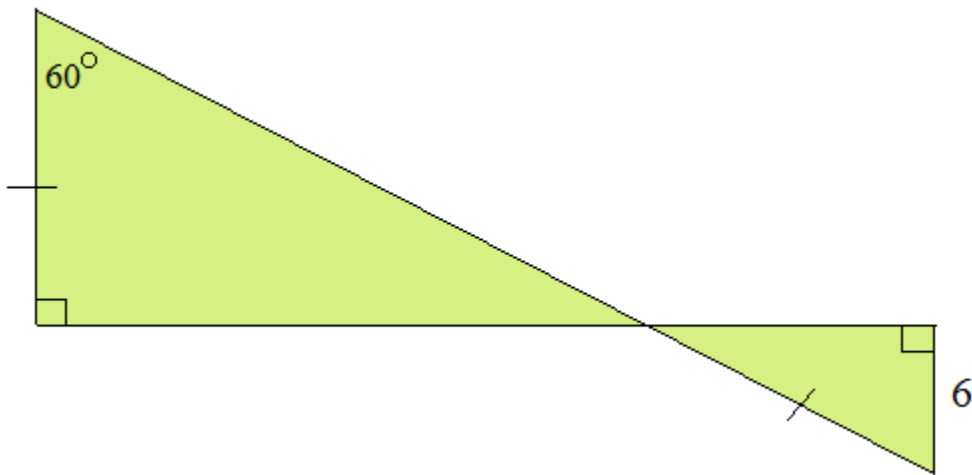


Special Right Triangles Review

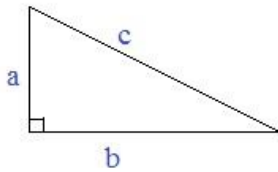
Notes, Examples, Puzzle, and Practice Quiz (with Solutions)



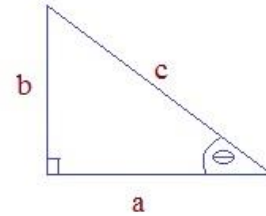
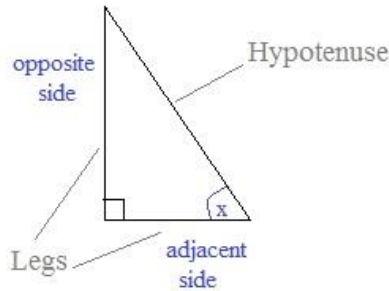
****What is the area inside the green triangles?**

Topics include 30-60-90, 45-45-90, Pythagorean Triples, and more.

Right Triangle Review Notes:



Pythagorean Theorem
 $a^2 + b^2 = c^2$



Trigonometry Relations

$$\sin \theta = \frac{b}{c} \quad \csc \theta = \frac{c}{b}$$

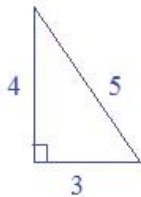
$$\cos \theta = \frac{a}{c} \quad \sec \theta = \frac{c}{a}$$

$$\tan \theta = \frac{b}{a} \quad \cot \theta = \frac{a}{b}$$

Utilizing the Pythagorean Theorem or Trig Identities can determine angle and side measurements of any right triangle. However, "Special Right Triangles" have features that make calculations easy!!

Special Right Triangles:

"Sides"

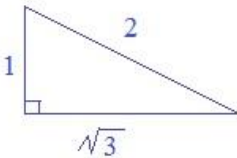


3 - 4 - 5
Right Triangle

Others include: 5 - 12 - 13
 7 - 24 - 25
 8 - 15 - 17

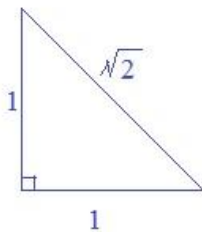
Note:
 -- Pythagorean Theorem confirms
 $3^2 + 4^2 = 5^2$
 -- Any multiple of 3-4-5 will work!
 Examples: 30-40-50 or 15-20-25

"Angles:"



30 - 60 - 90
Right Triangle

Note:
 -- Pythagorean Theorem and trig relations confirm
 (ex: $\sin 30^\circ = 1/2 = .5$)
 -- Any ratio of 1 - $\sqrt{3}$ - 2 will work.
 $\longrightarrow X - \sqrt{3}X - 2X$

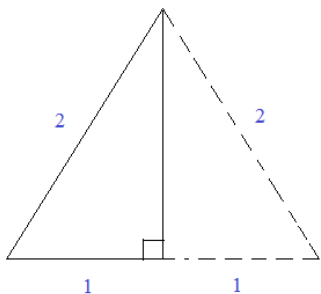
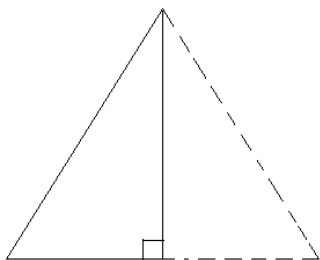


45 - 45 - 90
Right Triangle

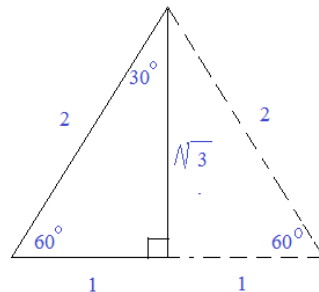
Note:
 -- Pythagorean theorem and trig relations confirm
 -- Congruent sides imply congruent (opposite) angles
 -- any ratio of 1 - 1 - $\sqrt{2}$ will work.
 $\longrightarrow X - X - \sqrt{2}X$

Creating the 30-60-90 triangle....

Take the left half of an equilateral triangle...

