrac{1}{-6}$ ✓

$\dfrac{(1/2)}{3} + \dfrac{1}{2} = \dfrac{1}{3(1/2)}$

$\dfrac{1}{6} + \dfrac{3}{6} = \dfrac{2}{3}$ ✓

7) $\dfrac{1}{c-3} = \dfrac{c}{4}$

cross multiply

$c^2-3c = 4$

$c^2-3c-4 = 0$

$(c-4)(c+1) = 0$

$\boxed{c = 4, -1}$

$\dfrac{1}{(4)-3} = \dfrac{(4)}{4}$ ✓

$\dfrac{1}{(-1)-3} = \dfrac{(-1)}{4}$ ✓

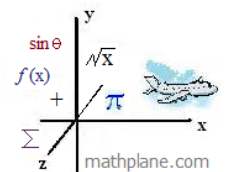**8)** $\dfrac{5}{x-2} = \dfrac{5x+10}{x^2}$

cross multiply

$5x^2 = 5x^2 - 10x + 10x - 20$

$5x^2 = 5x^2 - 20$

$\boxed{\text{No Solution!}}$

1) A girl can wash the windows in 45 minutes.
Working with her brother, they can wash the windows in 20 minutes.

Working by himself, how fast can the brother wash the windows?

the girl's rate is $\dfrac{1 \text{ window}}{45 \text{ minutes}}$

the brother's rate is $\dfrac{1 \text{ window}}{x \text{ minutes}}$

amount   amount
girl   brother

$$1 = \frac{1}{45}t + \frac{1}{x}t \qquad \frac{\text{window}}{\text{amount}} = \text{rate x time}$$

let t = 20 minutes

$$1 = \frac{20}{45} + \frac{20}{x}$$

$$\boxed{x = 36 \text{ minutes}}$$

$$\frac{25}{45} = \frac{20}{x}$$

2) Mac wants to fill a fuel tank.
Pump 1 can fill a tank in 5 hours.
Pump 2 can fill a tank in 8 hours.

At 1:00 pm, Mac starts filling the tank with Pump 1...
Later in the afternoon, Pump 2 becomes available. So, Mac starts using it to
fill the fuel tank -- as he continues to use Pump 1. (So, both are filling the tank.)

At 5:00, the fuel tank is full.

What time did Mac begin using Pump 2?

$$\frac{1 \text{ tank}}{5 \text{ hours}}(4 \text{ hours}) + \frac{1 \text{ tank}}{8 \text{ hours}}(t) = 1 \text{ tank}$$

$$\frac{4}{5}(\text{tank}) + \frac{1 \text{ tank}}{8}(\text{time}) = 1 \text{ tank}$$

$$\frac{4}{5} + \frac{1}{8}t = 1$$

$$\frac{5t}{40} = \frac{8}{40} \qquad t = 1.6 \text{ hours} \quad (1 \text{ hr. } 36 \text{ min})$$

$$\boxed{3:24 \text{ PM, Pump 2 began...}}$$

3) My garden hose can fill the swimming pool in 3 hours.
Or, a high-powered tank can fill the pool in 40 minutes.

At 1:00, I began filling the pool, using the garden hose.
An hour later, the tank arrived. So, at 2:00,
we continued filling the pool with both the hose and the tank.

What time did we fill the pool?

Approach 1:

Garden hose: in 1 hour, garden hose can fill 1/3 of the pool....

So, at 2:00,

$$\frac{1}{180}(t) + \frac{1}{40}(t) = \frac{2}{3}$$

$$2t + 9t = 240$$

$$t = 21.8$$

2:00 + 22 minutes ----> $\boxed{2:22}$

Approach 2:

Altogether: $\dfrac{1}{180}(t) + \dfrac{1}{40}(t - 60) = 1$

pool rate x time   tank rate x time

$$2(t) + 9(t - 60) = 360$$

$$11t = 900$$

$$t = 81.8$$

1:00 + 82 minutes ----> $\boxed{2:22 \text{ pm}}$

4) At 9:00 am, Danny goes out for his daily jog.
   He runs due north at 7 miles per hour. Then, turns around and runs due south at a pace of 8 miles per hour.
   If he returns to his starting spot at 9:50 am, how far did he run?

