AAYTAN ACADEMY
STD-10(Basic)
Answer Key of Most IMP Questions

A **Write the answer of the following questions.** [Each carries 1 Mark] **[21]**

1. Two players, Sangeeta and Reshma, play a tennis match. It is known that the probability of Sangeeta winning the match is 0.62. What is the probability of Reshma winning the match ?

➤ Let S and R denote the events that Sangeeta wins the match, respectively.

Given that the probability of Sangeeta's winning = P(S) = 0.62

∴ The probability of Reshma's

winning = P(R)

= 1 – P(S)

= 1 – 0.62

= 0.38

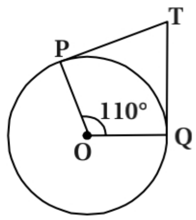
2. In Figure, if TP and TQ are the two tangents to a circle with centre O so that ∠POQ = 110°, then ∠PTQ is equal to

(A) 60°

(B) 70°

(C) 80°

(D) 90°



Ans. (B) 70°

➤ TP and TQ are tangents to the circle with centre O.

∠POQ = 110°

➠ OP ⊥ PT, OQ ⊥ TQ

∴ ∠P = 90°, ∠Q = 90°

➠ The sum of all angles of a quadrilateral is 360°.

∠P + ∠Q + ∠T + ∠O = 360°

∴ 90° + 90° + ∠T + 110° = 360°

∴ ∠T = 360° – 290° = 70°

∴ ∠PTQ = 70°

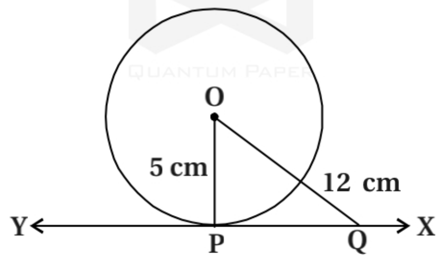
➤ Alternate (B) is true.

3. A tangent PQ at a point P of a circle of radius 5 cm meets a line through the centre O at a point Q so that OQ = 12 cm. Length PQ is :

(A) 12 cm (B) 13 cm (C) 8.5 cm (D) $\sqrt{119}$ cm

Ans. (D) $\sqrt{119}$ cm

➤



Here, OP ⊥ YX : ∠OPQ = 90°, OQ is hypotenuse

$OQ^2 = OP^2 + PQ^2$

∴ $(12)^2 = (5)^2 + PQ^2$

∴ $144 = 25 + PQ^2$

∴ $144 - 25 = PQ^2$

∴ $119 = PQ^2$

∴ $PQ = \sqrt{119}$

➤ Alternate (D) $\sqrt{119}$ cm is true.

4. $9 \sec^2 A - 9 \tan^2 A$ = ...

(A) 1 (B) 9 (C) 8 (D) 0

Ans. (B) 9

➤ $9\sec^2 A - 9 \tan^2 A$

$= 9(\sec^2 A - \tan^2 A)$

$= 9(1)$ $(\because \sec^2 A - \tan^2 A = 1)$

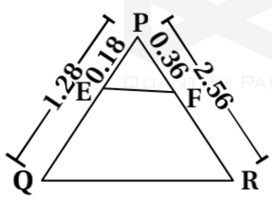
$= 9$

Thus, Alternate (B) will come.

5. E and F are points on the sides PQ and PR respectively of a Δ PQR. For each of the following cases, state whether EF || QR : PQ = 1.28 cm, PR = 2.56 cm, PE = 0.18 cm and PF = 0.36 cm

➤ We have PE = 0.18 cm, PQ = 1.28 cm, PF = 0.36 cm, PR = 2.56 cm.

➤



➤ $\frac{PE}{PQ} = \frac{0.18}{1.28} = \frac{18}{128} = \frac{9}{64}$

➤ $\frac{PF}{PR} = \frac{0.36}{2.56} = \frac{36}{256} = \frac{9}{64}$

∴ $\frac{PE}{PQ} = \frac{PF}{PR}$

∴ EF || QR

∴ If a line divided to sides of triangle in same ratio the line is parallel to third side.

6. Check whether the following are quadratic equation : $(x - 2)^2 + 1 = 2x - 3$

➤ Therefore, $(x - 2)^2 + 1 = 2x - 3$ can be rewritten as

∴ $x^2 - 4x + 4 + 1 = 2x - 3$

i.e., $x^2 - 6x + 8 = 0$

It is of the form $ax^2 + bx + c = 0$

Therefore, the given equation is a quadratic equation.

7. Find the nature of the roots of the following quadratic equations. If the real roots exist, find them : $2x^2 - 3x + 5 = 0$

➤ Comparing the equation $2x^2 - 3x + 5 = 0$

with $ax^2 + bx + c = 0$, we have

$a = 2,\ b = -3,\ c = 5$

➤ Discriminant D $= b^2 - 4ac$

$= (-3)^2 - 4(2)(5)$

$= 9 - 40$

$\therefore$ D $= -31 < 0$

So, the equation has no real roots. i.e. the real roots do not exist.

8. If discriminant of $2x^2 + 5x - k = 0$ is 81 then k = ...

(A) 5 (B) 7 (C) –7 (D) –5

➠ (B) 7

9. Mode – Median = ... × (Median – Mean) (2, 3, 4)

➠ 2

10. If $13x + 19y = 90$ and $19x + 13y = 70$ then $x + y$ = ...

(A) 4 (B) 5 (C) 6 (D) 7

➠ (B) 5

11. If $\cos A = \frac{4}{5}$, $\tan A$ = ... $\left(\frac{3}{5}, \frac{3}{4}, \frac{4}{3}\right)$

➠ $\frac{3}{4}$

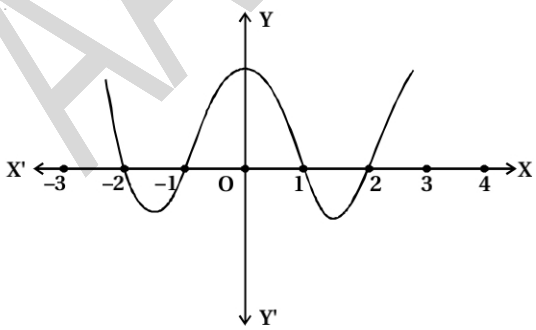
12. The number of zeroes for the given polynomial between 0 and 4 is ....

(A) 3

(B) 6

(C) 2

(D) 4



➠ (C) 2

13. True or False : Mona has probability $\frac{1}{80}$ of getting 80 marks out of 80 in Maths question paper.

➠ False

14. Given that HCF (306, 657) = 9, find LCM (306, 657).

➠ Required L.C.M. = 22,338

15. True or False : If $b^2 - 4ac > 0$, than quadratic equation has no real solution.

➠ False

16. Which one is true for the linear pair of equations graph when we have two intersecting lines and unique solution ?

(A) $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ (B) $\frac{a_1}{a_2} = \frac{b_1}{b_2}$ (C) $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ (D) $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

➠ (A) $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

17. If $a = 5$, $d = 8$ then S_{20} = ...

(A) 2160 (B) 2060 (C) 810 (D) 1620

➠ (D) 1620

18. In ΔABC and ΔMNO, ∠M = ∠A, ∠O = ∠B then ΔMNO ~ .... is similarity.

(A) ΔABC (B) ΔACB (C) ΔCBA (D) ΔBCA

➠ (B) ΔACB

19. Distance between the points (0, 5) and (–5, 0) is .... ($5\sqrt{5}$, 5, $5\sqrt{2}$)

➠ $5\sqrt{2}$

20. Number of tangents that can be drawn from point on a circle. (one, two, three)

➠ 1

21. Total surface area of a ₹ 10 coin is .... ($3\pi r(h + r)$, $\pi r(h + r)$, $2\pi r(h + r)$)

➠ $2\pi r(h + r)$ 12) 53.5

B Write the answer of the following questions. [Each carries 2 Marks] [72]

22. A lot consists of 144 ball pens of which 20 are defective and the others are good. Nuri will buy a pen if it is good, but will not buy if it is defective. The shopkeeper draws one pen at random and gives it to her. What is the probability that (i) She will buy it ? (ii) She will not buy it ?

