Truck tire-terrain interaction modelling and testing: literature survey

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Abstract: The interaction between tire and terrain is the primary factor affecting the efficiency of the ride. The terrain on which the vehicle operates can range between dry or wet road to soil or clayey depending on the vehicle application whether it is off-road or on-road. The terrain properties affect the vehicle ride significantly and thus it is highly important to investigate this aspect. This paper focuses on studying the tire-terrain interaction from several aspects. The truck tires used in this study are previously modelled and validated by previous research students. The terrains used are modelled in a virtual performance software Pam-Crash. The terrains include the hard surface (road); soils such as sand and clayey; snow; and water. The tire-terrain interaction is modelled in Pam-Crash using contact definition and several parameters are collected. The hydroplaning speed of the tire is studied at different conditions for the inflation pressure, vertical load and water depth. The rolling resistance over several terrains is computed and compared. The soil mixing/layering concept is presented and investigated. This work is a preliminary step for an expanded investigation that will be applied during this research.

Keywords: truck tire; tire-terrain interaction; smoothed-particle hydrodynamics; finite element analysis; soil modelling and calibration; hydroplaning; MATLAB/simulink.


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1 Tire modelling and validation

The need for tires started since the Palaeolithic era, where people used round logs to move heavy objects more easily (The Evolution of the Wheel, 2006). People placed logs and sled under a heavy object and dragged the sled over one log to next. Sumerian, the first urban civilisation in the historical region of southern Mesopotamia, is assumed to be the first who used of the solid wheel transportation (Anthony, 2010). Wooden wheels had been manufactured for over thousand years with different spokes, leather tops and iron strip-tops, in chronological order. In 1839, Goodyear (1853) discovered the vulcanisation process. The vulcanisation process is the process of heating raw rubber with sulphur to transform sticky natural rubber to a firm but pliable material. Later, in 1845, a Scottish engineer, Robert William Thomson, perceived an idea of air-inflated or pneumatic bicycle tires. Robert expected that the pneumatic tires could overcome the limitation of the solid rubber tires (The History of Pneumatic Devices; Pneumatic Devices – Pneumatic Tube, 2006). Following in 1889, a bicyclist brought a punctured bicycle tire to the Michelin brothers, André and Édouard Michelin, to fix (Harp, 2001). Michelin brothers manufactured a detachable pneumatic tire that could save time and effort. The brothers’ attempt was accepted and, within a few years, the Michelin firm achieved extraordinary growth by serving the early stage of the automotive industry.

As tires are the preliminary elements connecting the vehicle to the ground, it is significantly important to understand their dynamics. Thus, tires have a powerful impact on the vehicle’s performance. While designing a tire, several objectives should be taken into consideration. Tires should be able to provide directional and handling stability for the vehicle. In addition, the structure of tires should support cushion vehicle’s ride over unreliable surfaces.

Considerable types of tires exist depending on the application. The pneumatic tires are widely used for automobile and bicycle applications. A pneumatic tire is defined as
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an air inflated structure that can absorb shocks. Thus, tires isolate vibrations to attenuate vibration arising from rough terrains. Pneumatic tires are composed of distinct components such as the carcass, belt plies, tread, under-tread, side wall and beads. A cross section of a pneumatic tire with the parts labelled is shown in Figure 1. The most vital component in the tire is the carcass as it is influential on the performance characteristics of the tire.

Several attempts to model tires have been made using several virtual environments. Previous research students developed several truck tires using finite element analysis (FEA) technique in the virtual environment Pam-Crash. In 2006, Chae (2006) developed and validated a nonlinear finite element tire model. Later in 2009, Slade (2009) developed an off-road rigid ring model for truck tire using FEA. On the other side, few studies have employed mesh-less smoothed particle hydrodynamics (SPH) (Lescoe, 2010; Goodin and Priddy, 2016) and discrete element (DE) (Nakashima et al., 2010; Melanz et al., 2016) methods for modelling tyre-soil interactions. One year later, Lescoe (2010) developed a soil model in a tire-soil interaction using FEA and smoothed-particle hydrodynamics (SPH). The objective of the interaction analysis is to predict the contact force/moment properties. Similarly, Dhillon (2013) applied the SPH method to develop and calibrate soil models by conducting pressure-sinkage and shear-strength test simulations. The SPH material used by Dhillon is elastic-plastic-hydrodynamic material type in conjunction with a linear pressure volume equation of state.

Figure 1 Cross section of typical radial tire (see online version for colours)

1.1 Tire parameters definition

This section describes the conventional tire axis system, forces and moments definitions that are widely accepted. For this study, three forces and three moments are described and shown in Figure 2. The primary role of the tire is to support sprung mass, resist external
disturbances and transmit braking and driving torques from the vehicle to the road surface (Chae, 2006). Thus, the forces and moments along all three axes are continuously applied during vehicle operation. The forces applied to the tire are the longitudinal force \( F_x \), the lateral force \( F_y \) and the vertical force \( F_z \). Similarly, the moments applied are overturning moment \( M_x \), rolling resistance moment \( M_y \) and self-aligning moment \( M_z \).

In the case that the tire is not in contact with the ground, the interaction forces and moments are not applied.