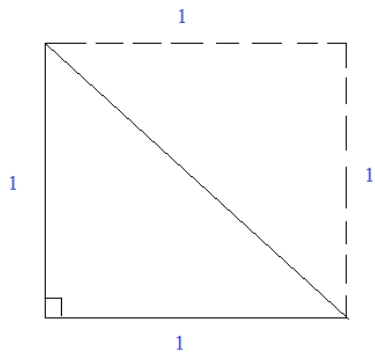
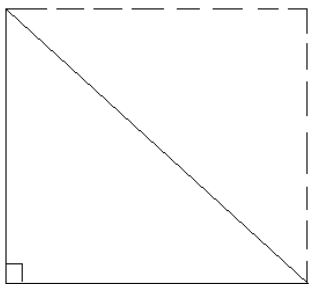


using Pythagorean Theorem
and, properties of triangles

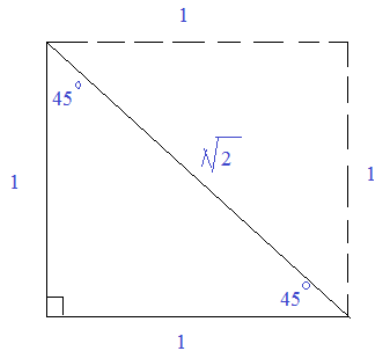


Creating the 45-45-90 triangle...

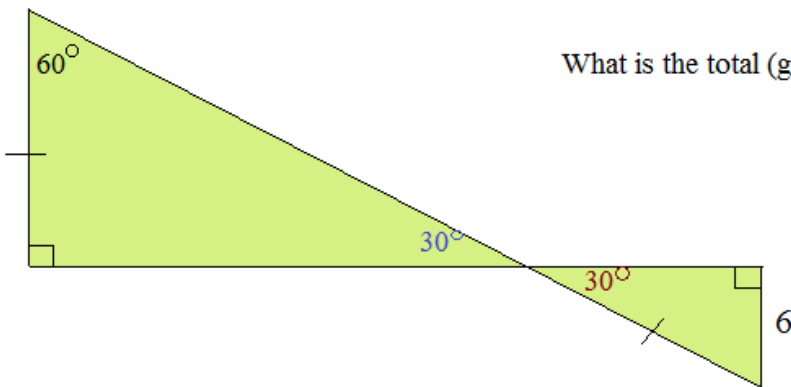
Take half of a square...



using Pythagorean Theorem
and, properties of triangles



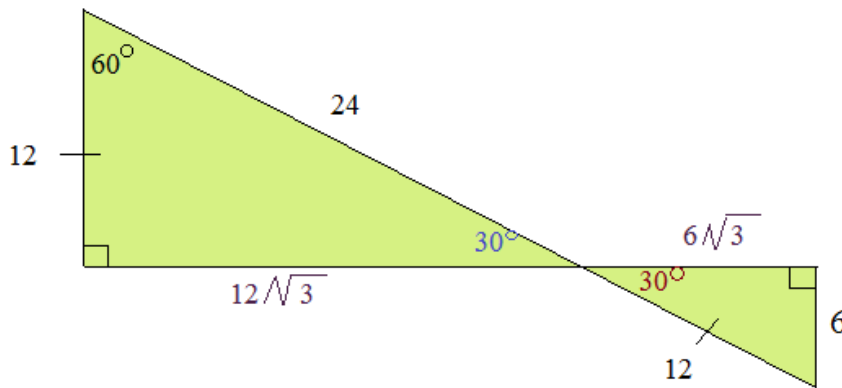
What is the total (green) area inside the 2 triangles?



Since one angle is 60 degrees, the other angle is 30 degrees.. (sum of interior angles of triangle = 180°)
30-60-90 right triangle...

Then, the small angle in the small triangle is 30 degrees. (vertical angles congruent)
So, another 30-60-90 right triangle.

In 30-60-90 right triangle, the hypotenuse = 2(small leg)



Small leg in big triangle is also 12. (congruent segments)

Ratio of sides of 30-60-90 triangles ----> $x, x\sqrt{3}, 2x$

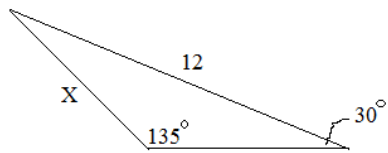
Area of triangle: $\frac{1}{2}$ (base)(height)

small triangle: $18\sqrt{3}$

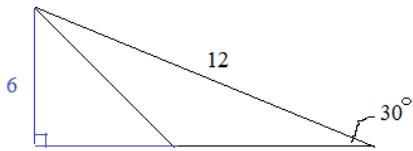
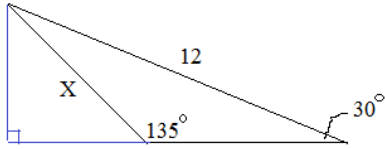
large triangle: $72\sqrt{3}$

Total green area: $90\sqrt{3} \approx 155.88$

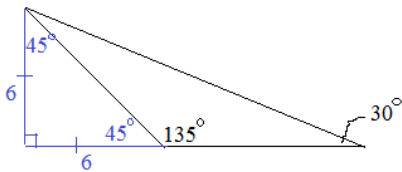
Example: Find X



Drop an altitude, creating another triangle....



30-60-90 right triangle: small side is 1/2 of hypotenuse... therefore, side opposite 30 degree angle is 6...