➤ A lot consists of 144 balls pens.

∴ The number of all possible outcomes is 144

➤ There are 20 defective pens.

∴ There are 144 – 20 = 124 good pens.

Event A : Nuri will buy a pen.

Nuri will buy a pen if it is good and there are 124 good pens.

$$\therefore \ P(A) = \frac{\text{The number of favourable outcomes to event A}}{\text{The Total number of outcomes}}$$

$$= \frac{124}{144} = \frac{31}{36}$$

➤ **Event B : Nuri will not buy a pen.**

Nuri will not buy a pen if a pen is defective and there are 20 defective pens.

$$\therefore \ P(B) = \frac{\text{The number of favourable outcomes to event B}}{\text{The number of all possible outcomes}}$$

$$= \frac{20}{144}$$

$$\therefore \ P(B) = \frac{5}{36}$$

23. Find the median of the following data which gives the marks, out of 50, obtained by 100 students in a test (Ungrouped data)

Marks obtained	20	29	28	33	42	38	43	25
Number of students	6	28	24	15	2	4	1	20

➤ Table

Marks obtained	Number of students (frequency) (f_i)	Cumulative frequency (cf)
20	6	6
25	20	6 + 20 = 26
28	24	26 + 24 = 50
29	28	50 + 28 = 78
33	15	78 + 15 = 93
38	4	94 + 4 = 97
42	2	97 + 2 = 99
43	1	99 + 1 = 100
Total	100	

➤ Here n = 100 which is even.

$$\text{Median} = \frac{\left(\frac{n}{2}\right)^{\text{th}} + \left(\frac{n}{2}+1\right)^{\text{th}} \text{observation}}{2}$$

$$\text{Median} = \frac{\left(\frac{100}{2}\right)^{\text{th}} + \left(\frac{100}{2}+1\right)^{\text{th}} \text{observation}}{2}$$

$$= \frac{50^{\text{th}} \text{ observation} + 51^{\text{th}} \text{ observation}}{2}$$

50^{th} observation from the cumulative frequency is 28. 51^{th} observation from the cumulative frequency is 29.

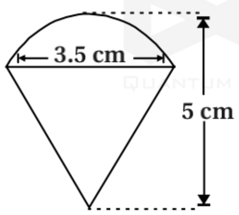
$\therefore$ Median = $\frac{28 + 29}{2} = \frac{57}{2}$ = 28.5.

Hence, the median is 28.5

The median mark 28.5 conveys the information that about 50% students obtained marks less than 28.5 and another 50% students obtained mark more than 28.5.

24. Rasheed got a playing top (*lattu*) as his birthday present, which surprisingly had no colour on it. He wanted to colour it with his crayons. The top is shaped like a cone surmounted by a hemisphere (see Fig). The entire top is 5 cm in height and the diameter of the top is 3.5 cm. Find the area he has to colour. (Take $\pi = \frac{22}{7}$)

➤ Diameter d = 3.5



$\therefore$ Radius $r = \frac{3.5}{2}$

$= \frac{35}{20}$ cm

(i) The curved surface area of hemisphere

The curved surface area of hemisphere

$= \frac{1}{2}\ (4\pi r^2)$

$= 2\pi r^2$

$= \left(2 \times \frac{22}{7} \times \frac{3.5}{2} \times \frac{3.5}{2}\right)\ \text{cm}^2$

$= \left(2 \times \frac{22}{7} \times \frac{35}{20} \times \frac{35}{20}\right)\ \text{cm}^2$

$= \frac{77}{4}\ \text{cm}^2$

$= 19.25\ \text{cm}^2$

(ii) The curved surface area of the cone

The height of the cone = The height of the top h – height (radius of the hemisphere)

$= \left(5 - \frac{3.5}{2}\right)\ \text{cm}$

$= (5 - 1.75)\ \text{cm}$

$= 3.25\ \text{cm}$

The slant height of the cone.

$l = \sqrt{r^2 + h^2}$

$= \sqrt{\left(\frac{3.5}{2}\right)^2 + (3.25)^2}\ \text{cm}$

$= \sqrt{\frac{12.25}{4} + 10.5625}\ \text{cm}$

$= \sqrt{\frac{12.25 + 10.5625 \times 4}{4}}\ \text{cm}$

$= \sqrt{\frac{12.25 + 42.25}{4}}\ \text{cm}$

$= \sqrt{\frac{54.50}{4}}\ \text{cm}$

$= \sqrt{\frac{(7.3824)^2}{(2)^2}}\ \text{cm}$

$= \frac{7.3824}{2}\ \text{cm}$

$= 3.6912\ \text{cm}$

$= 3.7\ \text{cm}$ (Approx)

CSA of cone $= \pi r l$

$= \left(\frac{22}{7} \times \frac{3.5}{2} \times 3.7\right)\ \text{cm}^2$

$= \left(\frac{22}{7} \times \frac{35}{20} \times \frac{37}{10}\right)\ \text{cm}^2$

$= \left(\frac{11 \times 37}{20} = \frac{407}{20}\right)\ \text{cm}^2$

$= 20.35\ \text{cm}^2$

(iii) The surface area of the top

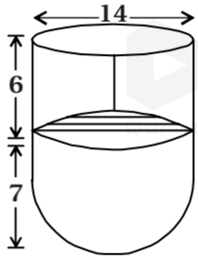
= The curved surface area of hemisphere + The curved surface area of cone

= $(19.25 + 20.35)\ cm^2$

= $39.6\ cm^2$ (Approx)

The total surface area of the top is not the sum of the total surface areas of the cone and hemisphere.

25. A vessel is in the form of a hollow hemisphere mounted by a hollow cylinder. The diameter of the hemisphere is 14 cm and the total height of the vessel is 13 cm. Find the inner surface area of the vessel. (take $\pi = \frac{22}{7}$)

➤ 

For hollow cylinder :

Diameter $d = 14$ cm $\Rightarrow$ radius $r = 7$ cm

Height $h = (13 - 7) = 6$ cm

The curved surface area of cylinder $= 2\pi rh$

$= 2 \times \frac{22}{7} \times 7 \times 6\ cm^2$

$= 264\ cm^2$

For hemisphere :

Diameter $d = 14$ cm $\Rightarrow$ Radius $r = \frac{d}{2} = \frac{14}{2} = 7$ cm

The curved surface area of hemisphere $= 2\pi r^2$

$= 2 \times \frac{22}{7} \times 7 \times 7\ cm^2$

$= 308\ cm^2$

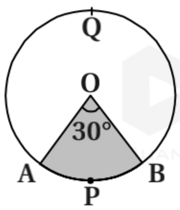
$\therefore$ The inner surface area of the vessel

= The curved surface area of cylinder + the curved surface area of hemisphere

= $(264 + 308)\ cm^2$

= $572\ cm^2$

26. Find the area of the sector of a circle with radius 4 cm and of angle 30°. Also, find the area of the corresponding major sector (Use $\pi = 3.14$).

➤ 

➤ Area of the sector (area of minor sector OAPB)

Area of sector $= \frac{\theta}{360} \times \pi r^2$

$= \frac{30}{360} \times 3.14 \times 4 \times 4$

$= \frac{12.56}{3}$ cm^2 = 4.19 cm^2 (approx)

Area of the corresponding major sector $= \pi r^2$ – Area of minor sector OAPB

$= (3.14 \times 16) - (4.19)$ cm^2

$= (50.24) - (4.19)$ cm^2

$= 50.24 - 4.19$ cm^2

$= 46.05$

$= 46.1$ cm^2 (approx)

27. A car has two wipers which do not overlap. Each wiper has a blade of length 25 cm sweeping through an angle of 115°. Find the total area cleaned at each sweep of the blades. use $\pi = \frac{22}{7}$

➤ We have radius $r = 25$ cm

Angle of sector $\theta = 115°$

Total area cleaned at each sweep of two blades

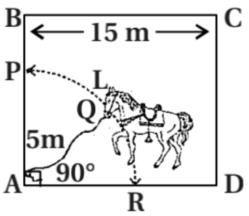
$= \left(\frac{\theta}{360} \times \pi r^2\right) \times 2$

$= \left(\frac{115}{360} \times \frac{22}{7} \times 25 \times 25\right) \times 2$ cm^2

$= \frac{23 \times 11 \times 25 \times 25}{18 \times 7}$ cm^2

$= \frac{158125}{126}$ cm^2

28. A horse is tied to a peg at one corner of a square shaped grass field of side 15 m by means of a 5 m long rope (see Figure). Find the increase in the grazing area if the rope were 10 m long instead of 5 m. (Use $\pi = 3.14$)



➤ If the length of the rope is increased to 10 m, the radius of new minor sector $r_1 = 10$ m and $\theta = 90°$.

