AAYTAN ACADEMY

STD-10(S)

Answer Key of Most IMP Questions

A **Write the answer of the following questions.** [Each carries 1 Mark] **[43]**

1. In a musical chair game, the person playing the music has been advised to stop playing the music at any time within 2 minutes after she starts playing. What is the probability that the music will stop within the first half-minute after starting ?

➤ Here the possible outcomes are all the numbers between 0 and 2. This is the portion of the number line from 0 to 2 (see figure)

Let E be the event that 'the music is stopped within the first half-minute'.

The outcomes favourable to E are points on the number line from 0 to $\frac{1}{2}$.

The distance from 0 to 2 is 2, while the distance from 0 to $\frac{1}{2}$ is $\frac{1}{2}$

Since all the outcomes are equally likely, we can argue that, of the total distance of 2, the distance favourable to the event E is $\frac{1}{2}$.

So, $P(E) = \dfrac{\text{Distance favourable to the event E}}{\text{Total distance in which outcomes can lie}}$

$= \dfrac{\frac{1}{2}}{2} = \dfrac{1}{4}$

2. India reached in Semifinal match in world cup cricket match. Other than India three more team Australia, Newzeland and England also plays the game then .... is the probability of an event that India win the world cup.

(A) $\frac{1}{4}$ (B) 4 (C) 8 (D) $\frac{1}{8}$

Ans. (A) $\mathbf{\frac{1}{4}}$

➤ Here there are four teams namely India, Australia, Newzeland and England out of four teams any one can win the world cup.

∴ Probability that India win world cup $= \frac{1}{4}$

3. Probability of complement of the impossible events is ....

(A) 1 (B) 0 (C) $\frac{2}{7}$ (D) $\frac{1}{7}$

Ans. (A) 1

➤ Impossible event ϕ has probability zero.

∴ Complement at ϕ zero is certain event has probability one.

∴ The Probability of complement of the impossible event is one.

4. If $P(A) : P(\overline{A}) = 4:1$ then $P(\overline{A}) =$ .... $\left(\frac{1}{4}, \frac{4}{5}, \frac{1}{5}\right)$

➤ Given that $P(A) : P(\bar{A}) = 4 : 1$.

$\therefore \frac{P(A)}{P(\bar{A})} = \frac{4}{1}$

$\therefore P(A) = 4P(\bar{A})$

➤ $\therefore 1 - P(\bar{A}) = 4P(\bar{A})$

$\therefore 1 = 5P(\bar{A})$

$\therefore P(\bar{A}) = \frac{1}{5}$

5. $\bar{A}$ is complement event of A. If $P(A) - P(\bar{A}) = 0.8$ then find P(A).

➤ $P(A) - P(\bar{A}) = 0.8$

$\therefore P(A) - [1 - P(A)] = 0.8$

$\therefore P(A) - 1 + P(A) = 0.8$

$\therefore 2P(A) = 0.8 + 1$

$\therefore 2P(A) = 1.8$

$\therefore P(A) = 0.9$

6. There is an empirical relationship between the three measures of central tendency $3M = Z + 2\bar{x}$ then $\frac{Z - M}{M - \bar{x}}$ = ...

(A) 0 (B) 1 (C) –2 (D) 2

➠ Ans : (D)

7. If the difference of mode and median is 24 then find the difference of median and mean.

➤ Mode – Median = 24

$\therefore$ Mode = Median + 24

Now Mode = 3 Median – $2\bar{x}$

$\therefore$ 24 + Median = 3 Median – $2\bar{x}$

$\therefore$ 24 + Median – 3 Median = – $2\bar{x}$

$\therefore$ 24 – 2 Median = – $2\bar{x}$

$\therefore$ Median – $\bar{x}$ = 12

so, the difference of Median and Mean is 12

8.

Class	5–14	14–23	23–32	32–41	41–50	50–59	59–68
Frequency	5	11	x	53	y	16	0

The cumulative frequency of the class 32 – 41 is 91 then find x.

➤ Given that the cumulative frequency of the class 32 – 41 is 91.

$\therefore$ The sum $5 + 11 + x + 53 = 91$

$\therefore 69 + x = 91$

$\therefore x = 91 - 69$

$\therefore x = 22$

9. For some data $\sum_{1}^{6} x_i = 270$ and $\sum_{1}^{11} x_i = 228$ then find 6^{th} observation.

➤ 6^{th} observation = $\sum_{1}^{6} x_i - \sum_{1}^{11} x_i$

$= 270 - 228$

$= 42$

10. Find the deviation of 3, 5, 6, 7, 8, 10, 11, 14 from mean.

➤ Mean $\bar{x} = \frac{\Sigma x_i}{n}$

$= \frac{3 + 5 + 6 + 7 + 8 + 10 + 11 + 14}{8}$

$= \frac{64}{8}$

$= 8$

➤ Deviation from mean

$= \frac{\Sigma|x_i - \bar{x}|}{n}$

$= \frac{|3-8|+|5-8|+|6-8|+|7-8|+|8-8|+|10-8|+|11-8|+|14-8|}{8}$

$= \frac{|-5| + |-3| + |-2| + |-1| + |0| + |2| + |3| + |6|}{8}$

$= \frac{5 + 3 + 2 + 1 + 0 + 2 + 3 + 6}{8}$

$= \frac{22}{8} = 2.75$

11. Find the mean of all factors of 12.

➤ Factors of 12 are 1, 2, 3, 4, 6 and 12.

$\bar{x} = \frac{\Sigma x_i}{n}$

$= \frac{1 + 2 + 3 + 4 + 6 + 12}{6}$

$= \frac{28}{6}$

$\therefore \bar{x} = \frac{14}{3}$

12. Find the value of $\Sigma(x_i - \bar{x})$.

➤ $\Sigma(x_i - \bar{x}) = 0$ ($\because$ The sum of the deviation of the observation from the mean is always zero)

13. The volume of the cone is 660 cm^3. If its radius is 6 cm find the slant height of the cone.

➤ Volume of the cone V = $\frac{1}{3}\pi r^2 h$

$\therefore 660 = \frac{1}{3} \times \frac{22}{7} \times 6 \times 6 \times h$

$\therefore h = \frac{660 \times 3 \times 7}{22 \times 6 \times 6} = 17.5$

Now $l^2 = h^2 + r^2$

$= \left(\frac{35}{2}\right)^2 + (6)^2$

$= \frac{1225}{4} + \frac{36}{1}$

$= \frac{1225 + 144}{4}$

$= \frac{1369}{4}$

$= \left(\frac{37}{2}\right)^2$

$\therefore \ l = \frac{37}{2}$

$\therefore \ l = 18.5$ cm

14. If perimeter and area of the circle are equal then diameter of the circle is .... unit.

(A) 4 (B) 2 (C) 8 (D) 1

➠ Ans : (A)

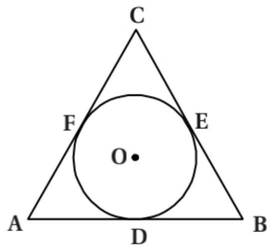
15. In the given figure, AB = 12 cm, BC = 8 cm and AC = 10 cm, then AD = ... cm.

(A) 5

(B) 4

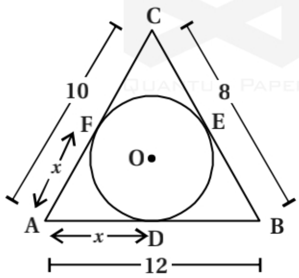
(C) 6

(D) 7



Ans. (D) 7

➤



➤ Assume that AD = x cm

$\therefore$ BD = 12 − x

BD = BE

$\therefore$ BE = 12 − x

C – E – B so,

CE + EB = BC

$\therefore$ CE + (12 − x) = 8

CE = 8 − 12 + x

CE = x − 4

But CE = CF

$\therefore$ x − 4 = CF

Now C – F – A so,

CF + FA = AC

$\therefore\ (x - 4) + AF = 10$

$\therefore\ AF = -x + 4 + 10$

$\therefore\ AF = 14 - x$

But AF = AD

$\therefore\ 14 - x = x$

$\therefore\ 14 = x + x$

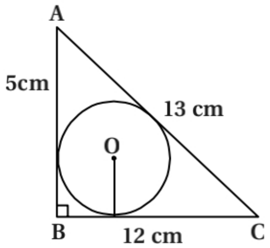
$\therefore\ 2x = 14$

$\therefore\ x = \frac{14}{2}$

$\therefore\ x = 7$

$\therefore$ AD = 7 cm

16. In ΔABC ∠B is a right angle. AB = 5 cm, BC = 15, find the radius of the circle which touches all the sides of ΔABC.



➤ $AC^2 = AB^2 + BC^2$

$= (5)^2 + (12)^2$

$= 25 + 144$

$= 169$

$\therefore\ AC = 13$ cm

Area of ΔABC = $\frac{1}{2}$ × perimeter of ΔABC × r

$\therefore\ \frac{1}{2} \times 5 \times 12 = \frac{1}{2} \times (5 + 12 + 13) \times r$

$\therefore\ 30 = \frac{1}{2} \times 30 \times r$

$\therefore\ 30 = \frac{30r}{2}$

$\therefore\ 30r = 60$

$\therefore\ r = 2$

➤ **Another method**

$AC^2 = AB^2 + BC^2$

$= (5)^2 + (12)^2$

$\therefore\ AC^2 = 169$

$\therefore\ AC = 13$

$$\text{Inradius} = \frac{\text{sum of two sides formed right angle} - \text{hypotenuse}}{2}$$

Inradius = $\frac{(5+12)-(13)}{2} = \frac{17-13}{2} = \frac{4}{2} = 2$ cm

17. sin 2A = 2 sin A is true when A = ...

(A) 0° (B) 30° (C) 45° (D) 60°

Ans. (A) 0°

A = 0° then sin 2A = sin2(0°) = sin 0 = 0

2 sinA = 2sin 0° = 2 × 0 = 0

If A = 0° than sin 2A = sin2 (0)

= sin 0°

= 0

and 2 sin A = 2 sin 0° = 0

i.e. sin 2 A = 2 sin A

Hence, alternate (A) will come.