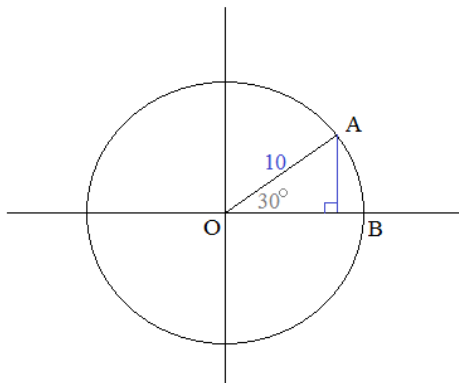
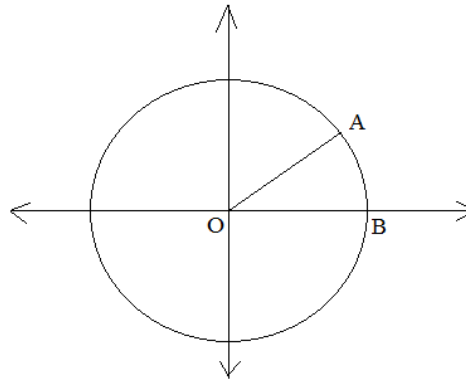
(supplementary angles) We know the left triangle is a 45-45-90 right triangle: hypotenuse is leg $\cdot \sqrt{2}$ therefore, $X = 6\sqrt{2}$

Example: Here is a circle that is centered on the origin. If the radius is 10 and $\angle AOB$ is 30° ,

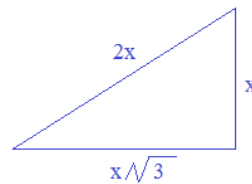
what is the coordinate of B?

(10, 0)

what is the coordinate of A?



30-60-90 triangle



$AB = 5$

$OB = 5\sqrt{3}$

$(5\sqrt{3}, 5)$

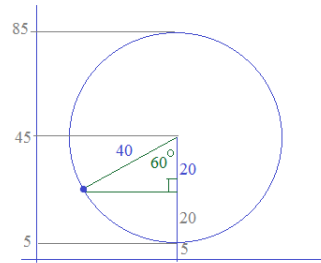
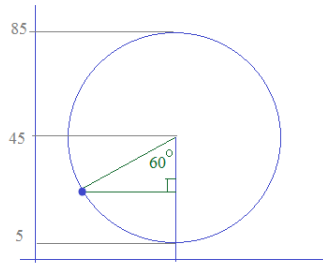
Example: A giant Ferris Wheel is 80 feet in diameter and rotates 1 time every 3 minutes.
The middle of the wheel is 45 feet above the ground.

Assuming a rider gets on and starts the ride at the bottom of the wheel, what is the rider's height

a) after 30 seconds

since diameter is 80 feet, the radius is 40 feet.
Also, the middle of the wheel is 45 feet high.

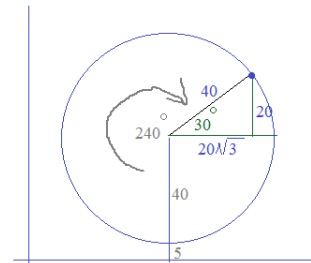
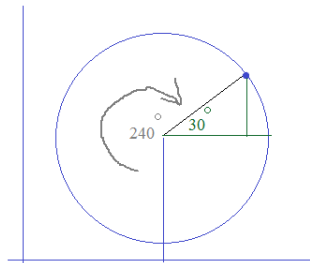
After 30 seconds, the wheel has turned 1/6 of the way ----> 60 degrees



The rider is 25 feet high

b) after 2 minutes

After 2 minutes, the wheel has rotated 2/3 of the way ----> 240 degrees

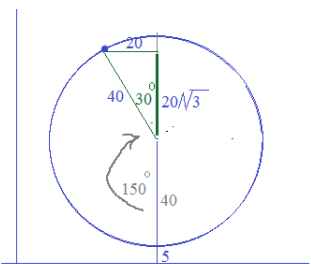
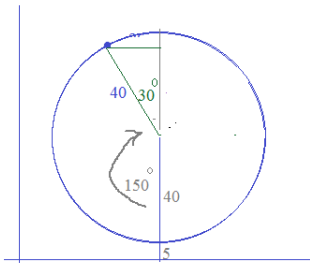


The rider is 65 feet high

c) after 1 minute 15 seconds

75 seconds out of 3 minutes ---->

$$\frac{75 \text{ seconds}}{180 \text{ seconds}} = \frac{150 \text{ degrees}}{360 \text{ degrees}}$$



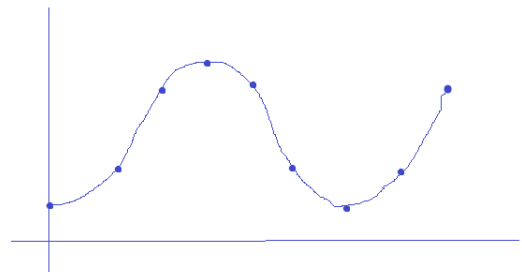
The rider is $45 + 20\sqrt{3}$ high

Fill in the table of values for the height as a function of time....

time (seconds)	0	30	60	90	120	150	180	210	240
height (feet)									

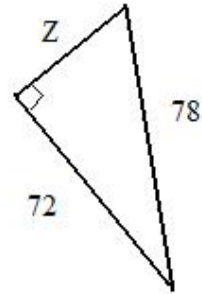
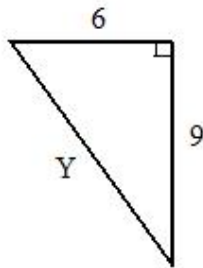
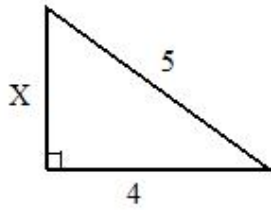
time (seconds)	0	30	60	90	120	150	180	210	240
height (feet)	5	25	65	85	65	25	5	25	65