$\therefore$ Area of new minor sector

$= \frac{\theta}{360} \times \pi r_1^2$

$= \frac{90}{360} \times 3.14 \times 10 \times 10$ m^2

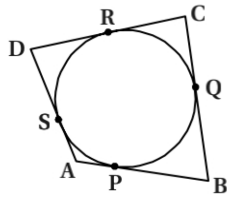
$= 78.5$ m^2

$\therefore$ Then, the increase in the grazing area

$= (78.5 - 19.625)$ m^2

$= 58.875\ m^2$

29. A quadrilateral ABCD isdrawn to circumscribe a circle (see Figure). Prove that AB + CD = AD + BC



➤ Let the sides AB, BC, CD and DA of a quadrilateral touch the circle at points P, Q, R and S respectively.

$\therefore$ AP = AS, DS = DR, CR = CQ, BQ = BP ....(i)

and A–P–B, B–Q–C, C–R–D and A–S–D ....(ii)

Now AB + CD = AP + PB + CR + RD

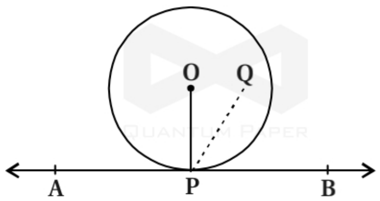
= AS + BQ + CQ + DS

= AS + DS + BQ + CQ

$\therefore$ AB + CD = AD + BC

30. Prove that the perpendicular at the point of contact to the tangent to a circle passes through the centre.

➤



AB is a tangent to a circle at a point P and O is the centre of the circle.

Let the perpendicular drawn from the point P to the tangent AB does not passes through the centre of the circle. Draw QP $\perp$ AB.

But the tangent at any point of the circle is perpendicular to the radius of the circle.

$\therefore$ OP $\perp$ AB $\therefore$ $\angle$OPB = 90° ....(i)

But QP $\perp$ AB (By construction)

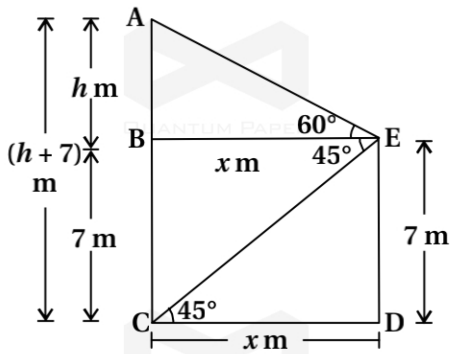
$\therefore$ $\angle$QPB = 90° ....(ii)

From results (i) and (ii), we have $\angle$OPB = $\angle$QPB

It is possible only when the point Q is coincide with the point O. Therefore, the perpendicular at the point of contact to the tangent to a circle passes through the centre.

31. From the top of a 7 m high building, the angle of elevation of the top of a cable tower is 60° and the angle of depression of its foot is 45°. Determine the height of the tower.

➤



➤ AC is a cable tower. A is its ED is a building ED = 7.

➤ BE = CD = x say

➤ In Δ ABC,

$\tan 60° = \frac{AB}{BE}$

$\therefore \sqrt{3} = \frac{h}{x}$

$\therefore h = \sqrt{3}\,x$

$\therefore h = \sqrt{3} \times 7$

$= 7\sqrt{3}$ m

In Δ CDE,

$\tan 45° = \frac{ED}{CD}$

$\therefore 1 = \frac{7}{x}$

$\therefore x = 7$ m

➤ A – B – C so, AB + BC = AC

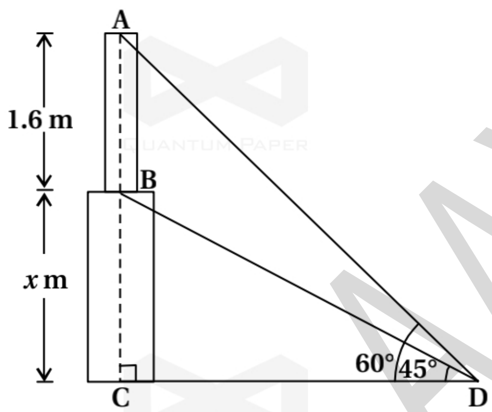
$h + 7 = AC$

$\therefore 7\sqrt{3} + 7 = AC$

$\therefore AC = 7\left(\sqrt{3} + 1\right)$

Therefore, the height of the cable tower is $7\left(\sqrt{3} + 1\right)$ m.

32. A statue, 1.6 m tall, stands on the top of a pedestal. From a point on the ground, the angle of elevation of the top of the statue is 60° and from the same point the angle of elevation of the top of the pedestal is 45°. Find the height of the pedestal.

➤ 

➤ Let us suppose that AB is a statue and BC is a pedestal. Given that AB = 1.6 m.

Suppose that the height of the pedestal be h m

$\therefore$ BC = h m

➤ D is a point on the ground.

From a point D, the angle of elevation of the top of the statue is 60° and from the same point the angle of elevation of the top of the pedestal is 45°.

$\therefore \angle ADC = 60°$ and $\angle BDC = 45°$

In right angle Δ BCD, $\tan 45° = \frac{BC}{CD}$

$\therefore 1 = \frac{BC}{CD}$

$\therefore$ BC = CD = say x m

In right angle Δ ACD, $\tan 60° = \frac{AC}{CD}$

$\therefore\ \sqrt{3} = \dfrac{AB + BC}{x}$

$\therefore\ \sqrt{3}\,x = 1.6 + x$

$\therefore\ \sqrt{3}\,x - x = 1.6$

$\therefore\ x\left[\sqrt{3} - 1\right] = 1.6$

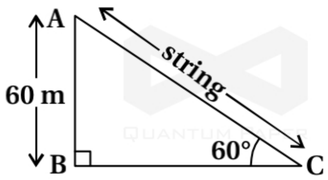
$= \dfrac{1.6}{\sqrt{3} - 1} \times \dfrac{\sqrt{3} + 1}{\sqrt{3} + 1}$

$= \dfrac{1.6\left(\sqrt{3} + 1\right)}{3 - 1}$

$= 0.8\ \left(\sqrt{3} + 1\right)$

Therefore, the height of the pedestal is $0.8\ \left(\sqrt{3} + 1\right)$ m.

33. A kite is flying at a height of 60 m above the ground. The string attached to the kite is temporarily tied to a point on the ground. The inclination of the string with the ground is 60°. Find the length of the string, assuming that there is no slack in the string.

➤ 

➤ Let A is the position of the kite AC is string

The kite is flying at a height above the ground.

AB = 60 m

➤ In right angle Δ ABC, ∠ACB

$\therefore\ \sin 60° = \dfrac{AB}{AC}$

$\therefore\ \dfrac{\sqrt{3}}{2} = \dfrac{60}{AC}$

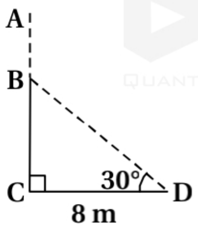
$\therefore\ AC = \dfrac{2 \times 60}{\sqrt{3}} = \dfrac{2 \times 3 \times 20}{\sqrt{3}}$

$\therefore\ \quad = 40\sqrt{3}$

Therefore, the length of the string is $40\sqrt{3}$ m.