18. If θ is an acute angle and b sinθ = a cosθ then $\frac{a\sin\theta - b\cos\theta}{a\sin\theta + b\cos\theta}$ = ...

(A) $\frac{a^2+b^2}{a^2-b^2}$ (B) $\frac{a^2-b^2}{a^2+b^2}$ (C) $\frac{a+b}{a-b}$ (D) $\frac{a-b}{a+b}$

Ans. (B) $\boldsymbol{\frac{a^2-b^2}{a^2+b^2}}$

➤ Here, b sinθ = a cosθ

$\therefore \frac{\sin\theta}{\cos\theta} = \frac{a}{b}$

$= \frac{a\sin\theta - b\cos\theta}{a\sin\theta + b\cos\theta}$

$= \frac{a\sin\theta}{\cos\theta} - \frac{b\cos\theta}{\cos\theta} \div \frac{a\sin\theta}{\cos\theta} + \frac{b\cos\theta}{\cos\theta}$

(Divide by cos θ)

$= a\left(\frac{a}{b}\right) - b \div a\left(\frac{a}{b}\right) + b$

$= \frac{a^2}{b} - b \div \frac{a^2}{b} + b$

$= \frac{a^2-b^2}{b} \div \frac{a^2+b^2}{b}$

$= \frac{a^2-b^2}{a^2+b^2}$

19. $\sin^2(3x + 30) + \cos^2(2x + 45) = 1$ then x = ... (15, 17, 14)

➤ $\sin^2(3x + 30) + \cos^2(2x + 45) = 1$

$\sin^2\theta + \cos^2\theta = 1$

$\therefore 3x + 30 = 2x + 45$

$\therefore 3x - 2x = 45 - 30$

$\therefore x = 15$

20. If $7\cos^2\theta + 3\sin^2\theta = 4$ then find $\cot\theta$.

➤ $7\cos^2\theta + 3\sin^2\theta = 4$

$\therefore 7\cos^2\theta + 3(1 - \cos^2\theta) = 4$

$\therefore 7\cos^2\theta + 3 - 3\cos^2\theta = 4$

$\therefore 4\cos^2\theta + 3 = 4$

$\therefore 4\cos^2\theta = 4 - 3$

$\therefore 4\cos^2\theta = 1$

$\therefore \cos^2\theta = \frac{1}{4}$

$\therefore \cos\theta = \frac{1}{2}$

$\therefore \cos 60° = \frac{1}{2}$

$\therefore \theta = 60°$

➤ $\cot\theta = \cot 60°$

$= \frac{1}{\sqrt{3}}$

$\therefore \cot\theta = \frac{1}{\sqrt{3}}$

21. Find the ratio in which the y-axis divides the line segment joining the points (5, – 6) and (–1, – 4). Also find the point of intersection.

➤ Let the ratio be $k : 1$ Then by the section formula, the coordinates of the point which divides AB in the ratio $k : 1$ are,

$$P(x, y) = \left(\frac{kx_2 + x_1}{k+1}, \frac{ky_2 + y_1}{k+1}\right)$$

$$P(x, y) = \left(\frac{(k \times -1) + 5}{k+1}, \frac{k(-4)+(-6)}{k+1}\right)$$

$$\therefore P(x, y) = \left(\frac{-k+5}{k+1}, \frac{-4k-6}{k+1}\right)$$

This point lies on the Y-axis so its X-coordinates is zero.

$$\therefore P(0, y) = \left(\frac{-k+5}{k+1}, \frac{-4k-6}{k+1}\right)$$

$$\therefore \frac{-k+5}{k+1} = 0$$

$\therefore -k + 5 = 0$

$\therefore -k = -5$

$\therefore k = 5$

Substitute $k = 5$ in $\frac{-4k-6}{k+1} = y$, We get

$$\therefore \frac{-4(5)-6}{5+1} = y$$

$$\therefore \frac{-20-6}{6} = y$$

-13

$\therefore\ y = \frac{\quad}{3}$

Hence the required ratio is 5 : 1 Putting the value of $k = 5$, we get the point of intersection as $\left(0, \frac{-13}{3}\right)$.

22. Find a relation between x and y such that the point (x, y) is equidistant from the points (7, 1) and (3, 5).

➤ Let P(x, y) be equidistant from the points A(7, 1) and B(3, 5). We are given that AP = BP

So $AP^2 = BP^2$.

$\therefore\ (x - 7)^2 + (y - 1)^2 = (x - 3)^2 + (y - 5)^2$

$\therefore\ x^2 - 14x + 49 + y^2 - 2y + 1 = x^2 - 6x + 9 + y^2 - 10y + 25$

$\therefore\ x^2 - 14x + 49 + y^2 - 2y + 1 - x^2 + 6x - 9 - y^2 + 10y - 25 = 0$

$\therefore\ -14x + 6x - 2y + 10y + 49 + 1 - 9 - 25 = 0$

$\therefore\ -8x + 8y + 16 = 0$

$\therefore\ -x + y + 2 = 0$ $\quad$ ($\because$ Divide by 8)

$\therefore\ x - y - 2 = 0$

$\therefore\ x - y = 2$

Thus, $x - y = 2$. Which is the required relation.

23. The distance of the point P(–5, 7) from the origin is .... $\left(74, 49, \sqrt{74}\right)$

➤ $OP = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

$= \sqrt{(-5 - 0)^2 + (7 - 0)^2}$

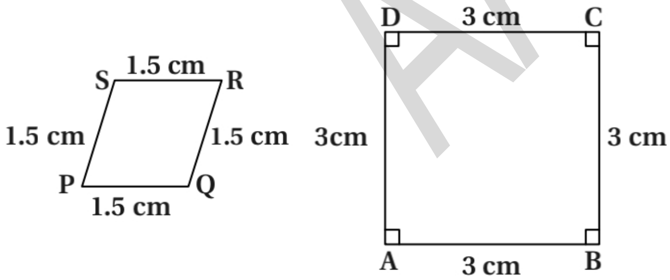
$= \sqrt{25 + 49}$

$\therefore\ OP = \sqrt{74}$

24. If $A\left(\frac{m}{2}, 5\right)$ is the midpoint of the line segment joining Q(–6, 7) and R(–2, 3) then find the value of m.

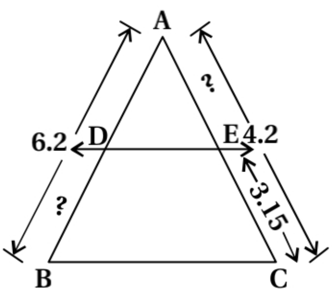
➠ $m = -8$

25. State whether the following quadrilaterals are similar or not :



➤ On observing the given figures we find that Their corresponding sides are proportional but their corresponding angles are not equal.

26. In Δ ABC parallel line to BC intersect AB and AC at D and E respectively then AB = ..., DB = ... and AE = ... (5.64, 2.55, 1.55; 1.55, 4.65, 1.05; 22.5, 1.05, 5.64)



➤ AC = AE + EC

$4.2 = AE + 3.15$

$AE = 4.2 - 3.15 = 1.05$

➤ $\frac{AD}{AB} = \frac{AE}{AC}$

$\therefore \frac{AD}{6.2} = \frac{1.05}{4.2}$

$\therefore AD = \frac{1.05 \times 6.2}{4.2}$

$\therefore AD = 1.55$

➤ $AB = AD + DB$

$\therefore 6.2 = 1.55 + DB$

$\therefore DB = 6.2 - 1.55$

$\therefore DB = 4.65$

27. The arithmetic sum of first n odd natural number is $n(n + 1)$.

➤ False.

➤ First n odd natural number are 1, 3, 5, 7, 9, ... n

$a = 1,\ d = 3 - 1 = 2$

$S_n = \frac{n}{2}[2a + (n - 1)d]$

$= \frac{n}{2}[2 \times 1 + (n - 1)2]$

$= \frac{n}{2}[2 + 2n - 2]$

$= \frac{n}{2}[2n]$

$= \frac{n}{2} \times [2n]$

$\therefore S_n = n^2$

28. The arithmetic sum of first n even number is n^2.

➤ False

➤ First n even natural numbers are 2, 4, 6, 8 .... n

$a = 2,\ d = 4 - 2 = 2$

$S_n = \frac{n}{2}[2a + (n - 1)d]$

$\therefore S_n = \frac{n}{2}\ [2 \times 2 + (n - 1)2]$

$= \frac{n}{2}\ [4 + 2n - 2]$

$= \frac{n}{2}\ [2n + 2]$

$= \frac{n}{2}\ [2 \times (n + 1]$

$= \frac{n}{2} \times 2 \times (n + 1)$

$= n(n + 1)$

$\therefore S_n = n^2 + n$

29. For an arithmetic sequence if $Sn = 5n^2 - 3n$ then $a_n = 10n - 8$.

➤ True

$Sn = 5n^2 - 3n$

Now $an = Sn - Sn - 1$

$= 5n^2 - 3n - [5(n - 1)^2 - 3(n - 1)]$

$= 5n^2 - 3n - [5(n^2 - 2n + 1) - 3n + 3)]$

$= 5n^2 - 3n - [5n^2 - 10n + 5 - 3n + 3]$

$= 5n^2 - 3n - [5n^2 - 13n + 8]$

$= 5n^2 - 3n - 5n^2 + 13n - 8$

$\therefore an = 10n - 8$

30. $2k + 1$, 13, $5k - 3$ are consecutive terms of an AP then find k.

(A) 17 (B) 13 (C) 4 (D) 9

Ans. (C) 4

➤ In an AP the common difference between two terms is same.

$\therefore a_2 - a_1 = a_3 - a_2$

$\therefore (13) - (2k + 1) = (5k - 3) - (13)$

$\therefore -2k + 12 = 5k - 3 - 13$

$\therefore -2k + 12 - 5k + 16 = 0$

$\therefore -7k + 28 = 0$

$\therefore 7k = 28$

$\therefore k = 4$

31. $1 + 2 + 3 + ... + 50 + 49 + 48 + ... + 1 =$ ....

(A) 2499 (B) 2500 (C) 2501 (D) 2599

➠ Ans : (B)

32. ...... of the following mathematician gives the general formula for solution of quadratic equation.

(A) Bhaskaracharya (B) Euclid (C) Sridharcharya (D) Aryabhatta

Ans. (C) Sridharcharya

33. Roots of the quadratic equation $\frac{x}{k} = \frac{k}{x}$ are ......

(A) $k, -k$ (B) $-k, -k$ (C) k, k (D) $k^2, -k^2$

Ans. (A) $k, -k$

➤ $\frac{x}{k} = \frac{k}{x}$

$\therefore x^2 = k^2$

$\therefore x = \pm k$

Hence roots of the equation are k and $-k$.

34. If $x + \frac{1}{x} = 2$ then $x^{2019} + \frac{1}{x^{2020}} =$ .... (2, –1, 1)

➤ $x + \frac{1}{x} = 2$

$\therefore \frac{x^2 + 1}{x} = 2$

$\therefore x^2 + 1 = 2x$

$\therefore x^2 - 2x + 1 = 0$

$\therefore\ (x - 1)^2 = 0$

$\therefore\ x - 1 = 0$

$\therefore\ x = 1$

➤ $x^{2019} + \frac{1}{x^{2020}} = (1)^{2019} + \frac{1}{(1)^{2020}}$

$= 1 + \frac{1}{1}$

$= 2$

35. If 3 is the solution of the quadratic equation $2x^2 - kx + 9 = 0$ then find the value of k.

➤ $2x^2 - kx + 9 = 0$

$\therefore\ 2(3)^2 - k(3) + 9 = 0$

$\therefore\ 2(9) - 3k + 9 = 0$

$\therefore\ 18 - 3k + 9 = 0$

$\therefore\ 27 = 3k$

$\therefore\ k = 9$

36. For a quadratic equation $ax^2 + bx + c = 0$, $a \neq 0$ if $b^2 - 4ac < 0$ then write the nature of the roots.

➤ No real roots.