↑
3 minutes
one rotation

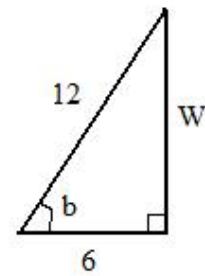
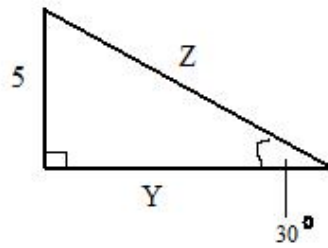
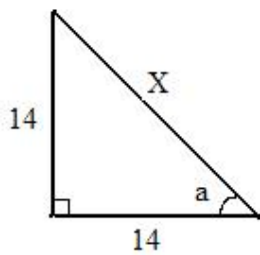


Right Triangles: Finding sides and angles (without a calculator!)

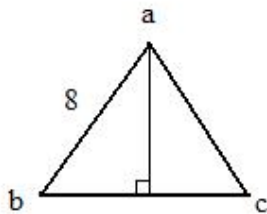
1)



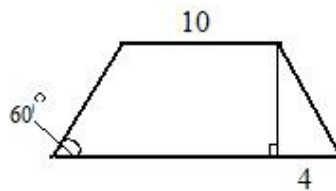
2)



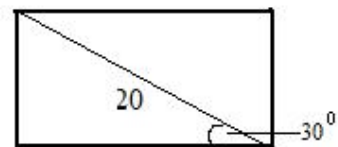
3)



Find the altitude of equilateral $\triangle abc$



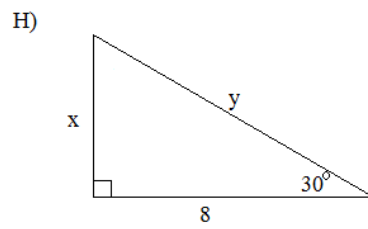
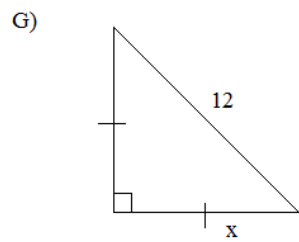
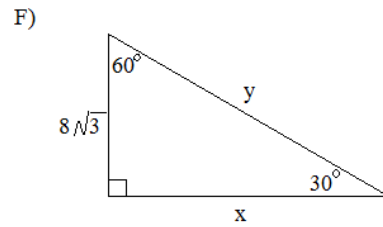
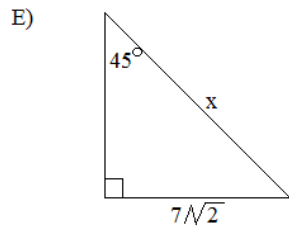
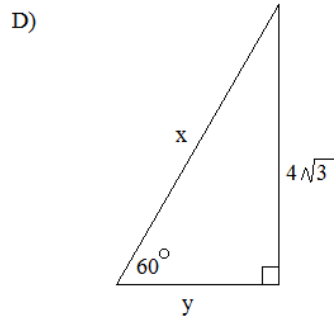
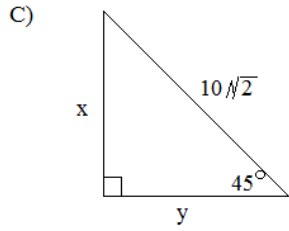
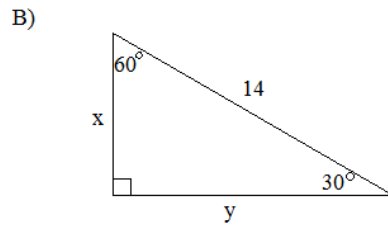
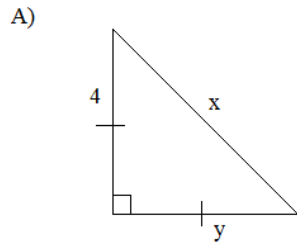
Find the area of the above isosceles trapezoid



Given the diagonal is 20. Find the area of the above rectangle.

Special Right Triangles

In each triangle, find x and y . (calculator is NOT necessary)



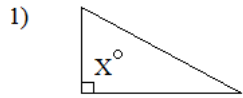
Hidden Message

Hint: Why are most of the questions "correct"?

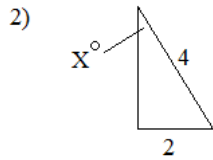
Find the measures of X.
Then, convert to letters to
discover the hidden message.

Letter Key:

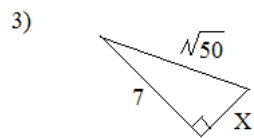
0	1	2	3	4	5	6	7	8	9
A	E	G	H	I	L	N	R	S	T



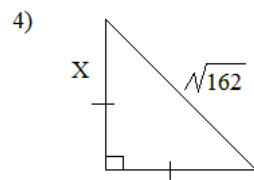
0 → _____



0 → _____



→ _____



→ _____

5) X-24-25 is a special one

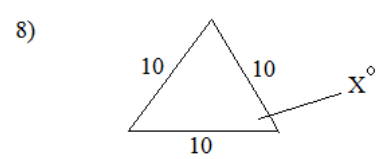
→ _____

6) 3-X-5 is a special one

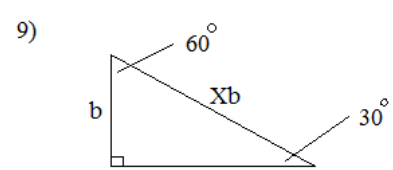
→ _____

7) Number of obtuse angles in a right triangle?