SOLUTIONS

distance = (rate)(time)

North: distance = $(\dfrac{7 \text{ miles}}{60 \text{ minutes}})$(time)

$d_N = \dfrac{7 \text{ miles}}{60 \text{ min.}}$ (t)

South: distance = $(\dfrac{8 \text{ miles}}{60 \text{ minutes}})$(time)

$d_S = \dfrac{8 \text{ miles}}{60 \text{ min.}}$ (50 min. − t)

Since the distance north is the same as the distance south,
we set the equations equal to each other...

$\dfrac{7 \text{ miles}}{60 \text{ min.}}$ (t) = $\dfrac{8 \text{ miles}}{60 \text{ min.}}$ (50 min − t)

(units cancel)

$\dfrac{7}{60} t = \dfrac{400 \text{ min}}{60} - \dfrac{8}{60} t$

(multiply by 60; then, collect 'like' terms)

15t = 400 minutes

t = 26 2/3 minutes

therefore, Danny runs 26 2/3 minutes North
and runs 23 1/3 minutes South...

distance north = (26 2/3)(7/60) = 3.11 miles   total: 6.22 miles
distance south = (23 1/3)(8/60) = 3.11 miles

5) Xavier runs 14 miles and walks 18 miles in 5 hours.
   If he is able to run twice as fast as he can walk, how fast does he walk?

distance = rate(time)

walking = w
running = j

$t_j = \dfrac{14 \text{ miles}}{2r}$

time = $\dfrac{\text{distance}}{\text{rate}}$

$t_w = \dfrac{18 \text{ miles}}{r}$

$t_j + t_w$ = 5 hours

$\dfrac{14 \text{ miles}}{2r} + \dfrac{18 \text{ miles}}{r}$ = 5 hours

multiply by 2r...

Xavier walks 5 miles per hour

runs 10 miles per hour..

14 miles + 36 miles = 2r(5 hours)

50 miles = 10r(hours)

check: run 14 miles at 10 miles per hour...
         1.4 hours...
         walk 18 miles at 5 miles per hour..
         3.6 hours...
         5 hours total ✓

r = 5 miles per hour   2r = 10 miles per hour

6) On an oval track, a rider can go around in 30 seconds...
   Another rider can go around one lap in 40 seconds...

   If the first rider passes the second rider, how long will it take to pass the rider again?!?!

method 1: set up rate equation

$\dfrac{1}{30} t - \dfrac{1}{40} t$ = 1

4t - 3t = 120

t = 120 seconds..

method 2: intuitively analyze

Assume the track is 120 feet per lap...

30 seconds/lap   4 feet per second

40 seconds/lap   3 feet per second

So, rider 1 is gaining 1 foot per second..
Since lap is 120 feet...
The rider will gain 120 feet in 120 seconds..
So, it will take 2 minutes to lap the rider again..

1) $x(x-3)^{-1} + x(x+3)^{-1} = 18(x^2-9)^{-1}$

$$\frac{x}{(x-3)} + \frac{x}{(x+3)} = \frac{18}{(x^2-9)}$$

SOLUTIONS

$$\frac{x^2+3x + x^2-3x}{(x+3)(x-3)} = \frac{18}{(x^2-9)}$$

$$2x^2 = 18$$

$$x = 3, -3$$

BUT, these are not in the domain!

Therefore, there is no solution...

2) $\frac{4}{x} - \frac{4}{x-6} = \frac{24}{6x-x^2}$

$$\frac{4}{x} + \frac{4}{6-x} = \frac{24}{x(6-x)}$$

$$24 - 4x + 4x = 24$$

$$24 = 24$$

Identity! so, all values of x would work...

EXCEPT 0 and 6 (because equations would be undefined)

All real numbers, except 0 and 6

3) Given the rational equation $\frac{x}{x-2} + \frac{x+3}{x+2} = \frac{a}{x^2-4}$

When this equation is solved for x, one of the solutions of the quadratic equation is extraneous.... List all possible values for a....
                    Also, identify the solutions of x.