37. If the roots of the equation $12x^2 - 25x + k = 0$ are reciprocal of each other then find k.

➤ $12x^2 - 25x + k = 0$

comparing the equation with $ax^2 + bx + c = 0$

$a = 12,\ b = -25,\ c = k$

➤ Roots are reciprocal of each other

$\therefore\ a = c$

$\therefore\ 12 = k$

$\therefore\ k = 12$

38. $51x + 49y = 150$ and $49x + 51y = 50$ then $x - y =$ .... (Without finding x, y) (100, 150, 50)

➤ $51x + 49y = 150$

$49x + 51y = 50$

$2x - 2y = 100$

$\therefore\ x - y = 50$ ($\because$ Taking 2 as common)

39. x years age the age of Virat Kohli was y years. What will be his age after 2 years ?

➤ x years age the age of Virat Kohli was y years.

$\therefore$ His present age $= (x + y)$ years.

$\therefore$ His age after 2 years $= (x + y + 2)$ years.

40.

	(A)	(B)
i)	The graph of $x = 0$ is	(a) X-axis and Y-axis
ii)	The graph of $y = 0$ is	(b) X-axis (c) Y-axis

➠ (i – c), (ii – b)

41. α, β and γ are the roots ofcubic polynomial $ax^3 + bx^2 + cx + d$ then $\frac{1}{\alpha\beta}+\frac{1}{\beta\gamma}+\frac{1}{\gamma\alpha}$ = ...

(A) $\frac{b}{d}$ (B) $\frac{d}{b}$ (C) $\frac{c}{d}$ (D) $-\frac{c}{d}$

Ans. (A) $\boldsymbol{\frac{b}{d}}$

➤ $\frac{1}{\alpha\beta}+\frac{1}{\beta\gamma}+\frac{1}{\gamma\alpha} = \frac{\gamma+\alpha+\beta}{\alpha\beta\gamma}$

$$= \frac{-\frac{b}{a}}{-\frac{d}{a}} = \frac{b}{a}\times\frac{a}{d} = \frac{b}{d}$$

42. α and β are roots of polynomial $p(x) = x^2 - 3x + 2m$ such that $\alpha + \beta = \alpha \cdot \beta$ then m = ...$\left(\frac{2}{3}, \frac{1}{3}, \frac{3}{2}\right)$

➤ Compare the polynomial $x^2 - 3x + 2m$ with $ax^2 + bx + c$

∴ We get $a = 1$, $b = -3$, $c = 2m$

$\alpha + \beta = \alpha\beta$ (Given)

∴ $-\frac{b}{a} = \frac{c}{a}$

∴ $-b = c$

∴ $3 = 2m$

∴ $m = \frac{3}{2}$

43. $a = pq^2$ and $b = p^3q$ where p and q are prime numbers HCF (a, b) = LCM (a, b) = ... (p^2q^3, p^3q^2, p^2q^2)

➤ $a = pq^2$ and $b = p^3q$

∴ HCF $(a, b) = pq$

∴ LCM $(a, b) = p^3q^2$

B **Write the answer of the following questions.** [Each carries 2 Marks] **[42]**

44. One card is drawn from a well-shuffled deck of 52 cards. Find the probability of getting

(i) a king of red colour (ii) a face card

(iii) a red face card (iv) the jack of hearts

(v) a spade (vi) the queen of diamonds

➤ One card is drawn from a well shuffled deck of 52 cards so the number of all possible outcomes is 52

(i) Let Event A : Getting a king of red colour.

There are 2 Kings of red colour, heart king and diamond king.

∴ The number of outcomes favourable to event A is 2.

∴ $P(A) = \frac{2}{52} = \frac{1}{26}$

(ii) Event B : Getting a face cards.

There are 12 facecards (4 kings + 4 Queens + 4 Jacks)

∴ The number of outcomes favourable to event B is 12.

∴ $P(B) = \frac{12}{52} = \frac{3}{13}$

(iii) Event C : Getting a red face card

There are 6 red face cards.

(2 Kings + 2 Queens + 2 Jacks of Heart and Diamond respectively.)

∴ The number of outcomes favourable to event C is 6.

∴ $P(C) = \frac{6}{52} = \frac{3}{26}$

(iv) Event D : Getting the jack of hearts

There is only one Jack of heart.

∴ The number of outcomes favourable to event D is 1.

∴ $P(D) = \frac{1}{52}$

(v) Event E : Getting a spade.

There are 13 spade cards.

∴ The number of outcomes favourable to event E is 13.

∴ $P(E) = \frac{13}{52} = \frac{1}{4}$

(vi) Event F : Getting a queen of diamonds.

There is only one card of queen of diamond

∴ The number of outcomes favourable to event F is 1.

∴ $P(F) = \frac{1}{52}$

45. Marks obtained out of 50, by 100 students in a test are given in the frequency table given below. Find the median of the data.

Marks obtained	20	29	28	33	42	38	43	25
No. of students	6	28	24	15	2	4	1	20

➠ **median = 28.5**

46. A hemispherical depression is cut out from one face of a cubical wooden block such that the diameter l of the hemisphere is equal to the edge of the cube. Determine the surface area of the remaining solid.(take $\pi = \frac{22}{7}$)

➤ Let the side of a cube be l

∴ The maximum diameter of the hemisphere = l

∴ The radius of the hemisphere = $\frac{l}{2}$

The surface area of the hemisphere $A = 2\pi r^2$

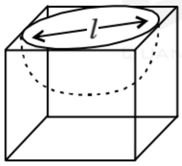
$$= 2\pi \left(\frac{l}{2}\right)^2$$

$$= \frac{\pi l^2}{2}$$

∴ The area of the base of the hemisphere $= \pi r^2$

$$= \pi \left(\frac{l}{2}\right)^2$$

$$= \frac{\pi l^2}{4}$$



The surface area of the cube = $6l^2$

The surface area of the remaining solid

= The surface area of the cube + the surface area of the hemisphere – the area of the base of the hemisphere

$= 6l^2 + \frac{\pi l^2}{2} - \frac{\pi l^2}{4}$

$= \frac{24l^2 + 2\pi l^2 - \pi l^2}{4}$

$= \frac{24l^2 + \pi l^2}{4}$

$= l^2 \left(\frac{24 + \pi}{4}\right)$

$= \frac{1}{4} l^2 (\pi + 24)$ (unit)^2

47. Area of a circle is 616 subtends an angle of 60° at the centre then find the length of the arc.

➤ Let radius of circle is r

$\therefore$ Area A = πr^2

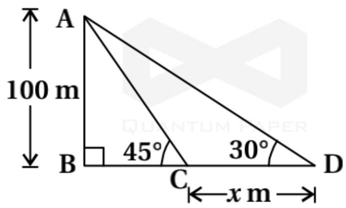
$\therefore\ r^2 = \frac{A}{\pi} = \frac{616 \times 7}{22} = 196 = (14)^2$

$\therefore\ r = 14$

Now $r = 14$, $\theta = 60$, l(length of arc) = ?

Length of arc $l = \frac{\pi r\theta}{180} = \frac{22}{7} \times \frac{14 \times 60}{180} = \frac{44}{3}$

48. When the angle of elevation of the sun decreases from 30° to 45°, the length of the shadow of the building decreases x m. If the height of the building is 100 m, find the value of x.

➤ 

➤ In Δ ABC,

$\tan 45° = \frac{AB}{BC}$

$\therefore\ 1 = \frac{100}{BC}$

$\therefore$ BC = 100 m

In Δ ABD,

$\tan 30° = \frac{AB}{BD}$

$\therefore\ \frac{1}{\sqrt{3}} = \frac{100}{BD}$

$\therefore$ BD = $\sqrt{3} \times 100$

BD = BC + CD

$\therefore\ \sqrt{3} \times 100 = 100 + CD$

$\therefore\ \sqrt{3} \times 100 - 100 = CD$

$\therefore\ 100\left(\sqrt{3}-1\right) = CD$

$\therefore\ 100\left(\sqrt{3}-1\right) = x$

$\therefore$ The value of x is $100\left(\sqrt{3}-1\right)$ m.

49.

	(A)	(B)
i)	One of the angle of a right angle triangle is 30°. The length of its opposite side is .... times its hypotenuse.	(a) two
ii)	One of the angle of right angle triangle is 60°. The length of its opposite side is .... times its hypotenuse.	(b) half
		(c) $\frac{\sqrt{3}}{2}$

➠ (i – b), (ii – c)

50. Name the type of quadrilateral formed, if any, by the following points, and give reasons for your answer : (–3, 5), (3, 1), (0, 3), (–1, –4)

➤ A(–3, 5), B(3, 1), C(0, 3) and D(–1, –4) are given points.

Each side,

$$AB = \sqrt{[(x_1)-(x_2)]^2 + [(y_1)-(y_2)]^2}$$

$$= \sqrt{[(-3)-(3)]^2 + [5-(1)]^2}$$

$$= \sqrt{(-6)^2 + (4)^2}$$

$$= \sqrt{36+16}$$

$$= \sqrt{52}$$

$$\therefore AB = 2\sqrt{13}$$

$$BC = \sqrt{[(3)-(0)]^2 + [(1)-(3)]^2}$$

$$= \sqrt{(3)^2 + (-2)^2}$$

$$= \sqrt{9+4}$$

$$\therefore BC = \sqrt{13}$$

$$CD = \sqrt{[(0)-(-1)]^2 + [[(3)-(-4)]^2}$$

$$= \sqrt{(0+1)^2 + (3+4)^2}$$

$$= \sqrt{(1)^2 + (7)^2}$$

$$= \sqrt{1+49}$$

$$= \sqrt{50}$$

$$\therefore CD = 5\sqrt{2}$$

$$DA = \sqrt{[(-1)-(-3)]^2 + [(-4)-(5)]^2}$$

$$= \sqrt{(2)^2 + (-9)^2}$$

$= \sqrt{4 + 81}$

$\therefore$ DA = $\sqrt{85}$

Diagonals :

$$AC = \sqrt{[(-3) - (0)]^2 + [5 - (3)]^2}$$

$$= \sqrt{(-3)^2 + (2)^2}$$

$$= \sqrt{9 + 4}$$

$\therefore$ AC = $\sqrt{13}$

$$BD = \sqrt{[(3) - (-1)]^2 + [(1) - (-4)]^2}$$

$$= \sqrt{(4)^2 + (5)^2}$$

$$= \sqrt{16 + 25}$$

$\therefore$ BD = $\sqrt{41}$

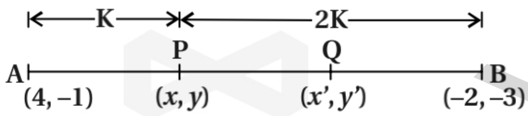
AC + BC = AB

$\sqrt{13} + \sqrt{13} = 2\sqrt{13}$

Thus, the points A, B and C are collinear.

Out of the given four points, three of them are collinear. So they do not form a quadrilateral.

51. Find the coordinates of the points of trisection of the line segment joining (4, –1) and (–2, –3).

➤ 

A(4, –1) and B(–2, –3) are given points.

Let the points P and Q divide AB into three equal parts.

P divides AB from A in the ratio A : 2.