→ _____



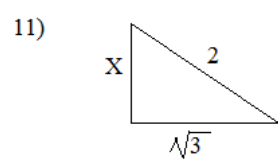
0 → _____



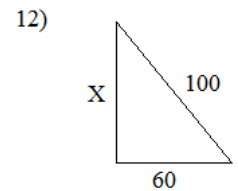
→ _____

10) X-12-13 is a special one

→ _____



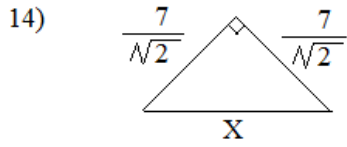
→ _____



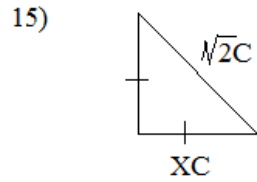
0 → _____

13) Number of right angles in an equilateral triangle?

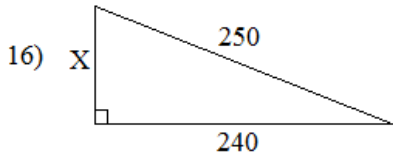
→ _____



→ _____



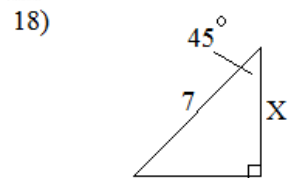
→ _____



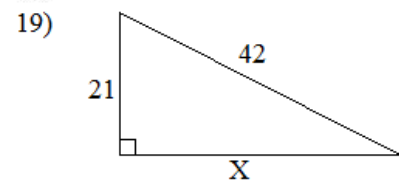
0 → _____

17) Number of right angles in a square?

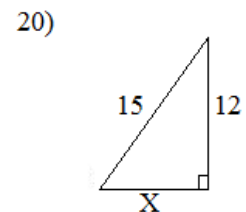
→ _____



$\frac{7\sqrt{\square}}{2}$ → _____



$21\sqrt{\square}$ → _____

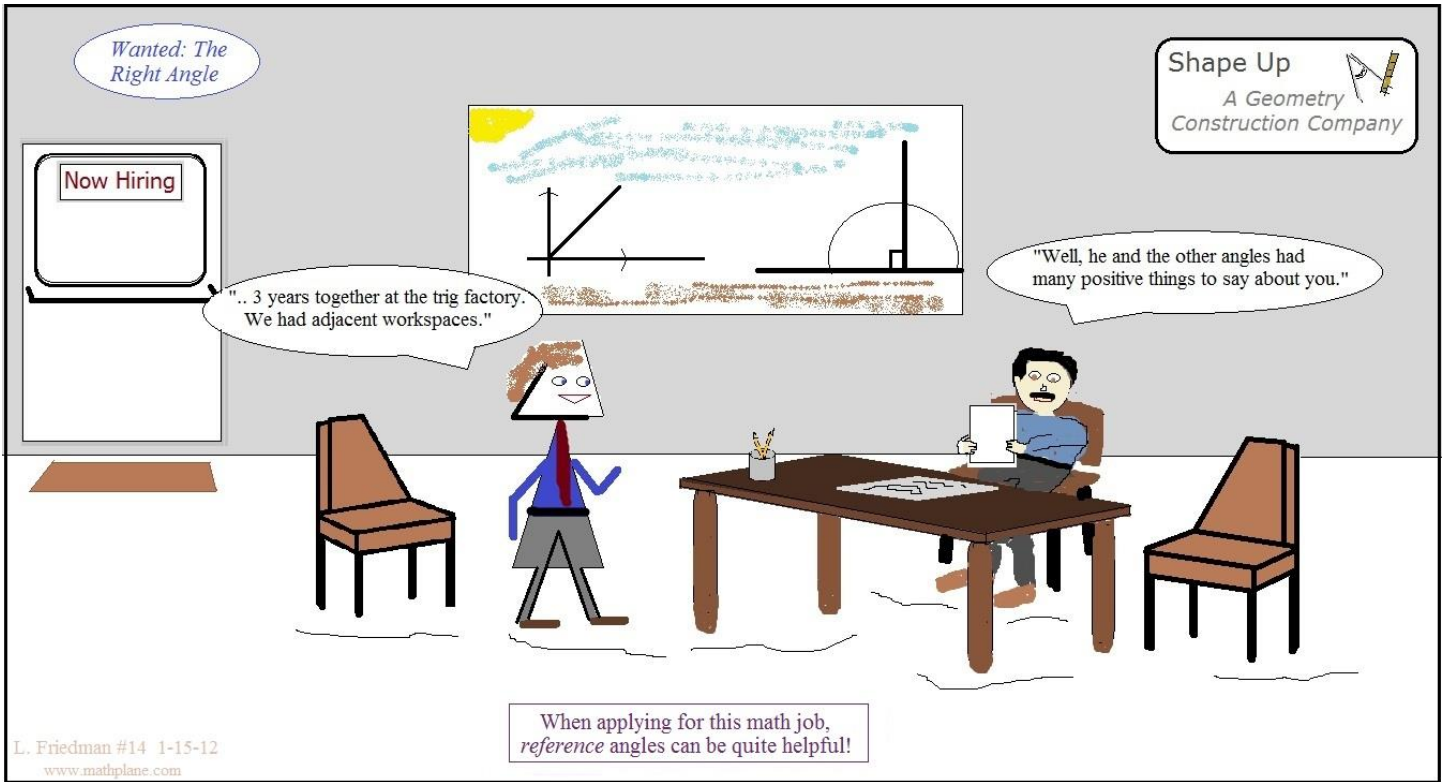


→ _____

Letter Key:									
0	1	2	3	4	5	6	7	8	9
A	E	G	H	I	L	N	R	S	T

Hidden Message

Hint: Why are most of the questions "correct"?