34. A tree breaks due to storm and the broken part bends so that the top of the tree touches the ground making an angle 30° with it. The distance between the foot of the tree to the point where the top touches the ground is 8 m. Find the height of the tree.

➤ Let us suppose that AC denotes a tree which breaks at B due to storm. Its top A touches the ground at D.



$\therefore$ AB = BD and CD = 8m

BD makes an angle 30° with the ground.

$\therefore \angle BDC = 30°$

In Δ BCD, $\tan 30° = \frac{BC}{CD}$

$\therefore \frac{1}{\sqrt{3}} = \frac{BC}{8}$

$\therefore BC = \frac{8}{\sqrt{3}}$ m

➤ In Δ BCD $\cos 30° = \frac{CD}{BD}$

$\therefore \frac{\sqrt{3}}{2} = \frac{8}{BD}$

$\therefore BD = \frac{16}{\sqrt{3}}$

➤ $BD = AB = \frac{16}{\sqrt{3}}$

➤ The height of the tree,

AC = AB + BC

$= \frac{16}{\sqrt{3}} + \frac{8}{\sqrt{3}}$

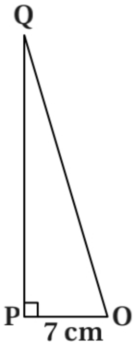
$= \frac{24}{\sqrt{3}}$

$= \frac{3 \times 8}{\sqrt{3}}$

$= 8\sqrt{3}$ m

Therefore, the height of the tree is $8\sqrt{3}$ m.

35. In Δ OPQ, right-angled at P, OP = 7 cm and OQ – PQ = 1 cm (see Fig.). Determine the values of sin Q and cos Q.



➤ In Δ OPQ $OQ^2 = OP^2 + PQ^2$

$\therefore (1 + PQ)^2 = OP^2 + PQ^2$

$\therefore 1 + PQ^2 + 2PQ = OP^2 + PQ^2$

$\therefore 1 + 2PQ = 7^2$

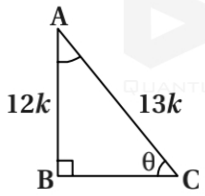
$\therefore$ PQ = 24 cm

OQ = 1 + PQ = 24 + 1 cm = 25 cm

So, $\sin Q = \frac{7}{25}$ and $\cos Q = \frac{24}{25}$

36. Given $\sec \theta = \frac{13}{12}$ calculate all other trigonometric ratios.

➤ Let Δ ABC is a right triangle in which $\angle B$ is right angle



Let $\angle A = \theta$

$$\sec\theta = \frac{13}{12} \quad \text{....(i)}$$

But $\sec\theta = \dfrac{AC}{AB}$ ....(ii)

From results (i) and (ii), we have

$$\frac{AC}{AB} = \frac{13}{12}$$

$$\therefore \frac{AC}{13} = \frac{AB}{12} = k$$

$\therefore$ AC = 13k, AB = 12k

➤ By Pythagoras theorem,

$$AC^2 = AB^2 + BC^2$$

$$\therefore BC^2 = AC^2 - AB^2$$

$$= (13k)^2 - (12k)^2$$

$$= 169k^2 - 144k^2$$

$$= 25k^2$$

$\therefore$ BC = 5k

$$\sin\theta = \frac{BC}{AC} = \frac{5k}{13k} = \frac{5}{13} \qquad \operatorname{cosec}\theta = \frac{AC}{BC} = \frac{13k}{5k} = \frac{13}{5}$$

$$\cos\theta = \frac{AB}{AC} = \frac{12k}{13k} = \frac{12}{13} \qquad \sec\theta = \frac{AC}{AB} = \frac{13k}{12k} = \frac{13}{12}$$

$$\tan\theta = \frac{BC}{AB} = \frac{5k}{12k} = \frac{5}{12} \qquad \cot\theta = \frac{AB}{BC} = \frac{12k}{5k} = \frac{12}{5}$$

37. Evaluate the following : $\dfrac{\cos 45^\circ}{\sec 30^\circ + \operatorname{cosec} 30^\circ}$

➤ We know that $\cos 45^\circ = \dfrac{1}{\sqrt{2}}$, $\sec 30^\circ = \dfrac{2}{\sqrt{3}}$ and $\operatorname{cosec} 30^\circ = 2$

➤ $\cos 45^\circ \div \sec 30^\circ + \operatorname{cosec} 30^\circ$

$$\therefore \frac{1}{\sqrt{2}} \div \frac{2}{\sqrt{3}} + \frac{2}{1}$$

$$\therefore \frac{1}{\sqrt{2}} \div \frac{2 + 2\sqrt{3}}{\sqrt{3}}$$

$$\therefore \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2\left(1 + \sqrt{3}\right)}$$

$$\therefore \frac{\sqrt{3}}{\sqrt{2}} \times \frac{1}{2\left(1 + \sqrt{3}\right)}$$

$$\therefore \frac{\sqrt{3}}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} \times \frac{1}{2\left(1+\sqrt{3}\right)} \times \frac{1-\sqrt{3}}{1-\sqrt{3}}$$

$$\therefore \frac{\sqrt{6}}{2} \times \frac{1-\sqrt{3}}{2(1-3)}$$

$$= \frac{\sqrt{6}}{2} \times \frac{1-\sqrt{3}}{2(-2)}$$

$$= \frac{\sqrt{6}}{4} \times \frac{1-\sqrt{3}}{-2}$$

$$= \frac{\sqrt{6}}{4} \times \frac{-(1-\sqrt{3})}{2}$$

$$= \frac{\sqrt{6}}{4} \times \frac{\sqrt{3}-1}{2}$$

$$= \frac{\sqrt{6}\left(\sqrt{3}-1\right)}{8}$$

$$= \frac{\sqrt{18}-\sqrt{6}}{8}$$

$$= \frac{\sqrt{9 \times 2}-\sqrt{6}}{8}$$

$$= \frac{3\sqrt{2}-\sqrt{6}}{8}$$

38. If the points A(6, 1), B(8, 2), C(9, 4) and D(p, 3) are the vertices of a parallelogram, taken in order, find the value of p.

➤ We know that diagonals of a parallelogram bisect each other.

➤ So, the coordinates of the midpoint of

AC = coordinates of the midpoint of BD

$$\therefore \left(\frac{6+9}{2}, \frac{1+4}{2}\right) = \left(\frac{8+p}{2}, \frac{2+3}{2}\right)$$

$$\therefore \left(\frac{15}{2}, \frac{5}{2}\right) = \left(\frac{8+p}{2}, \frac{5}{2}\right)$$

$$\therefore \frac{15}{2} = \frac{8+p}{2}$$

$\therefore 30 = 16 + 2p$

$\therefore 30 - 16 = 2p$

$\therefore 14 = 2p$

$$\therefore \frac{14}{2} = p$$

$\therefore p = 7$

39. In what ratio does the point (–4, 6) divide the line segment joining the points A(–6, 10) and B(3, –8) ?

➤ Let P(–4, 6) divides AB internally in the ratio $m_1 : m_2$.