$\therefore \dfrac{AP}{PB} = \dfrac{k}{2k} = \dfrac{1}{2} = \dfrac{m}{n}$ where $k > 0$

$\therefore$ $m = 1$ and $n = 2$

A(4, –1) = A (x_1, y_1)

B(–2, –3) = B (x_2, y_2)

$\therefore$ x-coordinate of P

$$x = \frac{mx_2 + nx_1}{m + n}$$

$$= \frac{1(-2) + 2(4)}{1 + 2}$$

$$= \frac{-2 + 8}{3} = 2$$

$\therefore$ y-coordinate of P

$$y = \frac{my_2 + ny_1}{m + n}$$

$$= \frac{1(-3) + 2(-1)}{1 + 2}$$

$$= \frac{-3 - 2}{3} = \frac{-5}{3}$$

$P(x, y) = P\left(2, \dfrac{-5}{3}\right)$

➤ Now P–Q–B and PQ = QB = K so Q is the midpoint of $\overline{PB}$.

- Let the coordinates of Q is (x', y').
- Q is the midpoint of $\overline{PB}$ $P\left(2, -\frac{5}{3}\right)$ and $B(-2, -3)$

$$\therefore\ Q(x', y') = \left(\frac{x + (-2)}{2}, \frac{y + (-3)}{2}\right)$$

$$= \left(\frac{2 + (-2)}{2}, \frac{-\frac{5}{3} + (-3)}{2}\right)$$

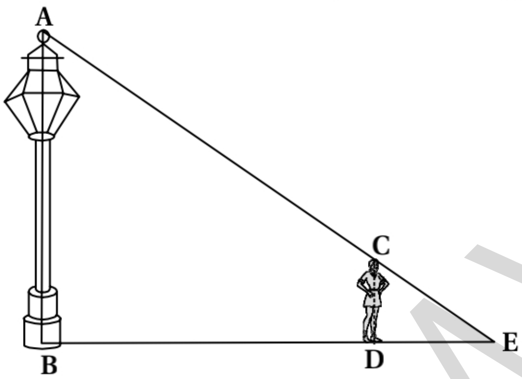
$$= \left(\frac{0}{2}, \frac{-5 - 9}{2 \times 3}\right)$$

$$= \left(0, \frac{-14}{6}\right)$$

$$= \left(0, \frac{-7}{3}\right)$$

Therefore, the required trisection points are $\left(2, \frac{-5}{3}\right)$ and $\left(0, \frac{-7}{3}\right)$.

52. A girl of height 90 cm is walking away from the base of a lamp-post at a speed of 1.2 m/s. If the lamp is 3.6 m above the ground, find the length of her shadow after 4 seconds.



- Let AB denote the lamp-post

and CD the girl after walking for 4 seconds away from the lamp-post. (see figure)

From the figure, you can see that DE is the shadow of the girl. Let DE be x meters.

- BD = 1.2 m/s = 4 m/s

Note : That in Δ ABE and Δ CDE

$\angle B = \angle D$ (Each is of 90° because lamp-post as well as the girl are standing vertical to the ground.)

and $\angle E = \angle E$ (same angle)

- So $\Delta ABE \sim \Delta CDE$ (AA similarity criterion)

Therefore, $\frac{BE}{DE} = \frac{AB}{CD}$

$\therefore\ \frac{4.8 + x}{x} = \frac{3.6}{0.9}$ [90 cm = 0.9 cm]

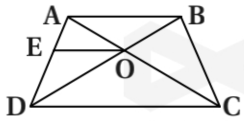
$\therefore\ 4.8 + x = 4x$

$\therefore\ 3x = 4.8$

$\therefore\ x = 1.6$

So, the shadow of the girl after walking 4 seconds is 1.6 m long.

53. The diagonals of a quadrilateral ABCD intersect each other at the point O such that $\frac{AO}{BO} = \frac{CO}{DO}$ Show that ABCD is a trapezium.

➤ 

➤ We have trapezium ABCD in which diagonals AC and BD Intersect each other at O such that.

$\therefore \frac{AO}{BO} = \frac{CO}{DO}$ (given)

$\therefore \frac{AO}{CO} = \frac{BO}{DO}$

➤ In Δ ADB Draw the EO || AB such that A–E–D and A–O–C.

$\therefore \frac{DE}{EA} = \frac{OD}{BO}$

$\therefore \frac{EA}{DE} = \frac{BO}{DO}$ ....(i)

But $\frac{AO}{CO} = \frac{BO}{DO}$ ....(ii) (given)

➤ From (i) and (ii)

$\frac{EA}{DE} = \frac{BO}{DO} = \frac{AO}{CO}$

So in Δ ADB, E ∈ AB and O ∈ AC (using converse of basic Proportionality theorem)

OE || DC and OE || AB

∴ AB || DC

∴ ABCD is a trapezium.

54. How many terms of the AP : 24, 21, 18,.... must be taken so that their sum is 78 ?

➤ Here, $a = 24$, $d = 21 - 24 = -3$, $S_n = 78$ We need to find n.

We know that, $S_n = \frac{n}{2}[2a + (n - 1)d]$

So, $78 = \frac{n}{2}[48 + (n - 1)(-3)] = \frac{n}{2}[51 - 3_n]$

or, $3n^2 - 51n + 156 = 0$

or, $n^2 - 17n + 52 = 0$

or, $(n - 4)(n - 13) = 0$

or, $n = 4$ or $n = 13$

Both values of n are admissible. So, the number of terms is either 4 or 13.

Remarks :

➤ In this case, the sum of the first 4 terms = the sum of the first 13 terms = 78.

➤ Two answers are possible because the sum of the terms from 5^{th} to 13^{th} will be zero. This is because a is positive and d is negative, so that some terms will be positive and some others negative, and will cancel out each other.

55. Check whether 301 is a term of the list of numbers 5, 11, 17, 23, . . .

➤ We have :

$a_2 - a_1 = 11 - 5 = 6$

$a_3 - a_2 = 17 - 11 = 6$

$a_4 - a_3 = 23 - 17 = 6$

As $a_{k+1} - a_k$ is the same for $k = 1, 2, 3$, etc., the given list of numbers is an AP.

➤ Now, $a = 5$, and $d = 6$

Let 301 be a term, say, the n^{th} term of this AP.

We know that

➤ $a_n = a + (n - 1)d$

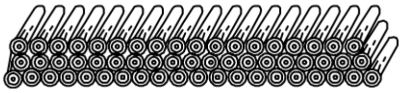
So, $301 = 5 + (n - 1) \times 6$

i.e., $301 = 6n - 1$

So, $n = \frac{302}{6} = \frac{151}{3}$

But n should be a positive integer. So, 301 is not a term of the given list of numbers.

56. 200 logs are stacked in the following manner : 20 logs in the bottom row, 19 in the next row, 18 in the row next to it and so on (see Fig.). In how many rows are the 200 logs placed and how may logs are in the top now ?



➤ we have,

20 logs in the bottom row

19 logs in the next row

18 logs in the row next to it and so on.

Total number of logs = 200

So, the terms are :

20, 19, 18, ...

which are in AP

$a = 20$, $d = 19 - 20 = -1$, and $S_n = 200$, $n = ?$

➤ $S_n = \frac{n}{2}[2a + (n - 1)d]$

$200 = \frac{n}{2}[2 \times 20 + (n - 1)(-1)]$

$\therefore 200 \times 2 = n[40 - n + 1]$

$\therefore 400 = n[-n + 41]$

$\therefore 400 = -n^2 + 41n$

$\therefore n^2 - 41n + 400 = 0$

$\therefore n^2 - 25n - 16n + 400 = 0$

$\therefore n(n - 25) - 16(n - 25) = 0$

$\therefore (n - 25)(n - 16) = 0$

$\therefore n - 25 = 0$ or $n - 16 = 0$

$\therefore n = 25$or $n = 16$

➤ If $n = 25$ then the number of logs in the top row is,

$a_n = a + (n - 1)d$

$\therefore\ a_{25} = 20 + (25 - 1)\ (-1)$

$= 20 - 24 = -4$ which is rejected.

If $n = 16$ then the number of logs in the top row is

$a_{16} = a + (16 - 1)\ d$

$= 20 + 15(-1)$

$= 20 - 15$

$= 5$

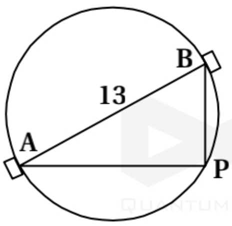
So, there are 5 logs in the top row and the 200 logs are placed in 16 rows

57. A pole has to be erected at a point on the boundary of a circular park of diameter 13 metres in such a way that the differences of its distances from two diametrically opposite fixed gates A and B on the boundary is 7 metres. Is it possible to do so ? If yes, at what distances from the two gates should the pole be erected ?

➤ Let us first draw the diagram (see Fig)

Let P be the required location of the pole. Let the distance of the pole from the gate B be x m, i.e., BP = x m. Now the difference of the distances of the pole from the two gates = AP – BP (or, BP – AP) = 7 m.

Therefore, AP = $(x + 7)$ m.



➤ Now, AB = 13m, and since AB is a diameter,

$\angle APB = 90°$ (Why ?)

Therefore, $AP^2 + PB^2 = AB^2$ (By Pythagoras theorem)

i.e., $(x + 7)^2 + x^2 = (13)^2$

i.e., $x^2 + 14x + 49 + x^2 = 169$

i.e., $2x^2 + 14x - 120 = 0$

So, the distance 'x' of the pole from gate B satisfies the equation $x^2 + 7x - 60 = 0$

➤ So, it would be possible to place the pole if this equation has real roots. To see if this is so or not, let us consider its discriminant. The discriminant is

$b^2 - 4ac = 72 - 4 \times 1 \times (-60) = 289 > 0.$

So, the given quadratic equation has two real roots, and it is possible to erect the pole on the boundary of the park.

Solving the quadratic equation

$x^2 + 7x - 60 = 0$ by the quadratic formula, we get

$x = \dfrac{-7 \pm \sqrt{289}}{2} = \dfrac{-7 - 17}{2}$

Therefore, $x = 5$ or -12.

Since x is the distance between the pole and the gate B, it must be positive. Therefore, $x = -12$ will have to be ignored. So, $x = 5$.

Thus, the pole has to be erected on the boundary of the park at a distance of 5m from the gate B and 12m from the gate A.

58. Check whether the following are quadratic equation : $(x + 2)^3 = x^3 - 4$

➤ Here, LHS = $(x + 2)^3 = x^3 + 6x^2 + 12x + 8$

Therefore, $(x + 2)^3 = x^3 - 4$ can be rewritten as

$x^3 + 6x^2 + 12x + 8 = x^3 - 4$

i.e., $6x^2 + 12x + 12 = 0$ or $x^2 + 2x + 2 = 0$

It is of the form $ax^2 + bx + c = 0$

So, the given equation is a quadratic equation.

59. Find the root of the following quadratic equation by factorisation : $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$

➤ $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$

$\therefore \sqrt{2}x^2 + (2 + 5)x + 5\sqrt{2} = 0$

$\therefore \sqrt{2}x^2 + 2x + 5x + 5\sqrt{2} = 0$

$\therefore \sqrt{2}x(x + \sqrt{2}) + 5(x + \sqrt{2}) = 0$

$\therefore (x + \sqrt{2})(\sqrt{2}x + 5) = 0$

$\therefore x + \sqrt{2} = 0$ or $\sqrt{2}x + 5 = 0$

$\therefore x = -\sqrt{2}$ or $x = -\frac{5}{\sqrt{2}}$

Hence, $-\sqrt{2}$ and $\frac{-5}{\sqrt{2}}$ are the roots of the given equation $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$.