Now, go back:

If a = 8 $\frac{x}{x-2} + \frac{x+3}{x+2} = \frac{8}{x^2-4}$

$x(x+2) + (x-2)(x+3) = a$

$x^2 + 2x + x^2 + x - 6 = a$

$2x^2 + 3x - 6 = a$

***If a solution is extraneous, then x = 2 or x = -2!!

If x = 2, then a = 8

If x = -2, then a = -4

$2x^2 + 3x - 6 = 8$

$(2x+7)(x-2) = 0$ -7/2

If a = -4 $\frac{x}{x-2} + \frac{x+3}{x+2} = \frac{-4}{x^2-4}$

$2x^2 + 3x - 6 = -4$

$(2x-1)(x+2) = 0$ 1/2

4) Simplify and give restrictions...

$$\frac{x^2+6x+5}{x^2-4} \cdot \frac{x^2-1}{x^2+3x+2}$$

$$\frac{(x+1)(x+5)}{(x+2)(x-2)} \cdot \frac{(x+2)(x+1)}{(x+1)(x-1)}$$

$$\frac{(x+1)(x+5)}{(x+2)(x-2)} \cdot \frac{(x+2)(x+1)}{(x+1)(x-1)}$$

$$\frac{x^2+6x+5}{x^2-3x+2}$$

Restrictions:

denominators cannot = 0

x cannot equal 2, -2, -1

then, when dividing, x cannot equal 1 or -1

all reals except -2, -1, 1, 2

5) Find the missing expression:

$$\frac{x^2 + 6x - 7}{x^2 + 1} \div \frac{????}{(x + 10)(x + 3)} = \frac{x^2 + 17x + 70}{(x + 1)^2}$$

SOLUTIONS

$$\frac{(x + 7)(x - 1)}{(x + 1)(x - 1)} \cdot \frac{(x + 10)(x + 3)}{????} = \frac{(x + 7)(x + 10)}{(x + 1)(x + 1)}$$

we need an $(x + 1)$...
and, we
need an $(x + 3)$ to cancel...

$\boxed{(x + 1)(x + 3)}$

6) $$\frac{1}{x + 1} + \frac{x}{x - 6} - \frac{5x - 2}{x^2 - 5x - 6}$$

$$\frac{x - 6 + x^2 + x - 5x + 2}{(x + 1)(x - 6)} = \frac{x^2 - 3x - 4}{(x + 1)(x - 6)}$$

$$\boxed{\frac{x - 4}{x - 6}}$$

7) $$\frac{x + 1}{3} - \frac{x + 1}{x + 2} = 0 \qquad \frac{x + 1}{3} = \frac{x + 1}{x + 2}$$

since the numerators are the same, the obvious answer is

$$3 = x + 2 \qquad x = 1$$

But, wait! There is one more answer....

$$(x + 1)(x + 2) = 3(x + 1)$$

$$x^2 + 3x + 2 = 3x + 3$$

$$x^2 = 1 \qquad \boxed{x = 1 \text{ or } -1}$$

8) $$2 + \frac{5}{x - 4} = \frac{x + 1}{x - 4}$$

method 1: $$\frac{2(x - 4)}{(x - 4)} + \frac{5}{x - 4} = \frac{x + 1}{x - 4}$$

$$2x - 8 + 5 = x + 1$$

$$x = 4$$

$\boxed{\text{but, } x = 4 \text{ is not in the domain...} \\ \text{no solutions}}$