A(–6, 10) = A(x_1 y_1)

B(3, –8) = B(x_2 y_2)

➤ $P(x, y) = \left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)$

$\therefore P(-4, 6) = \left(\frac{m_1 \times 3 + m_2(-6)}{m_1 + m_2}, \frac{m_1(-8) + m_2(10)}{m_1 + m_2}\right)$

$\therefore P(-4, 6) = \left(\frac{3m_1 - 6m_2}{m_1 + m_2}, \frac{-8m_1 + 10m_2}{m_1 + m_2}\right)$

$\therefore \frac{3m_1 - 6m_2}{m_1 + m_2} = -4\infty$

$\therefore 3m_1 - 6m_2 = -4\ (m_1 + m_2)$

$\therefore 3m_1 - 6m_2 + 4m_1 + 4m_2 = 0$

$\therefore 7m_1 - 2m_2 = 0$

$\therefore 7m_1 = 2m_2$

$\therefore \frac{m_1}{m_2} = \frac{2}{7}$

$\therefore m_1 : m_2 = 2 : 7$

$\frac{-8m_1 + 10m_2}{m_1 + m_2} = 6$

$\therefore -8m_1 + 10m_2 = 6\ (m_1 + m_2)$

$\therefore -8m_1 + 10m_2 = 6m_1 + 6m_2$

$\therefore -8m_1 + 10m_2 - 6m_1 - 6m_2 = 0$

$\therefore -14m_1 + 4m_2 = 0$

$\therefore -14m_1 = -4m_2$

$\therefore 14m_1 = 4m_2$

$\therefore 7m_1 = 2m_2$

$\therefore \frac{m_1}{m_2} = \frac{2}{7}$

$\therefore m_1 : m_2 = 2 : 7$

So the point (–4, 6) divides the line segment joining the points A(–6, 10) and B(3, –8) in the ratio 2 : 7.

Alternate method :

The ratio $m_1 : m_2$ can also be written as $\frac{m_1}{m_2} : 1$ or $k : 1$ $P(x, y) = \left(\frac{kx_2 + x_1}{k + 1}, \frac{ky_2 + y_1}{k + 1}\right)$

Let (–4, 6) divides AB in the ratio $k : 1$ internally.

A(–6, 10) and B(3, –8)

$\therefore (-4, 6) = \left(\frac{3k - 6}{k + 1}, \frac{-8k + 10}{k + 1}\right)$

$\therefore -4 = \frac{3k - 6}{k + 1}$

$\therefore -4(k + 1) = 3k - 6$

$\therefore -4k - 4 = 3k - 6$

$\therefore -4k - 4 - 3k + 6 = 0$

$\therefore -7k + 2 = 0$

$\therefore -7k = -2$

$\therefore 7k = 2$

$\therefore k = \frac{2}{7}$

$\therefore k : 1 = 2 : 7$

$6 = \frac{-8k + 10}{k + 1}$

$\therefore 6(k + 1) = -8k + 10$

$\therefore 6k + 6 = -8k + 10$

$\therefore 6k + 6 + 8k - 10 = 0$

$\therefore 14k - 4 = 0$

$\therefore 14k = 4$

$\therefore 7k = 2$

$\therefore k = \frac{2}{7}$

$\therefore k : 1 = 2 : 7$

So, the point (–4, 6) divides the segment joining A(–6, 10) and B(3, –8) in the ratio 2 : 7.

40. Show that the points (1, 7), (4, 2), (–1, –1) and (–4, 4) are the vertices of a square.

➤ Let A (1, 7), B (4, 2), C (–1, –1) and D (–4, 4) be the given points.

$AB = \sqrt{(1 - 4)^2 + (7 - 2)^2}$

$= \sqrt{(-3)^2 + (5)^2}$

$= \sqrt{9 + 25}$

$= \sqrt{34}$

$BC = \sqrt{(4 + 1)^2 + (2 + 1)^2}$

$= \sqrt{(5)^2 + (3)^2}$

$= \sqrt{25 + 9}$

$= \sqrt{34}$

$CD = \sqrt{(-1 + 4)^2 + (-1 - 4)^2}$

$= \sqrt{(3) + (-5)^2}$

$= \sqrt{9 + 25}$

$= \sqrt{34}$

$DA = \sqrt{(1 + 4) + (7 - 4)^2}$

$= \sqrt{(5)^2 + (3)^2}$

$= \sqrt{25 + 9}$

$= \sqrt{34}$

$AC = \sqrt{(1 + 1)^2 + (7)^2}$

$= \sqrt{(2)^2 + (8)^2}$

$= \sqrt{4 + 64}$

$= \sqrt{68}$

$BD = \sqrt{(4 + 4)^2 + (2 - 4)^2}$

$= \sqrt{(8)^2 + (-2)^2}$

$= \sqrt{64 + 4}$

$= \sqrt{68}$

Since AB = BC = CD = DA and AC = BD all the four sides of the quadrilateral ABCD are equal and its diagonals AC and BD are also equal.

Therefore ABCD is a square.

Alternative solution : We find the four sides and one diagonal AC as above. Here,

$AD^2 + DC^2 = 34 + 34 = 68 = AC^2$

Therefore by the converse of pythagoras theorem, $\angle D = 90°$. A quadrilateral with all four sides equal and one angle 90° is a square. So, ABCD is a square.

41. Find the values of y for which the distance between the points P(2, –3) and Q(10, y) is 10 units.

➤ P(2, –3), Q(10, y)

P(x_1 y_1) Q(x_2, y_2)

$$\therefore\ PQ = \sqrt{[(x_1) - (x_2)]^2 + [(y_1) - (y_2)]^2}$$

$$= \sqrt{[(2) - (10)]^2 + [(-3) - (y)]^2}$$

$$= \sqrt{(2 - 10)^2 + (-3 - y)^2}$$

$$= \sqrt{(-8)^2 + [-1\ (3 + y)]^2}$$

$$= \sqrt{64 + (3 + y)^2}$$

$$= \sqrt{64 + 9 + 6y + y^2}$$

$$= \sqrt{y^2 + 6y + 73}$$

Now, PQ = 10

$$\therefore\ \sqrt{y^2 + 6y + 73} = 10$$

$$\therefore\ \left(\sqrt{y^2 + 6y + 73}\right)^2 = (10)^2 \text{ (squaring both sides)}$$

$\therefore\ y^2 + 6y + 73 = 100$

$\therefore\ y^2 + 6y - 27 = 0$

$\therefore\ y^2 - 3y + 9y - 27 = 0$

$\therefore\ y(y - 3) + 9\ (y - 3) = 0$

$\therefore\ (y - 3)\ (y + 9) = 0$

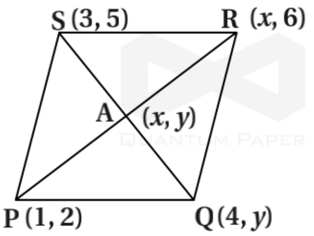
$\therefore\ y - 3 = 0$ or $y + 9 = 0$

$\therefore\ y = 3$or $y = -9$

Hence, the value of y is 3 and –9.

42. If (1, 2), (4, y), (x, 6) and (3, 5) are the vertices of a parallelogram taken in order, find x and y.

➤



PQRS is a parallelogram. Its diagonals PR and SQ bisect each other.

$\therefore$ The midpoint of PR = The midpoint of SQ.

$$\therefore\ \left(\frac{1 + x}{2}, \frac{2 + 6}{2}\right) = \left(\frac{3 + 4}{2}, \frac{5 + y}{2}\right)$$

$$\therefore\ \left(\frac{1 + x}{2}, 4\right) = \left(\frac{7}{2}, \frac{5 + y}{2}\right)$$

$\therefore \frac{1 + x}{2} = \frac{7}{2}$ and $4 = \frac{5 + y}{2}$

$\therefore 1 + x = 7$ $\therefore 8 = 5 + y$

$\therefore x = 6$ $\therefore y = 3$

Thus, $x = 6$ and $y = 3$

43. Find the sum of the first 22 terms of the AP : 8, 3, –2, . . .

➤ Here, $a = 8$, $d = 3 - 8 = -5$, $n = 22$.

We know that,

➤ $S_n = \frac{n}{2} [2a + n - 1)d]$

➤ Therefore, $= \frac{22}{2} [2 \times 8 + (22 - 1)(-5)]$

$= 11 [16 + (21)(-5)]$

$= 11 [16 - 105]$

$= 11 (-89]$

$= -979$

So, the sum of the first 22 terms of the AP is – 979.

44. Which term of the AP : 3, 8, 13, 18, ..., is 78 ?

➤ Here First term $a = 3$

Common difference $d = 8 - 3 = 5$

Let the n^{th} term of an A. P. be 78.