60. Priyanka's mother is 20 years older than her. The product of their ages 5 years from now will be 300 we would to like to find Priyanka's present age keep if in mind, construct the quadratic equation.

➠ Quadratic equation $x^2 + 30x - 175 = 0$

61.

	(A)	(B)
i)	Which Indian Mathematician gave the formula to find the solution of the quadratic equation $ax^2 + bx + c = 0$ by perfect square method.	(a) Brahmagupta
ii)	Who gave the method to solve the quadratic equation in Hindu religion ?	(b) Sridharacharya
		(c) Bhaskaracharya.

➠ (i – b), (ii – c)

62. For what value of k will the following pair of linear equations have infinitely many solutions. $kx + 3y - (k - 3) = 0$; $12x + ky - k = 0$

➠ $k = 6$

63. Find the zeroes of the following quadratic polynomial and verify the relationship between the zeroes and the coefficients : $6x^2 - 3 - 7x$

➤ $\therefore 6x^2 - 7x - 3$ (Arrang in decending power of x)

Let $p(x) = 6x^2 - 7x - 3$

$= 6x^2 - (9 - 2)\ x - 3$

$= 6x^2 - 9x + 2x - 3$

$= 3x(2x - 3) + 1(2x - 3)$

$= (2x - 3)\ (3x + 1)$

➤ To find the zeroes of $p(x)$, we have $p(x) = 0$

$p(x) = (2x - 3)\ (3x + 1)$

$0 = (2x - 3)\ (3x + 1)$

$\therefore\ 2x - 3 = 0$	$3x + 1 = 0$
$\therefore\ 2x = 3$	$\therefore\ 3x = -1$
$\therefore\ x = \frac{3}{2}$	$\therefore\ x = \frac{-1}{3}$

Hence, $\frac{3}{2}$ and $-\frac{1}{3}$ are the zeroes of the given polynomial $6x^2 + 7x - 3$.

➤ $p(x) = 6x^2 - 7x - 3$ comparing with $ax^2 + bx + c$ we get $a = 6$, $b = -7$, $c = -3$

Sum of the zeroes $= \left(\frac{3}{2}\right) + \left(-\frac{1}{3}\right) = \frac{-(-7)}{6} = \frac{-b}{a} = -\frac{(\text{coefficient of } x)}{\text{coefficient of } x^2}$

Product of the zero $= \frac{3}{2} \times \frac{-1}{3} = \frac{-3}{6} = \frac{c}{a} = \frac{\text{constant term}}{\text{coefficient of } x^2}$

64. Prove that $3 + 2\sqrt{5}$ is irrational.

➤ Let $x = 3 + 2\sqrt{5}$ is rational.

$\therefore\ \frac{x - 3}{2} = \sqrt{5}$ is rational.

Let us assume that $\sqrt{5} = \frac{a}{b}$

(Where HCF $(a, b) = 1$, $a, b \in$ N)

$\therefore\ a^2 = 5b^2$ ....(i)

$\therefore\ 5|a^2$

$\therefore\ 5|a$

$\therefore$ Let $a = 5a_1$; $a_1 \in$ N

$\therefore\ 25a_1^2 = a^2 = 5b^2$ ($\because$ Putting $a = 5a_1$)

$\therefore\ b^2 = 5a_1^2$

$\therefore\ 5|b^2$

$\therefore\ 5|b$

Thus $5|a$ and $5|b$

But this is contradiction

as HCF $(a, b) = 1$

$\therefore$ So we conclude that

$\sqrt{5}$ is irrational

$\therefore\ 3 + 2\sqrt{5}$ is irrational

C **Write the answer of the following questions. [Each carries 3 Marks]**

65. A petrol pump owners wants to analyses the daily need of diesel at the pump. For this he collected the data of vehicles visited in 1 hour. The following frequency distribution table shows the classification of the number of vehicles and quantity of diesel filled in them.

Diesel filled (in lit)	3-5	5-7	7-9	9-11	11-13
No. of vehicles	5	10	10	7	8

1) Average diesel required for a vehicle is

(A) 8.15 l (B) 6 l (C) 10600 l (D) 5.5 l

2) If approximately 2000 vehicles comes daily at the petrol pump, then how much liters of diesel two pump should have ?

(A) 16200 l (B) 16300 l (C) 10600 l (D) 15000 l

3) The sum of upper and lower limit of median class is ....

(A) 22 (B) 10 (C) 16 (D) None of above

4) If the median of given data is 8 l then mode will be equal to :

(A) 7.5 l (B) 7.7 l (C) 5.7 l (D) 8 l

➠ **1) (A) 8.15 l**

➤ The frequency distribution table from the given data can be drawn as

Class	class mark x_i	frequency (f_i)	$x_i f_i$
3-5	4	5	20
5-7	6	10	60
7-9	8	10	80
9-11	10	7	70
11-13	12	8	96
Total		$\Sigma f_i = 40$	$\Sigma x_i f_i = 326$

Mean $= \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{326}{40} = 8.15$ litres

2) (B) 16300 l

➤ If 2000 vehicles comes daily and average quantity of diesel required for a vehicle is 8.15 litres, then total quantity of diesel required

$= 2000 \times 8.15$

$= 16300$ litres

3) (C) 16

➤ Here, $n = 40$ and $\frac{n}{2} = 20$

c. f. for the distribution are 5, 15, 25, 32, 40. Now, *c.f.* Just greater than 20 is 25 which is corresponding to the class interval 7 – 9

So median class is 7 – 9

∴ Required sum of upper limit and lower limit = 7 + 9 = 16

4) (D) 8 l

➤ Because the value of mean, mode and median are equal for same data.

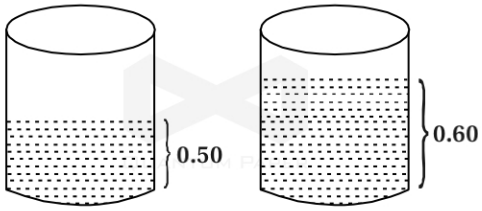
66. 1000 litres = ... cubic metre $\left(1, 10, \frac{1}{2}\right)$

➤ 1000 litres = 1 kilo litre

1 k l = 1 cubic meter.

∴ 1000 l = 1 cubic meter.

67. A cylinder is filled half with water. If we add 10 l water in it, the height of the water is increased up to 0.60 part what is the capacity of the cylinder ?

➤ 

A cylinder is filled half with water i.e. the water level is up to 0.5. If we add 10 l water in it, the height of the water is increased up to 0.60 part i.e. It is increased (0.60-0.5) = 0.10

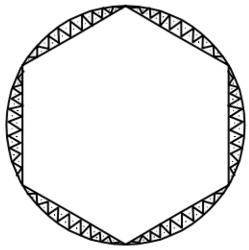
For 0.1 part → 10 l water

∴ For 1 part → (?)

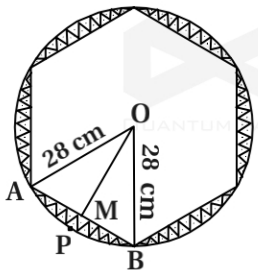
∴ $\frac{10}{1} \times \frac{1}{0.10} = 10 \times 10 = 100\ l$

Therefore the capacity of the cylinder is 100 l

68. A round table cover has six equal designs as shown in Figure. If the radius of the cover is 28 cm, find the cost of making the designs at the rate of ₹ 0.35 per cm^2. (Use $\sqrt{3} = 1.7$)



➤



Suppose AB is a chord. $\angle AOB = \frac{360°}{6} = 60°$

Area of minor sector OAPB :

$$\text{Area of minor sector OAPB} = \frac{\theta}{360} \times \pi r^2$$

$$= \frac{60}{360} \times \frac{22}{7} \times (28)^2\ cm^2$$

$$= \frac{1232}{3}\ cm^2$$

$$= 410.67\ cm^2$$

➤ Draw OM ⊥ AB.

∴ ΔOMA and ΔOMB become right angled triangle.

In ΔAOB, OA = OB (equal radius of one circle)

∴ OM = OM (Common side)

∴ ∠OMA ≅ ∠OMB (∵ 90°)

∴ Δ OMA ≅ Δ OMB (RHS)

$\therefore$ AM = BM = $\frac{1}{2}$ AB

and $\angle$AOM = $\angle$BOM

$\therefore \angle AOM = \angle BOM = \frac{1}{2} \angle AOB$

$= \frac{1}{2} \times 60°$

$\therefore \angle AOM = \angle BOM = 30°$

➤ In ΔOMA

$\cos 30° = \frac{OM}{OA}$

$\therefore \frac{\sqrt{3}}{2} = \frac{OM}{28}$

$\therefore OM = \frac{\sqrt{3} \times 28}{2}$

$\therefore OM = 14\sqrt{3}$ cm

In ΔOMA,

$\sin 30° = \frac{AM}{OA}$

$\therefore \frac{1}{2} = \frac{AM}{28}$

$\therefore AM = \frac{28}{2}$

$\therefore AM = 14$

$\therefore 2AM = 28$

$\therefore AB = 28$ cm

➤ **Area of Δ AOB**

Area of Δ AOB $= \frac{1}{2} \times AB \times OM$

$= \frac{1}{2} \times 28 \times 14\sqrt{3}$ cm^2

$= 196\sqrt{3}$ cm^2

$= 196 \times 1.7$ cm^2

$= 333.2$ cm^2

➤ **Area of minor segment APB** = Area of minor segment APB

= Area of minor sector OAPB – Area of ΔAOB

= 410.67 – 333.2

$\therefore$ Area of minor segment = 77.47 cm^2

1 Area of minor segment = 77.47 cm^2

$\therefore$ 6 Area of minor segment = (?)

= 77.47 × 6 = 464.82 cm^2

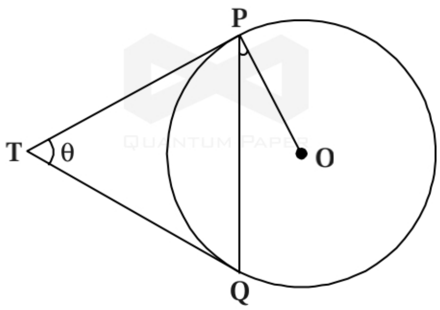
➤ **Cost :**

1 cm^2 = ₹ 0.35 (cost)

$\therefore$ 464.82 cm^2 = (?)

= 0.35 × 464.82 = ₹ 162.68

69. Two tangents TP and TQ are drawn to a circle with centre O from an external point T. Prove that $\angle PTQ = 2\angle OPQ$.

➤ 

Given : A circle with centre O, an external point T and two tangents TP and TQ to the circle are given where P and Q are points of contact.

To prove : $\angle PTQ = 2\angle OPQ$

Proof :

➠ Let $\angle PTQ = \theta$

The lengths of the tangents drawn from an external point to the circle are equal.