method 2: $$2 = \frac{x + 1}{x - 4} - \frac{5}{x - 4}$$
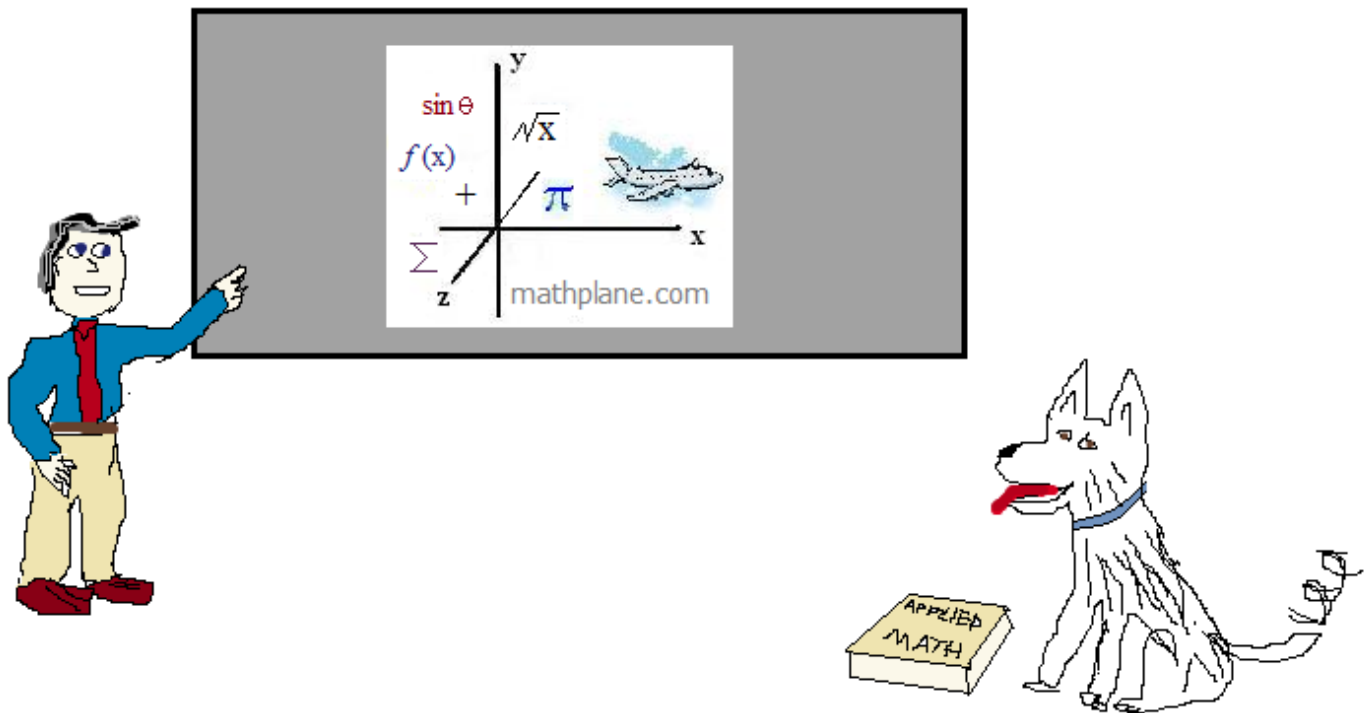
$$2 = \frac{x - 4}{x - 4}$$

$$2 = 1 \qquad \boxed{\text{no solutions}}$$

Thanks for visiting. (Hope it helps!)

If you have questions, suggestions, or requests, let us know.

Cheers

And, Mathplane.ORG for mobile and tablet

Also, stores at Teacherspayteachers and TES

One more….

Work – Rate Question

A girl can wash 10 windows in 45 minutes...
Working with her brother, together, they can wash 5 windows in 10 minutes.

Working by himself, how fast can the brother wash 15 windows?

ANSWER on the next page….

A girl can wash 10 windows in 45 minutes...
Working with her brother, together, they can wash 5 windows in 10 minutes.

Working by himself, how fast can the brother wash 15 windows?

$$\text{output} = \text{rate} \cdot \text{time}$$

(set up a work-rate equation)

$$\frac{5 \text{ windows}}{10 \text{ minutes}} (\text{time}) = \frac{10 \text{ windows}}{45 \text{ minutes}} (\text{time}) + \frac{15 \text{ windows}}{x \text{ minutes}} (\text{time})$$

together output   girl's output   brother's output

Solve for x

common denominator of 90 minutes; divide by 'windows'

$$\frac{45(t)}{90 \text{ minutes}} = \frac{20(t)}{90 \text{ minutes}} + \frac{15(t)}{x \text{ minutes}}$$

multiply by 'minutes'; combine terms

$$\frac{25t}{90} = \frac{15t}{x}$$

cross multiply ; divide by t

$$1350 = 25x$$

$$x = 54$$

> The brother can wash
> 15 windows in 54 minutes...