➤ $a_n = 78$

$\therefore a + (n - 1)d = 78$

$\therefore 3 + (n - 1)(5) = 78$

$\therefore 3 + 5n - 5 = 78$

$\therefore 5n = 78 + 2$

$\therefore 5n = 80$

$\therefore n = 16$

Hence, the 16^{th} term of the given AP is 78.

45. In an AP : Given $a_{12} = 37$, $d = 3$ find a and S_{12}.

➤ Here, $a_{12} = 37 = l$, $n = 12$, $d = 3$, $a = ?$

Now, $a_n = a + (n - 1)d$

$\therefore a_{12} = a + (12 - 1)\ 3$

$\therefore 37 = a + 33$

$\therefore a = 4$

➤ $S_n = \frac{n}{2}(a + l)$

$S_{12} = \frac{12}{2}(4 + 37)$

$\therefore\ S_{12} = 6\ (41) = 246$

Thus, $a = 4$ and $S_{12} = 246$

46. Find two numbers whose sum is 27 and product is 182.

➤ Let one number be x and product of two number is 182.

So, the other number is $\frac{182}{x}$

➤ Also the sum of two numbers is 27

$\therefore\ x + \frac{182}{x} = 27$

$\therefore\ x^2 + 182 = 27x$

$\therefore\ x^2 - 27x + 182 = 0$

$\therefore\ x^2 - (14 + 13)x + 182 = 0$

$\therefore\ x^2 - 14x - 13x + 182 = 0$

$\therefore\ x(x - 14) - 13(x - 14) = 0$

$\therefore\ (x - 14)\ (x - 13) = 0$

$\therefore\ x - 14 = 0$ or $x - 13 = 0$

$\therefore\ x = 14$ or $x = 13$

Thus, If one number is 14 the other number is $\frac{182}{14} = 13$.

Or If one number is 13 then the other number is $\frac{182}{13} = 14$.

Verification :

The sum of 14 and 13 is 27

and their product is $13 \times 14 = 182$.

47. Find the root of the following quadratic equation by factorisation : $100x^2 - 20x + 1 = 0$

➤ $100x^2 - 20x + 1 = 0$

$\therefore\ (10x)^2 - 20x + (1)^2 = 0$

$\therefore\ (10x)^2 - 2(10x)(1) + (1)^2 = 0$

$\therefore\ (10x - 1)^2 = 0$

$\therefore\ (10x - 1)\ (10x - 1) = 0$

$\therefore\ 10x - 1 = 0$ or $10x - 1 = 0$

$\therefore\ 10x = 1$ or $10x = 1$

$\therefore\ x = \frac{1}{10}$ or $x = \frac{1}{10}$

Hence, $\frac{1}{10}$ and $\frac{1}{10}$ are two equal roots of the given equation $100x^2 - 20x + 1 = 0$.

48. Solve the following pair of equations by substitution method :

$7x - 15y = 2$ ....(i)

$x + 2y = 3$ ....(ii)

➤ **Step 1 :** Pick either of the equations given and write one variable in terms of the other.

➤ Let consider equation (i) and write in terms of x or y.

OR

Consider equation (ii) and write in terms of x or y Let us consider the equation (ii) $x + 2y = 3$

Write in terms of x

$x + 2y = 3$ (ii)

write in terms of $x = -2y + 3$ ....(iii)

➤ **Step 2 :** Substitute the value of x in equation (i) we get $7x - 15y = 2$

$\therefore\ 7(-2y + 3) - 15y = 2$

$\therefore\ -14y + 21 - 15y = 2$

$\therefore\ -29y = 2 - 21$

$\therefore\ -29y = -19$

$\therefore\ y = \frac{19}{29}$

➤ **Step 3 :** Substitute the value of y in equation (iii) we get $x = 3 - 2y = 3 - 2\left(\frac{19}{29}\right) = \frac{49}{29}$

Substituting $x = \frac{49}{29}$ and $y = \frac{19}{29}$, you can verify that both the equations (i) and (ii) are satisfied.

Note : We can use any one of the equations and write one variable in terms of other. It depend upon the equation that which equation is selected ? Which variable is substituted ?

49. On comparing the ratios $\frac{a_1}{a_2}$, $\frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$, find out whether the following pair of linear equation are constitstent, or inconsistent.

$\frac{3}{2}x + \frac{5}{3}y = 7;\ \ 9x - 10y = 14$

➤ comparing $\frac{3}{2}x + \frac{5}{3}y - 7 = 0$ with

$a_1x + b_1y + c_1 = 0$, we have

$a_1 = \frac{3}{2}$, $b_1 = \frac{5}{3}$ and $c_1 = -7$

➤ comparing $9x - 10y - 14$ with

$a_2x + b_2y + c_2 = 0$, we have

$a_2 = 9,\ b_2 = -10,\ c_2 = -14$

$\frac{a_1}{a_2} = \frac{\frac{3}{2}}{9} = \frac{3}{2} \times \frac{1}{9} = \frac{1}{6}\quad \frac{b_1}{b_2} = \frac{\frac{5}{3}}{-10} = \frac{-5}{3} \times \frac{1}{10} = -\frac{1}{6}$

$\frac{c_1}{c_2} = \frac{-7}{-14} = \frac{1}{2}$

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

$\therefore$ The line representing the pairs of linear equations intersect at a point. So they are consistent. There is unique solution.

50. Solve $2x + 3y = 11$ and $2x - 4y = -24$ and hence find the value of 'm' for which $y = mx + 3$.

➤ $2x + 3y = 11$ ....(i)

$2x - 4y = -24$ ....(ii)

$\therefore\ 2x + 3y = 11$

$\therefore\ 2x = 11 - 3y$

$\therefore\ x = \dfrac{11 - 3y}{2}$ ....(iii)

➤ $x = \dfrac{11 - 3y}{2}$ patting this value of x in equation (ii)

$2x - 4y = -24$

$\therefore\ 2\left[\dfrac{11 - 3y}{2}\right] - 4y = -24$

$\therefore\ 11 - 3y - 4y = -24$

$\therefore\ 11 - 7y = -24$

$\therefore\ -7y = -35$

$\therefore\ y = 5$

➤ Putting $y = 5$ in equation (iii)

$x = \dfrac{11 - 3y}{2}$

$= \dfrac{11 - 3(5)}{2}$

$= \dfrac{-4}{2}$

$\therefore\ x = -2$

Thus the solution of the given equations is $x = -2$ and $y = 5$.

➤ $y = mx + 3$

$\therefore\ 5 = m \times (-2) + 3$

$\therefore\ 5 = -2m + 3$

$\therefore\ 5 - 3 = -2m$

$\therefore\ 2 = -2m$

$\therefore\ -2m = 2$

$\therefore\ m = -1$

51. Solve the following pair of linear equations by the elimination method and the substitution method : $3x - 5y - 4 = 0$ and $9x = 2y + 7$

➤ $3x - 5y - 4 = 0$ | $9x = 2y + 7$

$\therefore\ 3x - 5y = 4$ ....(i) | $\therefore\ 9x - 2y = 7$ ....(ii)

➤ $3x - 5y = 4$ ....(i)

$9x - 2y = 7$ ....(ii)

Multiply equation (i) by 3 and then subtract equation (ii) from it, we get

$9x - 15y = 12$

$9x - 2y = 7$

$$\begin{array}{ccc} - & + & - \\ \hline \end{array}$$

$-13y = +5$

$\therefore \; y = \frac{-5}{13} = \frac{-5}{13}$ ....(iii)

➤ Putting $y = \frac{-5}{13}$ in equation (i) we get

$3x - 5y = 4$

$\therefore \; 3x - 5\left(\frac{-5}{13}\right) = 4$

$\therefore \; 3x - 5\left(\frac{-5}{13}\right) = 4$

$\therefore \; 3x + \frac{25}{13} = 4$

$\therefore \; 3x = 4 - \frac{25}{13}$

$\therefore \; 3x = \frac{52 - 25}{13}$

$\therefore \; 3x = \frac{27}{13}$

$\therefore \; 39x = 27$

$\therefore \; x = \frac{9}{13}$

Thus, $x = \frac{9}{13}$ and $y = \frac{-5}{13}$

Method of substitution :