$\therefore$ TP = TQ $\Rightarrow \angle TQP = \angle TPQ$.

$\therefore$ ΔTPQ is an isosceles triangle.

➠ In ΔTPQ,

$\angle TPQ + \angle TQP + \angle PTQ = 180°$

$\therefore \angle TPQ + \angle TPQ + \angle PTQ = 180°$ [$\therefore \angle TQP = \angle PTQ$]

$\therefore 2\angle TPQ + \angle PTQ = 180°$

$\therefore 2\angle TPQ = 180° - \angle PTQ$

$\therefore \angle TPQ = \dfrac{180° - \angle PTQ}{2}$

$\therefore \angle TPQ = 90° - \dfrac{\angle PTQ}{2}$

$\therefore \angle TPQ = 90° - \dfrac{1}{2}\angle PTQ$

$\therefore \angle TPQ = 90° - \dfrac{1}{2}\theta$

$\therefore \angle TPQ = \angle TQP = 90° - \dfrac{1}{2}\theta$

The tangent at any point of a circle is perpendicular to the radius through the point of contact.

$\therefore \angle OPT = 90°$

$\therefore \angle OPQ = \angle OPT - \angle TPQ$

$= 90° - \left[90° - \dfrac{1}{2}\theta\right]$

$= 90° - 90° + \dfrac{1}{2}\theta$

$\therefore \angle OPQ = \dfrac{1}{2}\theta$

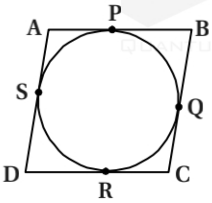
$\therefore \angle OPQ = \dfrac{1}{2} \times \angle PTQ$

∴ 2∠OPQ = ∠PTQ

∴ ∠PTQ = 2∠OPQ

70. Prove that the parallelogram circumscribing a circle is a rhombus.

➤ This sides AB, BC, CD and DA of a parallelogram ABCD touch the circle at the points P, Q, R and S respectively. AP and AS are tangents to the circle from the external point A.



∴ AP = AS ....(i)

similarly BP = BQ ....(ii)

CR = CQ ....(iii)

DR = DS ....(iv)

Adding the results (i) to (iv) we get

AP + BP + CR + DR = AS + BQ + CQ + DS

∴ (AP + BP) + (CR + DR) = (AS + DS) + (BQ + CQ)

∴ AB + CD = AD + BC ....(v)

But □ ABCD is a parallelogram

∴ AB = CD and BC = AD ....(vi)

From result (v) and (vi) we have

AB + AB = AD + AD

∴ 2AB = 2AD

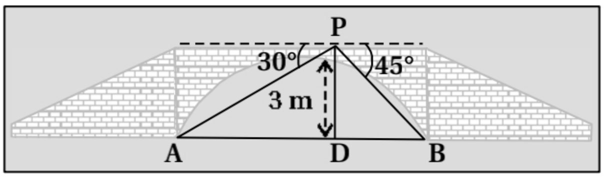
∴ AB = AD (∵ Adjacent sides are equal)

In a parallelogram, AB = AD

∴ AB = BC = CD = AD

∴ □ ABCD is a rhombus.

71. From a point on a bridge across a river, the angles of depression of the banks on opposite sides of the river are 30° and 45°, respectively. If the bridge is at a height of 3 m from the banks, find the width of the river.



➤ A and B represent points on the bank on opposite side of the river. AB is width of the river.

➤ P is a point on the bridge at a height of 3m i.e. DP = 3m.

➤ The width of river = AB = ?

➤ AB = AD + DB

In Δ APD right angle at D,

$\tan 30° = \frac{PD}{AD}$

$\therefore \frac{1}{\sqrt{3}} = \frac{3}{AD}$ m

$\therefore AD = 3\sqrt{3}$ m

In a right angle Δ PBD

$\tan 45° = \frac{PD}{BD}$

$\therefore 1 = \frac{PD}{DB}$

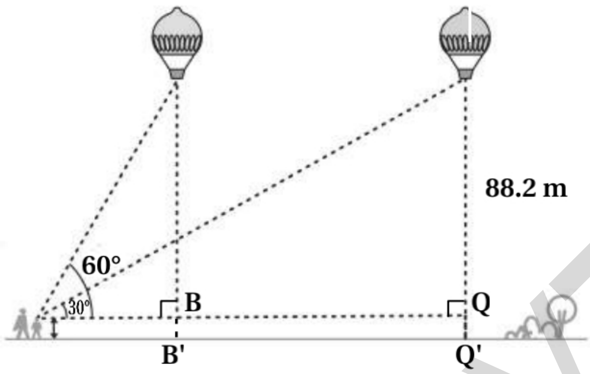
$\therefore BD = PD = 3$ m

➤ AB = AD + DB

$= 3\sqrt{3} + 3 = 3\left(\sqrt{3} + 1\right)$ m

Therefore, the width of the river is $3\left(1 + \sqrt{3}\right)$ m.

72. A 1.2 m tall girl spots a balloon moving with the wind in a horizontal line at a height of 88.2 m from the ground. The angle of elevation of the balloon from the eyes of the girl at any instant is 60°. After some time, the angle of elevation reduces to 30° (see Figure). Find the distance travelled by the balloon during the interval.



➤ The angles of elevation of the balloon are 60° and 30° respectively.

Here, AB' = PQ' = 88.2 m

BB' = QQ' = 1.2 m

$\therefore$ PQ = PQ' − QQ'

= 88.2 − 1.2 = 87 m

PQ = AB = 87 m

In Δ ABO,

$\tan 60° = \frac{AB}{OB}$

$\therefore \sqrt{3} = \frac{87}{OB}$

$\therefore OB = \frac{87}{\sqrt{3}}$ ....(i)

In Δ PQO

$\tan 30° = \frac{PQ}{OQ}$

$\therefore \frac{1}{\sqrt{3}} = \frac{87}{OQ}$

$\therefore OQ = 87 \times \sqrt{3}$ ...(ii)

The distance travelled by the balloon during the interval,

BQ = OQ − OB

$= 87\sqrt{3} - \frac{87}{\sqrt{3}}$

$= 87\left(\sqrt{3} - \frac{1}{\sqrt{3}}\right)$

$= 87\left(\frac{3-1}{\sqrt{3}}\right)$

$= \frac{87 \times 2}{\sqrt{3}}$

$= \frac{87 \times 2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

$= \frac{87 \times 2 \times \sqrt{3}}{3}$

$= 58\sqrt{3}$ m

Therefore, the distance travelled by the balloon is $58\sqrt{3}$ m.

73. Prove that $\frac{\sin\theta - \cos\theta + 1}{\sin\theta + \cos\theta - 1} = \frac{1}{\sec\theta - \tan\theta}$ using the identity $\sec^2\theta = 1 + \tan^2\theta$

➤ L.H.S. $= \frac{\sin\theta - \cos\theta + 1}{\sin\theta + \cos\theta - 1}$

$= \sin\theta - \cos\theta + 1 \div \sin\theta + \cos\theta - 1$

$= \sin\theta - \cos\theta + 1 \div \sin\theta + \cos\theta - 1$

$= \frac{\sin\theta}{\cos\theta} - \frac{\cos\theta}{\cos\theta} + \frac{1}{\cos\theta} \div \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\cos\theta} - \frac{1}{\cos\theta}$

$= \tan\theta - 1 + \sec\theta \div \tan\theta + 1 - \sec\theta$

$= \sec\theta + \tan\theta - 1 \div 1 - (\sec\theta - \tan\theta 1)$

$= \frac{\sec\theta + \tan\theta - 1}{1 - (\sec\theta - \tan\theta)}$

$= \frac{\sec\theta + \tan\theta - 1}{(\sec^2\theta - \tan^2\theta) - (\sec\theta - \tan\theta)}$

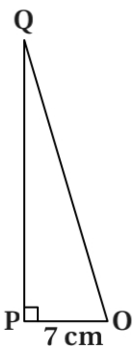
$= \frac{\sec\theta + \tan\theta - 1}{(\sec\theta - \tan\theta)(\sec\theta + \tan\theta) - (\sec\theta - \tan\theta)}$

$= \frac{(\sec\theta + \tan\theta - 1)}{(\sec\theta - \tan\theta)(\sec\theta + \tan\theta - 1)}$

$= \frac{1}{\sec\theta - \tan\theta}$

= R.H.S.

74. In Δ OPQ, right-angled at P, OP = 7 cm and OQ – PQ = 1 cm (see Fig.). Determine the values of sin Q and cos Q.



➤ In Δ OPQ $OQ^2 = OP^2 + PQ^2$

$\therefore (1 + PQ)^2 = OP^2 + PQ^2$

$\therefore 1 + PQ^2 + 2PQ = OP^2 + PQ^2$

$\therefore\ 1 + 2PQ = 7^2$

$\therefore\ PQ = 24$ cm

OQ = 1 + PQ = 24 + 1 cm = 25 cm

So, $\sin Q = \dfrac{7}{25}$ and $\cos Q = \dfrac{24}{25}$

75. Prove the following identitie, where the angle involved is acute angle for which the expression is defined : $\dfrac{\tan\theta}{1-\cot\theta} + \dfrac{\cot\theta}{1-\tan\theta} = 1 + \sec\theta \cdot \operatorname{cosec}\theta$

[Hint : Write the expression in terms of sin θ and cos θ]

➤ L.H.S. $= \dfrac{\tan\theta}{1-\cot\theta} + \dfrac{\cot\theta}{1-\tan\theta}$

$\therefore$ L.H.S. $= \tan\theta \div 1 - \cot\theta + \cot\theta \div 1 - \tan\theta$

$= \dfrac{\sin\theta}{\cos\theta} \div 1 - \dfrac{\cos\theta}{\sin\theta} + \dfrac{\cos\theta}{\sin\theta} \div 1 - \dfrac{\sin\theta}{\cos\theta}$

$= \dfrac{\sin\theta}{\cos\theta} \div \dfrac{\sin\theta - \cos\theta}{\sin\theta} + \dfrac{\cos\theta}{\sin\theta} \div \dfrac{\cos\theta - \sin\theta}{\cos\theta}$

$= \dfrac{\sin\theta}{\cos\theta} \times \dfrac{\sin\theta}{\sin\theta - \cos\theta} + \dfrac{\cos\theta}{\sin\theta} \times \dfrac{\cos\theta}{\cos\theta - \sin\theta}$

$= \dfrac{\sin^2\theta}{\cos\theta(\sin\theta - \cos\theta)} + \dfrac{\cos^2\theta}{\sin\theta(\cos\theta - \sin\theta)}$

$= \dfrac{\sin^2\theta}{\cos\theta(\sin\theta - \cos\theta)} - \dfrac{\cos^2\theta}{\sin\theta(\sin\theta - \cos\theta)}$

$= \dfrac{\sin^3\theta - \cos^3\theta}{\sin\theta\cos\theta\,(\sin\theta - \cos\theta)}$

$= \dfrac{(\sin\theta - \cos\theta)\left(\sin^2\theta + \sin\theta\cos\theta + \cos^2\theta\right)}{\sin\theta\cos\theta\,(\sin\theta - \cos\theta)}$ [$\because$ using this formula $a^3 - b^3 = (a-b)\ (a^2 + ab + b^2)$]

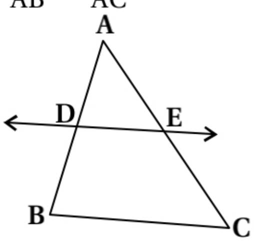
$= \dfrac{1 + \sin\theta\cos\theta}{\sin\theta\cos\theta} = \dfrac{1}{\sin\theta\cdot\cos\theta} + \dfrac{\sin\theta\cos\theta}{\sin\theta\cos\theta}$

$= \dfrac{1}{\sin\theta} \times \dfrac{1}{\cos\theta} + 1$

$= \operatorname{cosec}\theta \cdot \sec\theta + 1$

$= 1 + \sec\theta \cdot \operatorname{cosec}\theta =$ R.H.S.