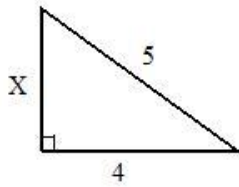


SOLUTIONS ->

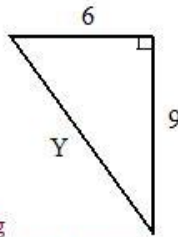
Right Triangles: Finding sides and angles (without a calculator!)

SOLUTIONS

1)



X = 3 because
3-4-5 special triangle

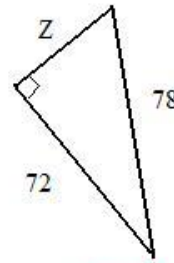


using
pythagorean theorem:

$$6^2 + 9^2 = Y^2$$

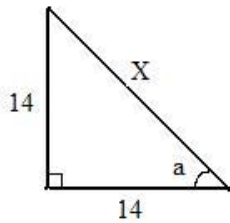
$$36 + 81 = Y^2$$

$$Y = \sqrt{117}$$

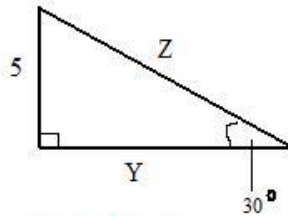


Z = 30 because 30-72-78 is
6 x (5-12-13) triangle OR,
using pythagorean theorem--
 $78^2 = 72^2 + Z^2$

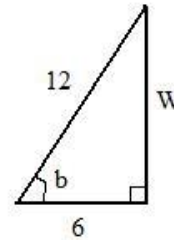
2)



45-45-90 triangle
therefore, $X = 14\sqrt{2}$
Or, $X = \sqrt{196 + 196}$
 $= 14\sqrt{2}$

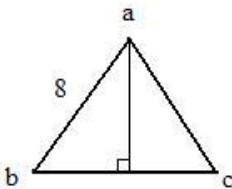


30-60-90 triangle
 $Z = 5 \times 2 = 10$
 $Y = 5 \times \sqrt{3}$
To check answer, try $5^2 + 5\sqrt{3}^2 = 10^2$
pythagorean theorem $100 = 100$



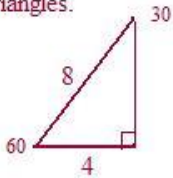
Since the hypotenuse is 2x one of the
legs, we can conclude it is a 30-60-90
triangle. Therefore, angle b is 60
degrees. And, side W is $6\sqrt{3}$

3)

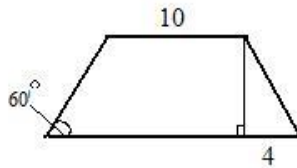


Find the altitude of
equilateral $\triangle abc$

An equilateral triangle has angles of
60-60-60.. this is helpful, since the
altitude produces two 30-60-90
triangles.



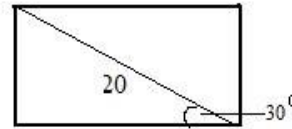
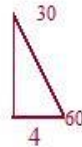
And, altitude is $4\sqrt{3}$



Find the area of the above
isosceles trapezoid

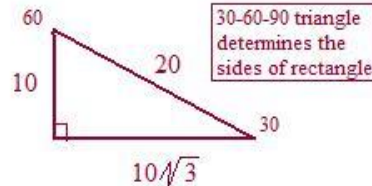
Area Trapezoid = $1/2 (b_1 + b_2) h$
base 1 = 10
base 2 = $4 + 10 + 4 = 18$
To find height, we consider the
30-60-90 right triangle.
---> height = $4\sqrt{3}$

Area of Trapezoid =
 $1/2 (10 + 18) 4\sqrt{3} =$
 $56\sqrt{3}$



Given the diagonal is 20.
Find the area of the
above rectangle.

We know the diagonal (hypotenuse)
and the angle formed by the diagonal.

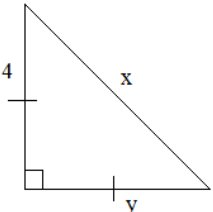


30-60-90 triangle
determines the
sides of rectangle

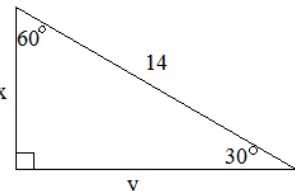
Area of rectangle is length x width =
 $10 \times 10\sqrt{3} = 100\sqrt{3}$

Special Right Triangles

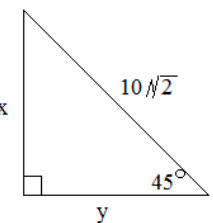
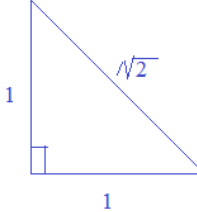
In each triangle, find x and y. (calculator is NOT necessary)

A)  2 congruent legs, so it is a 45-45-90 right triangle...

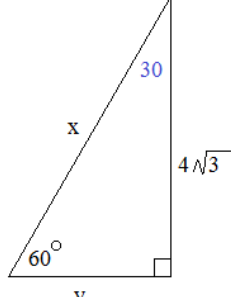
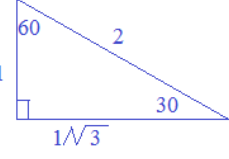
$y = 4$
 $x = 4\sqrt{2}$

B)  30-60-90 right triangle...
 small leg is 1/2 the hypotenuse..