$3x - 5y - 4 = 0$ ....(i)

$9x = 2y + 7$

$\therefore \; 9x - 2y - 7 = 0$ ....(ii)

➤ $9x - 2y - 7 = 0$

$\therefore \; 9x = 2y + 7$

$\therefore \; x = \frac{2y+7}{9}$ ....(iii)

➤ Putting $x = \frac{2y+7}{9}$ in equation (i), we get

$3x - 5y - 4 = 0$

$\therefore \; 3\left(\frac{2y + 7}{9}\right) - 5y - 4 = 0$

$\therefore \; \frac{2y + 7}{3} - 5y - 4 = 0$

$\therefore \; \frac{2y + 7}{3} = \frac{5y + 4}{1}$

$\therefore\ 2y + 7 = 3\ (5y + 4)$

$\therefore\ 2y + 7 = 15y + 12$

$\therefore\ 2y - 15y = 12 - 7$

$\therefore\ -13y = 5$

$\therefore\ y = \frac{-5}{13}$

➤ Putting $y = \frac{-5}{13}$ in equation (iii), we get

$$x = \frac{2y + 7}{9}$$

$$= \frac{2\left(\frac{-5}{13}\right) + 7}{9}$$

$$= \frac{\frac{-10}{13} + 7}{9}$$

$$= \frac{-10 + 91}{13 \times 9}$$

$$= \frac{81}{117}$$

$$\therefore\ x = \frac{9}{13}$$

Thus the solution of the given equations is $x = \frac{9}{13}$ and $y = \frac{-5}{13}$.

52. Find a quadratic polynomial each with the given numbers as the sum and product of its zeroes respectively : $-\frac{1}{4}, \frac{1}{4}$

➤ Let α and β are the zeroes of the quadratic polynomial sum of the zeroes $= \alpha + \beta = -\frac{b}{a} = -\frac{1}{4}$

Product of the zeroes $\alpha\beta = \frac{c}{a} = \frac{1}{4}$

➤ Required quadratic polynomial

$$= x^2 - (\alpha + \beta)\ x + \alpha\beta$$

$$= x^2 - \left(-\frac{1}{4}\right) x + \frac{1}{4}$$

$$= x^2 + \frac{x}{4} + \frac{1}{4}$$

$$= \frac{4x^2 + x + 1}{4}$$

$$= \frac{1}{4}\ (4x^2 + x + 1)$$

Hence, the required quadratic polynomial is $\frac{1}{4}\ (4x^2 + x + 1)$

53. Find the zeroes of the following quadratic polynomial and verify the relationship between the zeroes and the coefficients : $3x^2 - x - 4$

➤ Let $p(x) = 3x^2 - x - 4$

$= 3x^2 - (4 - 3)\ x - 4$

$= 3x^2 - 4x + 3x - 4$

$= x(3x - 4) + 1(3x - 4)$

$= (3x - 4)\ (x + 1)$

➤ To find the zeroes of $p(x)$, we take $p(x) = 0$,

$p(x) = (3x - 4)\ (x + 1)$

$\therefore\ 0 = (3x - 4)\ (x + 1)$

$\therefore\ 3x - 4 = 0$ or $x + 1 = 0$

$\therefore\ x = \frac{4}{3}$ or $x = -1$

Hence, $\frac{4}{3}$ and -1 are the zeroes of given polynomial $3x^2 - x - 4$.

➤ $p(x) = 3x^2 - x - 4$ comparing with $ax^2 + bx + c$, we get $a = 3$, $b = -1$, $c = -4$

Sum of the zeroes $= \left(\frac{4}{3}\right) + (-1) = \frac{4 - 3}{3} = \frac{1}{3}$

$$= \frac{-(-1)}{3} = -\frac{b}{a} = \frac{-(\text{coefficient of } x)}{\text{coefficient of } x^2}$$

Product of the zeroes $= \left(\frac{4}{3}\right) \times (-1) = \frac{-4}{3} = \frac{c}{a}$

$$= \frac{\text{constant term}}{\text{coefficient of } x^2}$$

54. Show that $3\sqrt{2}$ is irrational.

➤ Let us assume, to the contrary, that $3\sqrt{2}$ is rational

That is, we can find coprime a and b $(b \neq 0)$ such that $3\sqrt{2} = \frac{a}{b}$.

Rearranging, we get $\sqrt{2} = \frac{a}{3b}$

Since 3, a and b are integers, $\frac{a}{3b}$ is rational, and so $\sqrt{2}$ is rational.

But this contradicts the fact that $\sqrt{2}$ is irrational. So, we conclude that $3\sqrt{2}$ is irrational.

55. Show that $5 - \sqrt{3}$ is irrational.

➤ Let us assume, to the contrary, that $5 - \sqrt{3}$ is rational.

That is, we can find coprime a and b $(b \neq 0)$ such that $5 - \sqrt{3} = \frac{a}{b}$.

Therefore, $5 - \frac{a}{b} = \sqrt{3}$

Rearranging this equation, we get

$\sqrt{3} = 5 - \frac{a}{b} = \frac{5b - a}{b}$

Since a and b are integers, we get $5 - \frac{a}{b}$ is rational, and so $\sqrt{3}$ is rational.

But this contradicts the fact that $\sqrt{3}$ is irrational.

This contradiction has arisen because of our incorrect assumption that $5 - \sqrt{3}$ is rational.

So, we conclude that $5 - \sqrt{3}$ is irrational.

56. Prove that tangents drawn at the end points of a diameter are parallel to each other.

➠ Try Yourself

57. If $\tan\theta = \frac{4}{3}$ then $\frac{1 - \sin\theta}{1 + \sin\theta}$ = ... $\left(\frac{1}{9}, 9, \frac{3}{5}\right)$

➠ $\frac{1}{9}$

C **Write the answer of the following questions.** [Each carries 3 Marks] **[18]**

58. A carton consists of 100 shirts of which 88 are good, 8 have minor defects and 4 have major defects. Jimmy, a trader, will only accept the shirts which are good, but Sujatha, another trader, will only reject the shirts which have major defects. One shirt is drawn at random from the carton. What is the probability that

(i) it is acceptable to Jimmy ?

(ii) it is acceptable to Sujatha ?

➤ One shirt is drawn at random from the carton of 100 shirts. Therefore, there are 100 equally likely outcomes.

(i) The number of outcomes favourable (i.e., acceptable) to Jimmy = 88

Therefore, P (shirt is acceptable to Jimmy) $= \frac{88}{100} = 0.88$

(ii) The number of outcomes favourable to Sujatha = 88 + 8 = 96 (Why ?)

So, P (shirt is acceptable to Sujatha)

$= \frac{96}{100} = 0.96$

59. A student noted the number of cars passing through a spot on a road for 100 periods each of 3 minutes and summarised it in the table given below. Find the mode of the data :

Number of cars	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60	60 – 70	70 – 80
Frequency	7	14	13	12	20	11	15	8

➤ Here the maximum frequency is 20 in the interval 40 – 50.

∴ The modal class = 40-50

➤ The lower limit of the modal class $l = 40$

The frequency of the modal class $f_i = 20$

The frequency of the class preceding the modal class $f_0 = 12$

The frequency of the class succeeding the modal class $f_2 = 11$

The class size $h = 10$

$$\text{Mode} = l + \left[\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right] \times h$$

$$= 40 + \left[\frac{20 - 12}{2 \times 20 - 12 - 11}\right] \times 10$$

$$= 40 + \frac{80}{17}$$

$$= 40 + 4.7$$

$$= 44.7$$

= 44.7

Therefore, the mode of the given data is 44.7 cars.

60. The following table gives the distribution of the life time of 400 neon lamps :

Life time (in hours)	Number of lamps
1500 – 2000	14
2000 – 2500	56
2500 – 3000	60
3000 – 3500	86
3500 – 4000	74
4000 – 4500	62
4500 – 5000	48

Find the median life time of a lamp.

➤ We find cumulative frequency at first.