76. If a line intersects sides AB and AC of a Δ ABC at D and E respectively and is parallel to BC prove that $\dfrac{AD}{AB} = \dfrac{AE}{AC}$ (see figure)



➤ DE || BC (given)

$\therefore\ \dfrac{AD}{DB} = \dfrac{AE}{EC}$ (Theorem 6.1)

DB EC

or $\frac{\quad}{AD} = \frac{\quad}{AE}$

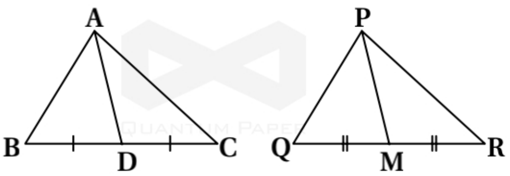
or $\frac{DB}{AD} + 1 = \frac{EC}{AE} + 1$

or $\frac{DB + AD}{AD} = \frac{EC + AE}{AE}$

or $\frac{AB}{AD} = \frac{AC}{AE}$ (A – D – B $\therefore$ AD + DB = AB and A–E–C $\therefore$ AE + EC = AC)

$\therefore \frac{AD}{AB} = \frac{AE}{AC}$

77. If AD and PM are medians of triangles ABC and PQR, respectively where Δ ABC ~ Δ PQR, prove that $\frac{AB}{PQ} = \frac{AD}{PM}$

➤ 

➤ We have Δ ABC ~ Δ PQR such that AD and PM are the median corresponding to the sides BC and QR respectively and the corresponding sides of similar triangles are proportional.

$\therefore \frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$ ....(i)

Corresponding Angle are also equal in two similar triangles.

$\angle A = \angle P, \angle B = \angle Q, \angle C = \angle R$ ....(ii)

➤ Since AD and PM are medians.

$\therefore$ BC = 2BD and QR = 2QM (iii)

From (i)

$\therefore \frac{AB}{PQ} = \frac{2BD}{2QM} = \frac{BD}{QM}$ ....(iii)

And $\angle B = \angle Q \Rightarrow \angle ABD = \angle PQM$ ....(iv)

➤ From (iii) and (iv) we have

Δ ABD ~ Δ PQM (SAS similarity)

Their corresponding sides are proportional

$\therefore \frac{AB}{PQ} = \frac{AD}{PM}$.

78. A manufacturer of TV sets produced 600 sets in the third year and 700 sets in the seventh year. Assuming that the production increases uniformly by a fixed number every year, find :

(i) the production in the 1^{st} year

(ii) the production in the 10^{th} year

(iii) the total production in first 7 years

➤ (i) Since the production increases uniformly by a fixed number every year, the number of TV sets manufactured in 1^{st}, 2^{nd}, 3^{rd}, ..., years will form an AP.

Let us denote the number of TV sets manufactured in the n^{th} year by a_n.

Then $a_3 = 600$ and $a_7 = 700$

or $a + 2d = 600$

and $a + 6d = 700$

Solving these equations, we get $d = 25$ and $a = 550$.

Therefore, production of TV sets in the first year is 550.

(ii) Now, $a_{10} = a + 9d = 550 + 9 \times 25 = 775$

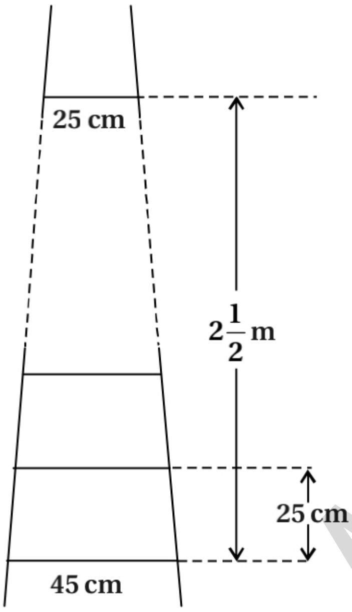
So, production of TV sets in the 10^{th} year is 775.

(iii) Also, $S_7 = \frac{7}{2}\ [2 \times 550 + (7 - 1) \times 25]$

$= \frac{7}{2}\ [1100 + 150] = 4375$

Thus, the total production of TV sets in first 7 years is 4375.

79. A ladder has rungs 25 cm apart (see Fig.). The rungs decrease uniformly in length from 45 cm at the bottom to 25 cm at the top. If the top and the bottom rungs are $2\frac{1}{2}$ m apart, what is the length of the wood required for the rungs ?



➤ The distance between two consecutive rungs = 25 cm.

Distance between top and bottom rungs = 2.5 m

= 250 cm

$\therefore$ Number of rungs $= \frac{250}{25} + 1$

$= 10 + 1 = 11$

The rungs decrease uniformly in length from 45 cm at bottom to 25 cm at the top. The Length of rungs form an AP and $T_{11} = 45$ cm ($\because$ there are 11 rungs)

➤ Let the Length of rung decreases x cm.

$\therefore\ 45 + (45 - x) + (45 - 2x) + ... + 25$ cm.

Her a = 45, $l = 25$, $n = 11$

➤ $S_n = \frac{n}{2}\ (a + l)$

$= \frac{11}{2}$ [45 + 25]

$= \frac{11}{2}$ [70]

= 385 cm

So, the length of the wood required for the rungs is 385 cm.

80. The sum of a two-digit number and the number obtained by reversing the digits is 66. If the digits of the number differ by 2, find the number. How many such numbers are there ?

➤ Let the ten's and the unit's digits in the first number be x and y respectively.

So, the first number may be written as

$10(x) + y = 10x + y$

➤ If we reverse the digits i.e. the ten's digit is y and the unit's digit is x, the number will be

$10(y) + x = 10y + x$.

➤ According to the given condition,

$(10x + y) + (10y + x) = 66$

$\therefore\ 11x + 11y = 66$

$\therefore\ 11\ (x + y) = 66$

$\therefore\ x + y = 6$ ....(i)

We are also given that the digits differ by 2 therefore

$x - y = 2$ ....(ii)

➤ Method of elimination

$x + y = 6$ (i)

$x - y = 2$ (ii)

Adding $2x = 8$

$\therefore\ x = 4$

➤ Substitute $x = 4$ in $x + y = 6$

$\therefore\ 4 + y = 6$

$\therefore\ y = 6 - 4 = 2$

➤ **Method of substitution :**

$x + y = 6$ ....(i)

$x - y = 2$ ....(ii)

➤ $x - y = 2 \Rightarrow x = y + 2$

➤ $x + y = 6$ (i)

$\therefore\ y + 2 + y = 6$

$\therefore\ 2y = 6 - 2 = 4$

$\therefore\ y = 2$

➤ Now $x + y = 6$ and $y = 2$

$\therefore\ x + 2 = 6$

$\therefore\ x = 4$

Thus, $(x, y) = (4, 2)$

$\therefore$ The required number is

$10x + y = 10(4) + 2 = 42$

➤ By reversing the digits the number is, $10y + x = 10(2) + 4 = 24$

Therefore, there are two numbers 42 and 24.

81. Draw the graphs of the equations $x - y + 1 = 0$ and $3x + 2y - 12 = 0$. Determine the coordinates of the vertices of the triangle formed by these lines and the X-axis, and shade the triangular region.

➤ $x - y + 1 = 0$

$\therefore\ -y = -x - 1$

$\therefore\ y = x + 1$

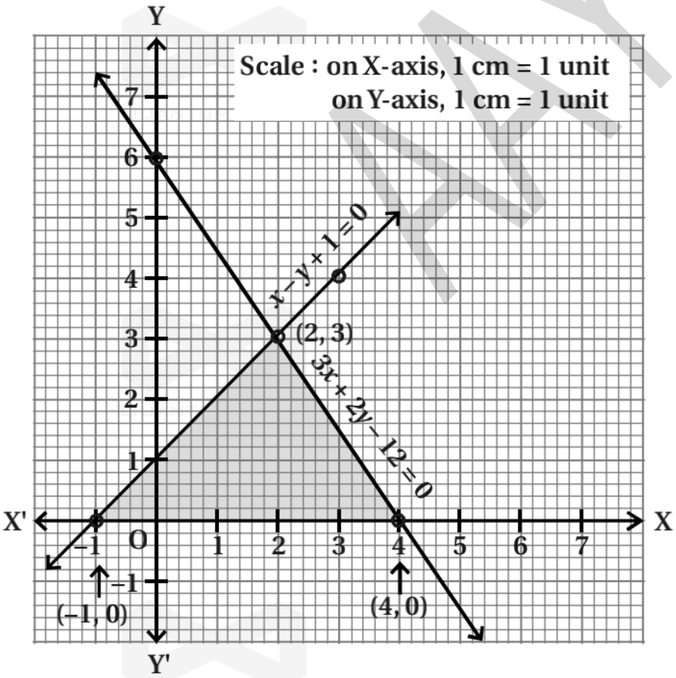
x	2	–1	3
$y = x + 1$	3	0	4

➤ $3x + 2y - 12 = 0$

$\therefore\ 2y = -3x + 12$

$\therefore\ y = \dfrac{-3x + 12}{2}$

x	2	4	0
$y = \dfrac{-3x + 12}{2}$	3	0	6



➤ The vertices of the triangle formed by these lines and the X-axis are (–1, 0) (4, 0) and (2, 3).

82. Ravi scored 40 marks in a test, getting 3 marks for each right answer and losing 1 mark for each wrong answer. Had 4 marks been awarded for each correct answer and 2 marks been deducted for each incorrect answer, then Ravi would have scored 50 marks. How many questions were there in the test ?

➠ right answer $x = 15$, wrong answer $y = 5$, Total number of questions = 20

83. Find the zeroes of the following quadratic polynomial and verify the relationship between the zeroes and the coefficients : $t^2 - 15$

➤ Let $p(t) = t^2 - 15$

$$= (t)^2 - \left(\sqrt{15}\right)^2$$

$$= \left(t - \sqrt{15}\right)\left(t + \sqrt{15}\right)$$

➤ To find the zeroes of $p(t)$ we take $p(t) = 0$.

$p(t) = \left(t - \sqrt{15}\right)\left(t + \sqrt{15}\right)$

$\therefore\ 0 = \left(t - \sqrt{15}\right)\left(t + \sqrt{15}\right)$

$\therefore\ t - \sqrt{15} = 0$ or $t + \sqrt{15} = 0$

$\therefore\ t = \sqrt{15}$ or $t = -\sqrt{15}$

Hence, $\sqrt{15}$ and $-\sqrt{15}$ are the zeroes of $t^2 - 15$.