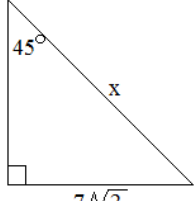
$x = 7$
 medium side is small $\cdot \sqrt{3}$
 $y = 7\sqrt{3}$

C)  

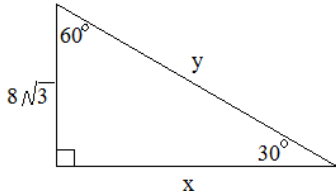
$x = 10$ $y = 10$

D)  

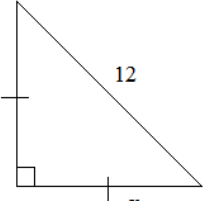
recognizing the ratios of the sides,
 $y = 4$ and $x = 8$

E)  $7\sqrt{2} \cdot \sqrt{2} = 14$

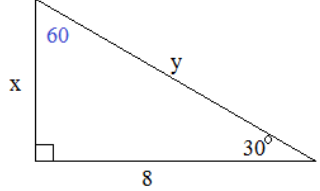
$x = 14$

F)  since the small leg is $8\sqrt{3}$,
 the big leg is $\sqrt{3} \cdot 8\sqrt{3} = 24 = x$

and, the hypotenuse is
 $2 \cdot 8\sqrt{3} = 16\sqrt{3} = y$

G)  $\frac{\sqrt{2}}{1} = \frac{12}{x}$

$x = \frac{12}{\sqrt{2}} = 6\sqrt{2}$

H)  $\frac{8}{x} = \frac{\sqrt{3}}{1}$

$\sqrt{3}x = 8$
 $x = \frac{8}{\sqrt{3}}$
 $y = 2 \cdot \frac{8}{\sqrt{3}} = \frac{16}{\sqrt{3}}$

Hidden Message

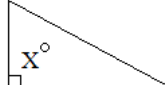
Hint: Why are most of the questions "correct"?

Find the measures of X.
Then, convert to letters to
discover the hidden message.

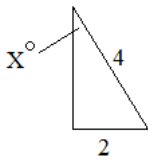
Letter Key:

0	1	2	3	4	5	6	7	8	9
A	E	G	H	I	L	N	R	S	T

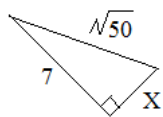
SOLUTIONS

- 

Right angles are 90 degrees.

0 → T
- 

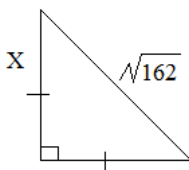
In a 30-60-90 right triangle, the length of the hypotenuse is *twice* the measure of the small side opposite the 30 degree angle.

0 → H
- 

Use Pythagorean Theorem:

$$X^2 + (7)^2 = (\sqrt{50})^2$$

$$X = 1$$

→ E
- 

Since 2 sides are congruent, this is an 45-45-90 triangle.

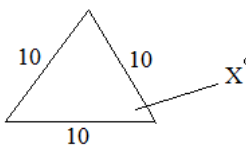
therefore, $X = \frac{\sqrt{162}}{\sqrt{2}} = \text{9}$

→ T
- X-24-25 is a special one

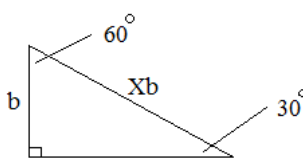
- 24 - 25 (Use Pythagorean Theorem to confirm)
- 3-X-5 is a special one

3 - - 5
- Number of obtuse angles in a right triangle?

There are NO obtuse angles in a right triangle.

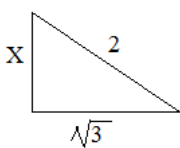
→ A
- 

All sides congruent, so it is an equilateral triangle.. And, all angles are 60 degrees.

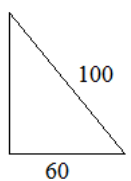
0 → N
- 

30-60-90 triangle, so hypotenuse is twice the length of the small side.

= 2
- X-12-13 is a special one

- 12 - 13 is a special right triangle
- 

X = 1
(use pythagorean thm or 30-60-90)

→ E
- 

3 - 4 - 5 ---> 60 - - 100
(multiply by 20)

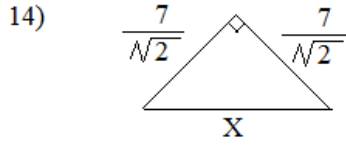
0 → S

13) Number of right angles in an equilateral triangle?

SOLUTIONS

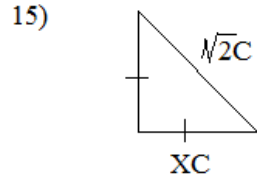
ZERO (all angles are 60 degrees)

0 → A



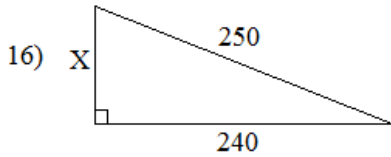
2 sides are congruent ---> 45-45-90
and, hypotenuse is $\sqrt{2} \cdot (\text{side})$ $X = 7$

7 → R



45-45-90 right triangle... $X = 1$

1 → E



divide by 10: $X - 24 - 25$
then, X would be 7 $X = 70$

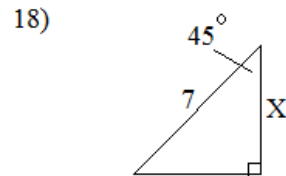
70 → R

17) Number of right angles in a square?