Life time (in hours)	**Number of lamps (f_i)**	**Cumulative frequency. (cf)**
1500 – 2000	14	14
2000 – 2500	56	70
2500 – 3000	60	130
3000 – 3500	86	216
3500 – 4000	74	290
4000 – 4500	62	352
4500 – 5000	48	400
Total	$n = 400$	

➤ Here, $n = 400 \therefore \frac{n}{2} = \frac{400}{2} = 200$

➤ The cumulative frequency just greater than 200 is 216 which lies in the interval 3000 – 3500

The median class = 3000 – 3500

➤ Lower limit of the median class $l = 3000$

Number of observations $n = 400$

The cumulative frequency of the class preceding the median class $cf = 130$

Frequency of the median class $f = 86$

Class size $h = 500$

$$\text{Median} = l + \frac{\frac{n}{2} - cf}{f} \times h$$

$$= 3000 + \left[\frac{200 - 130}{86}\right] \times 500$$

$$= 3000 + \frac{70}{86} \times 500$$

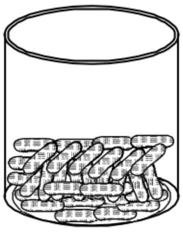
$$= 3000 + \frac{35000}{86}$$

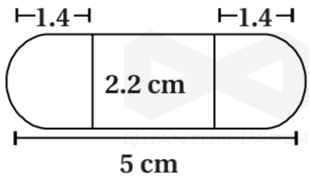
$$= 3000 + 406.98$$

= 3406.98

Therefore median life time of the lamp is 3406.98 hours.

61. A gulab jamun, contains sugar syrup up to about 30% of its volume. Find approximately how much syrup would be found in 45 gulab jamuns, each shaped like a cylinder with two hemispherical ends with length 5 cm and diameter 2.8 cm (see Figure). (take $\pi = \frac{22}{7}$)



➤ 

The radius of the cylindrical part of gulab jamun = Radius of hemisphere at both the ends.

Diameter = 2.8 cm

$\therefore$ radius = $\frac{2.8}{2}$ = 1.4 cm

The height of the cylindrical part of gulab jamun

H = Total height – 2 × radius of hemisphere

= 5 – 2 × 1.4

= 5 – 2.8 = 2.2 cm

The volume of a gulab jamun = the volume of cylinder + 2 × the volume of hemisphere

$$= \pi r^2 H + 2 \times \frac{2}{3}\pi r^3$$

$$= \pi r^2 \left(H + \frac{4}{3} r\right)$$

$$= \frac{22}{7} \times 1.4 \times 1.4 \left[2.2 + \frac{4}{3} \times 1.4\right] \text{ cm}^3$$

$$= 6.16\ [4.07]\ \text{cm}^3$$

$$= 25.0712\ \text{cm}^3$$

Hence, the volume of a gulab jamun = 25.0712

$\therefore$ the volume of 45 gulab jamun = 45 × 25.0712

= 1128.204 cm^3

A gulab jamun, contains sugar syrup up to about 30 % of its volume.

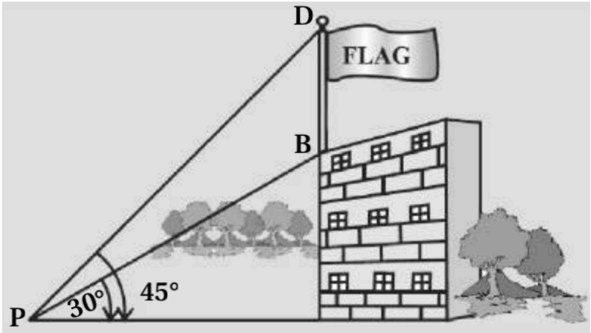
$\therefore$ The volume of sugar syrup contains in 45 gulab jamuns = 30% of the volume of 45 gulab jamun.

$$= \frac{30}{100} \times 1128.204$$

$$= 338.4612\ \text{cm}^3$$

$$= 338\ \text{cm}^3\ \text{(Approximate)}$$

62. From a point P on the ground the angle of elevation of the top of a 10 m tall building is 30°. A flag is hoisted at the top of the building and the angle of elevation of the top of the flagstaff from P is 45°. Find the length of the flagstaff and the distance of the building from the point P. (You may take $\sqrt{3}$ = 1.732)



➤ AB denotes the height of the building, BD the flagstaff and P the given point.

Δ PAB and Δ PAD are right angle triangles.

We are required to find DB and AP.

We know that AB = 10 m

In Δ PAB, $\tan 30° = \frac{AB}{AP}$

$\therefore \frac{1}{\sqrt{3}} = \frac{10}{AP}$

$\therefore AP = 10 \times \sqrt{3}$

$= 10 \times 1.732$

$= 17.32$ m

i.e. the distance of the building from P is 17.32 m

Let us suppose DB = x m

Then AD = $(10 + x)$m

In Δ PAD, $\tan 45° = \frac{AD}{AP}$

$\therefore I = \frac{10 + x}{10\sqrt{3}}$

$\therefore 10 + x = 10\sqrt{3}$

$\therefore x = 10\left(\sqrt{3} - 1\right)$

$= 10\ (1.732 - 1)$

$= 7.32$ m

So, the length of the flagstaff is 7.32 m.

63. Prove that $\sqrt{3}$ is irrational.

➤ Let us assume, to the contrary, that $\sqrt{3}$ is rational.

That is, we can find integers a and b ($\neq$ 0) such that $\sqrt{3} = \frac{a}{b}$.

Suppose a and b have a common factor other than 1, then we can divide by the common factor, and assume that a and b are coprime.

So, $b\sqrt{3} = a$.

Squaring on both sides, and rearranging, we get $3b^2 = a^2$.

Therefore, a^2 is divisible by 3, and by Theorem 1.3, it follows that a is also divisible by 3.

So, we can write $a = 3c$ for some integer c.

Substituting for a, we get $3b^2 = 9c^2$, that is, $b^2 = 3c^2$.

This means that b^2 is divisible by 3, and so b is also divisible by 3 (using Theorem 1.3 with $p = 3$).

Therefore, a and b have at least 3 as a common factor.

But this contradicts the fact that a and b are coprime.

This contradiction has arisen because of our incorrect assumption that $\sqrt{3}$ is rational. So, we conclude that $\sqrt{3}$ is irrational.

D **[4]**

64. The median of the following data is 525. Find the values of x and y, if the total frequency is 100.

Class interval	Frequency (f_i)
0 – 100	2
100 – 200	5
200 – 300	x
300 – 400	12
400 – 500	17
500 – 600	20
600 – 700	y
700 – 800	9
800 – 900	7
900 – 1000	4

➤

Class interval	Frequency (f_i)	Cumulative Frequency (cf)
0 – 100	2	2
100 – 200	5	7
200 – 300	x	$7 + x$
300 – 400	12	$19 + x$
400 – 500	17	$36 + x$
500 – 600	20	$56 + x$
600 – 700	y	$56 + x + y$
700 – 800	9	$65 + x + y$
800 – 900	7	$72 + x + y$
900 – 1000	4	$76 + x + y$

(i) It is given that $n = 100$.

So, $76 + x + y = 100$

$\Rightarrow x + y = 24$

(ii) The median is 525 which lies in the class 500 – 600.

(iii) So $l = 500$, $f = 20$, $cf = 36 + x$, $h = 100$

$$\text{Median} = l + \left[\frac{\frac{n}{2} - cf}{f}\right] \times h$$

$$\therefore\ 525 = 500 + \left[\frac{50 - 36 - x}{20}\right] \times 100$$

$$\therefore\ 525 - 500 = \frac{14 - x}{20} \times 100$$

$$\therefore\ 25 = (14 - x)\ 5$$

$$\therefore\ (14 - x) = \frac{25}{5}$$

$$\therefore\ 14 - x = 5$$

$$\therefore\ x = 14 - 5 = 9$$

$$\therefore\ x = 9$$

➤ From (1) $x + y = 24$

$\therefore\ 9 + y = 24 \quad \therefore\ y = 15$