➤ $p(t) = t^2 - 15$ comparing with $ax^2 + bx + c$, we get

$a = 1,\ b = 0,\ c = -15$

➤ Sum of the zeroes $= \left(\sqrt{15}\right) + \left(-\sqrt{15}\right) = 0$

$$= \frac{-(0)}{1} = -\frac{b}{a} = -\frac{(\text{coefficient of } t)}{\text{coefficient of } t^2}$$

➤ Product of the zeroes $= \left(\sqrt{15}\right) \times \left(-\sqrt{15}\right) = -15$

$$= \frac{-15}{1} = \frac{c}{a} = \frac{\text{constant term}}{\text{coefficient of } t^2}$$

D **Write the answer of the following questions.** [Each carries 4 Marks] **[20]**

84. (i) Find the probability that a year selected at random will contain 53 Sunday. Also find the probability that in a leap year.

(ii) February has 5 Friday.

➠ (i) $\frac{1}{7}$

(ii) $\frac{1}{7}$

85. A life insurance agent found the following data for distribution of ages of 100 policy holders. Calculate the median age, if policies are given only to persons having age 18 years onwards but less than 60 year.

Age (in years)	Number of policy holders
Below 20	2
Below 25	6
Below 30	24
Below 35	45
Below 40	78
Below 45	89
Below 50	92
Below 55	98
Below 60	100

➤ The given distribution is of less than type. So convert it into frequency distribution with cumulative frequency.

➤ There are 2 policy holders with age less than 20 years. So the frequency of the class 15 – 20 is 2

➤ There are 6 policy holders with age less than 25. So the number of policy holder in the interval 20 – 25 = The number of policy holders with age less than 25 – the number of policy holders with age less than 20 = 6 – 2 = 4.

➤ Similarly, we find the frequency for other class and get the following frequency distribution.

Age(in years) Class interval	The number of policy holders frequency (f_i)	Cumulative frequency (cf)
15 – 20	2	2
20 – 25	4	6
25 – 30	18	24
30 – 35	21	45
35 – 40	33	78
40 – 45	11	89
45 – 50	3	92
59 – 55	6	98
55 – 60	2	100
Total	n **= 100**	

➤ Here $n = 100 \Rightarrow \frac{n}{2} = \frac{100}{2} = 50$

➤ The cumulative frequency greater than 50 is 78 which lies in the interval 35 – 40.

$\therefore$ The median class is 35 – 40

➤ The lower limit of the median class is $l = 35$

Number of observations $n = 100$

The cumulative frequency of the class preceding the median class $cf = 45$

The frequency of the median class $f = 33$

Class size $h = 5$

➤ Median $= l + \dfrac{\frac{n}{2} - cf}{f} \times h$

$= 35 + \dfrac{50 - 45}{33} \times 5$

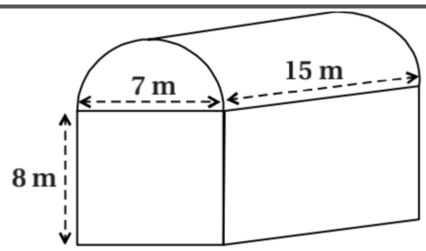
$= 35 + \dfrac{5}{33} \times 5$

$= 35 + 0.7575$

$= 35 + 0.76 = 35.76$

Therefore, the median age of policy holders is 35.76 year.

86. Shanta runs an industry in a shed which is in the shape of a cuboid surmounted by a half cylinder (see Fig.). If the base of the shed is of dimension 7 m × 15 m, and the height of the cuboidal portion is 8 m, find the volume of air that the shed can hold. Further, suppose the machinery in the shed occupies a total space of 300 m^3, and there are 20 workers, each of whom occupy about 0.08 m^3 space on an average. Then, how much air is in the shed ? (Take $\pi = \frac{22}{7}$)



➤ The volume of air inside the shed when there are no people or machinery is given by the volume of air inside the cuboid and inside the half cylinder, taken together.

Now the length of the cuboid

$l = 15$ m

Its breadth $b = 7$ m

and its height $h = 8$ m

Diameter of the half cylinder $d = 7$ m

$\therefore$ radius $r = \frac{7}{2}$ m,

Height of the half cylinder = 15 m

So, the required volume = volume of the cuboid + $\frac{1}{2}$ volume of the cylinder.

$= [l \times b \times h] + \left[\frac{1}{2}\pi r^2 h\right]$

$= [15 \times 7 \times 8] + \left[\frac{1}{2} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 15\right] m^3$

$= [840] + \left[\frac{11 \times 7 \times 15}{2 \times 2}\right] m^3$

$= [840] + \left[\frac{1155}{4}\right] m^3$

$= 840 + 288.75\ m^3$

$= 1128.75\ m^3$

The total space occupied by the machinery = $300\ m^3$

The total space occupied by the workers = $20 \times 0.08\ m^3 = 1.6\ m^3$

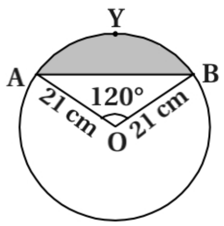
Therefore, the volume of the air, when there are machinery and workers.

$= [\text{Volume of the shed} - (300 + 1.6)]\ m^3$

$= [1128.75 - 301.6]\ m^3$

$= 827.15\ m^3$

87. Find the area of the segment AYB shown in figure, if radius of the circle is 21 cm and $\angle AOB = 120°$. (Use $\pi = \frac{22}{7}$)



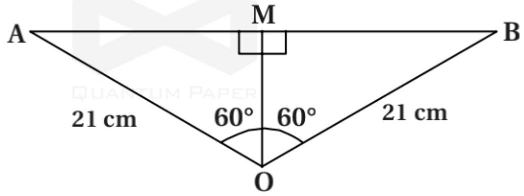
➤ **(i)** Area of minor sector OAYB

$$A = \frac{\theta}{360°} \times \pi r^2$$

$$= \frac{120°}{360} \times \frac{22}{7} \times 21 \times 21 \text{ cm}^2$$

$$= 462 \text{ cm}^2$$

➤ **(ii)** Area of Δ OAB,



Draw OM ⊥ AB

$\therefore$ ∠OMB = ∠OMA = 90°

OB = OA (hypotenuse) (Radius of one circle)

AM = MB (OA ⊥ AB) (By RHS congruence)

ΔAMO ≅ ΔBMO

M is midpoint of AB.

$$\angle AOM = \angle BOM = \frac{1}{2} \times \angle AOB$$

$$\angle AOM = \angle BOM = \frac{1}{2} \times 120 = 60$$

➤ **(iii)** Length of altitude OM

In ΔOMA,

$$\cos 60° = \frac{\text{Adjecent OM}}{\text{hypotenuse OA}}$$

$$\frac{1}{2} = \frac{OM}{21}$$

$$\therefore OM = \frac{21}{2} \text{ cm}$$

(iv) Length of chord AB

In ΔOMA,

$$\sin 60° = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\frac{\sqrt{3}}{2} = \frac{AM}{OA}$$

$$\frac{\sqrt{3}}{2} = \frac{AM}{21}$$

$$AM = \frac{\sqrt{3} \times 21}{2}$$

$$\therefore AB = 2AM$$

$$= 2 \times \frac{\sqrt{3} \times 21}{2}$$

$$\therefore AB = 21\sqrt{3} \text{ cm}$$

➤ **(v)** Area of triangle OAB

$$\text{Area of } \Delta OAB = \frac{1}{2} \times \text{Base} \times \text{Altitude}$$

$$= \frac{1}{2} \times AB \times OM$$

$$= \frac{1}{2} \times 21\sqrt{3} \times \frac{21}{2} \text{ cm}^2$$

$$= \frac{441}{4} \sqrt{3} \text{ cm}^2$$

➤ **(vi) Area of minor segment :**

Area of minor segment AYB = Area of the minor sector OAYB – Area of ΔOAB

$$= 462 - \frac{441}{4}\sqrt{3}\ cm^2$$

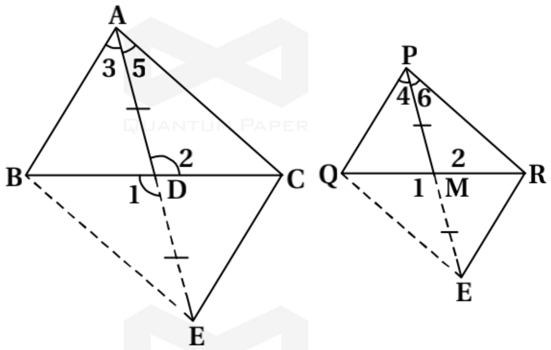
$$= 21 \times 22 - \frac{21 \times 21}{4}\sqrt{3}\ cm^2$$

$$= 21\left(22 - \frac{21}{4}\sqrt{3}\right)\ cm^2$$

$$= 21\left(\frac{88 - 21\sqrt{3}}{4}\right)\ cm^2$$

$$= \frac{21}{4}\left(88 - 21\sqrt{3}\right)\ cm^2$$

88. Sides AB and AC and median AD of a triangle ABC are respectively proportional to sides PQ and PR and median PM of another triangle PQR. Show that Δ ABC ~ Δ PQR.

➤ 

➤ **Solution :** We have two Δ ABC and Δ PQR such that AD and PM are medians corresponding to BC and QR respectively.

Produce AD up to point E such that AD = DE and similar produce PM up to N such that PM = MN.

Join EC and NR.

In Δ ADC and Δ EDB,

DC = DB. (D is midpoint by BC)

AD = DE (construction)

$\therefore \angle ADC = \angle BDE$ (verticle opposite angle)

$\therefore \Delta ADC \cong \Delta EDB$ (By S.A.S. similarity)

$\therefore$ AC $\cong$ EB... (i) (by C.P.C.T.)

Similarly we can prove Δ PMR $\cong$ Δ NMQ

$\therefore$ PR = NQ ....(ii) (CPCT)

➤ Now, $\frac{AB}{PQ} = \frac{AC}{PR} = \frac{AD}{PM}$

$\therefore \frac{AB}{PQ} = \frac{EB}{NQ} = \frac{AD}{PM}$ (From (i) and (ii))

$\therefore \frac{AB}{PQ} = \frac{EB}{NQ} = \frac{AD}{PM} = \frac{2AD}{2PM} = \frac{AE}{PN}$

$\therefore \Delta ABE \sim \Delta PQN$ (SSS similarity)

$\therefore \angle ABE = \angle PQN$ (C. P. C. Y)

$\therefore\ \angle 3 = \angle 4$ ....(iii)

Similarly we can prove $\angle 5 = \angle 6$ ....(iv)

➤ From (iii) and (iv)

$\angle 3 + \angle 5 = \angle 4 + \angle 6$

$\therefore\ \angle A = \angle P$

➤ In Δ ABC and Δ PQR,

$\frac{AB}{PQ} = \frac{AC}{PR}$ and $\angle A = \angle P$

$\therefore$ SAS similarity $\Delta ABC \sim \Delta PQR$.