Four right angles in a square

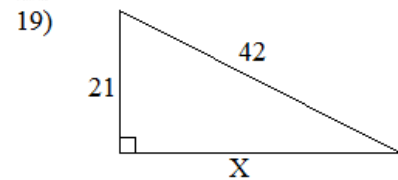


4 → I



(similar to $1 - 1 - \sqrt{2}$)
 $X = \frac{7}{\sqrt{2}}$

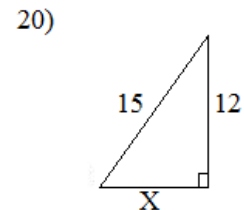
$\frac{7\sqrt{2}}{2}$ → G



Hypotenuse is 2x the small side..
therefore, $X = 21\sqrt{3}$

(similar to $1 - \sqrt{3} - 2$)

$21\sqrt{3}$ → H



Use Pythagorean Theorem
(or recognize similarity to 3-4-5)

$X = 9$ 9-12-15

9 → T

Letter Key:									
0	1	2	3	4	5	6	7	8	9
A	E	G	H	I	L	N	R	S	T

Hidden Message

Hint: Why are most of the questions "correct"?

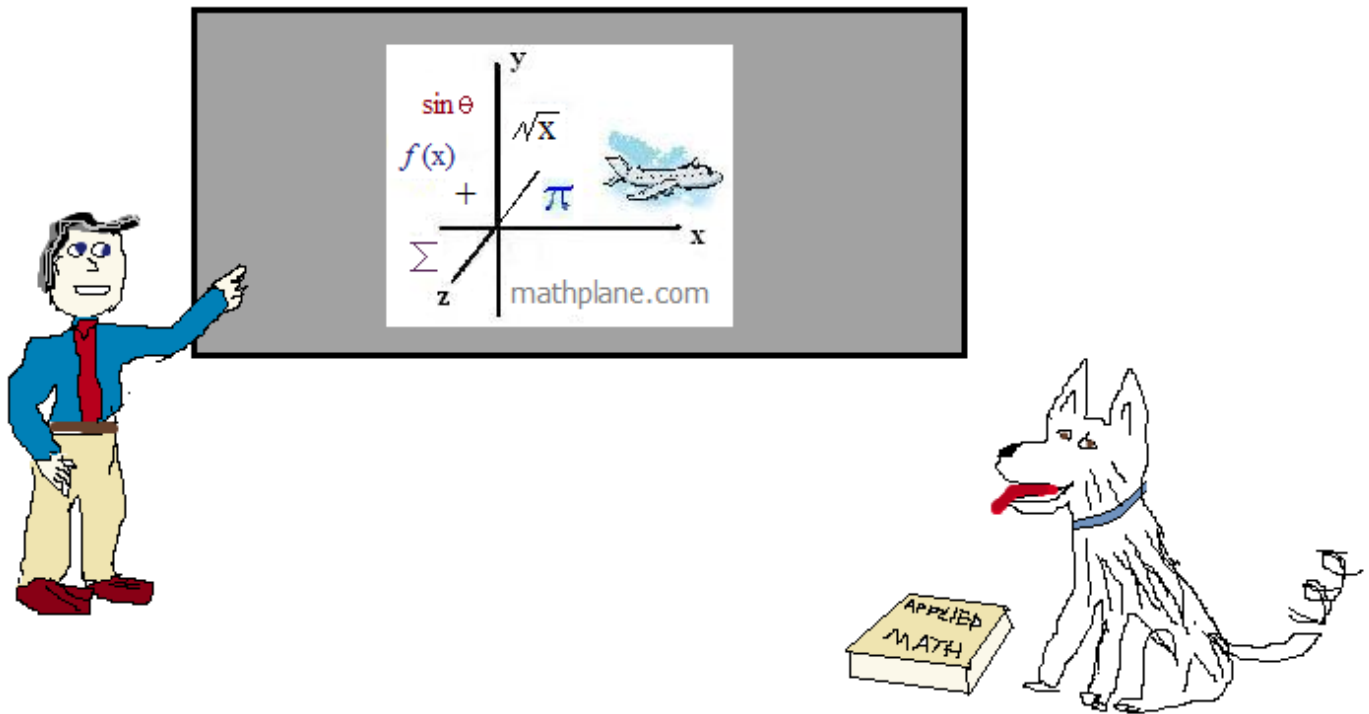
The triangles are "Right"

Thanks for visiting. (Hope it helped!)

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If you have questions, suggestions, or requests, let us know.

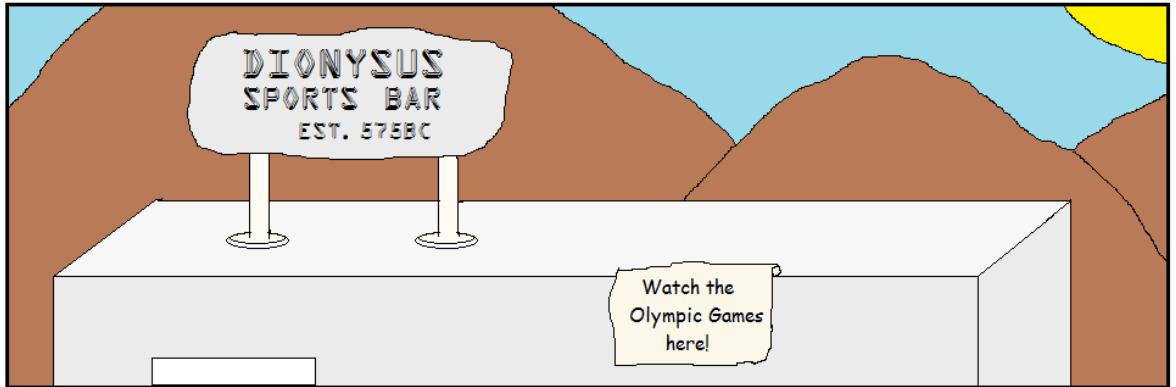
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