**Figure 2** Wheel axis, forces and moments definitions

**1.1.1 Longitudinal force**

The longitudinal force is generated when the tire is rolling or sliding along the longitudinal direction on a surface. The longitudinal force can be categorised into rolling resistance force, longitudinal frictional force and longitudinal reaction force. The rolling resistance force develops during the free rolling of the tire along a straight direction. The rolling resistant force is applied to the tire at the contact area against the tire rolling direction. The carcass deflection due to hysteresis in the tire materials is the primary cause of the rolling resistance. The surface condition also has an impact on the rolling resistance. Each road surface condition has a significant rolling resistance coefficient. The rolling resistance force divided by vertical tire load represents the rolling resistance coefficient. Generally, for truck tires, the coefficient of rolling resistance varies between 0.006 and 0.01 on a concrete or asphalt road. This coefficient is lower than that for passenger car tires due to larger tire diameter and higher inflation pressure. A higher rolling resistance is usually observed on wet surfaces than on dry surfaces. The rolling resistance coefficient can be determined using the equations provided by Wong (2008) for both radial-ply and bias-ply tires as shown in equations below:

Radial-ply truck tire: \( f_r = 0.006 + 0.23 \times 10^{-6} \times V^2 \)
Bias-ply truck tire: \( f_r = 0.007 + 0.45 \times 10^{-6} \times V^2 \)

where \( f_r \) is the rolling resistance coefficient and \( V \) is the tire velocity in km/h.

As slip occurs between the tire-road contact area, a longitudinal friction force develops. The longitudinal frictional force allows the vehicle to accelerate and decelerate. The slip is generated during accelerating and decelerating operations. There exists a difference in speed between the tire rolling speed and its travelling speed; this results in a slip between the tire’s tread and the surface. Equation (1) describes the slip ratio generated during braking. Slip ratio reaches 100% when the wheel locks due to high braking effort.

\[
\text{Slip Ratio(\%)} = 1 - \frac{R_e w}{v} \times 100 \tag{1}
\]

where

- \( R_e \) tire effective rolling radius
- \( w \) wheel angular velocity
- \( v \) tire speed.

**Figure 3** Cornering force developments at various slip angles

Longitudinal reaction force develops when the tire runs over surface obstacles such as steps, potholes, water drainage ditches, or speed bumps. The reaction force created due to constraints is applied to the tire in all three directions (longitudinal, lateral and vertical). The longitudinal force acts in the opposite direction of the tire moving direction at the tire-road contact area. The force magnitude depends significantly on the suspension characteristics of the vehicle. The force magnitude also depends on the operational conditions of the tire such as the vertical load, inflation pressure and speed.

1.1.2 Lateral force

While a vehicle is undertaking cornering operation or when it’s subjected to cross-wind, lateral forces are generated. The lateral force is considered a dynamic force due to the lateral acceleration of the vehicle. The lateral force is called cornering force when it is in reaction to a cornering manoeuvre. While undertaking a cornering manoeuvre, the lateral load is transferred thus a vertical load is exerted on the right tire. Therefore, higher cornering forces are also applied to the same right tires. The cornering force varies with respect to the slip angle as shown in Figure 3. While the vertical load is kept constant and the slip angle is increasing, the cornering force also increases. The previous statement is valid until the slip reaches a certain level where the cornering force reaches an asymptotic direction. While applying a cornering force the contact area between the tire and road also appears to change with respect to the slip angle. The shape of contact area varies with the slip angle as shown in Figure 4. In this case, the tire is originally rolling in the direction of the top of the page and is turning left. The adhesive area or so-called effective stationary contact always appears at the leading edge of the contact area as the tire rolls. Moreover, the slip is kept to the rear of the contact area as the resultant cornering force is applied to the tire centre.

Figure 4 Contact area shapes at different slip angles

1.1.3 Vertical force

The vertical contact force is applied between the tire and road surfaces while the tire is operating on a surface. This force is considered a static force due to gravity. The tire and sprung mass have vertical accelerations when the vehicle runs on a rough road. The vertical acceleration causes a dynamic vertical force acts on the tire-road contact area that can reach up to three times higher than a static vertical force. The normal contact pressure distribution in the contact area for a non-rolling stationary tire is shown in Figure 5 left. The tire geometry and boundary conditions are symmetric about the centre of the contact area; thus, the pressure distribution is also symmetric. Greater normal contact pressures are noticed under the sidewalls and centreline of the tire at rated vertical load and inflation pressure. This is due to higher vertical stiffness in those local areas. Figure 5 right shows the normal contact pressure distribution in the contact area for a rolling tire. Unlike the non-rolling stationary state, the boundary conditions are not symmetric about the centre of the contact area. In the rolling state, the leading portion of the tire is compressed and the trailing part of the tire near the contact area is extended.
1.1.4 Overturning moment

Due to non-symmetric vertical pressure distribution across the tire width in the contact area, a moment on the tire spindle about the longitudinal axis is formed; this moment is the overturning moment. The effect of the overturning moment is shown in Figure 6. The magnitude and direction of the contact forces are altered across the width of the contact area. Thus, resulting in a slight lateral offset from the non-steered centre line.

Figure 6  Overturning moment

Source: Yap (1991)

1.1.5 Rolling resistance moment

As the vertical contact pressure is distributed over the contact area unevenly, the vertical resultant reaction force tends to shift toward the leading edge. Thus, the moment can be
developed against the tire rotational direction. This moment is defined as the rolling resistance moment.

1.1.6 Vertical moment

The vertical moment or the self-aligning moment is defined as the moment acting on the tire spindle about the vertical axis. The vertical moment is determined by the non-symmetric contact force distribution on the tire surface contact plane. The pneumatic trail is a resultant of the cornering force acting on the tire with some offset behind. To restore the tire to its original un-steered orientation a vertical moment is applied. The self-aligning moment increases with the increase of the slip angle input.

1.2 FEA method and equation

FEA is a numerical method to solve engineering and mathematical problems. In 1909, Ritz (1909) established an efficient method for the approximation of problems in the field of deformable solids. Later in 1943, Courant (1943) proposed a particular linear function technique and applied the method to the solving torsion problems. The combination of both Ritz method and Courant modification is similar to FEA method. However, FEA method later proposed by Clough in 1960. Clough was the first to introduce the term ‘finite element’ in the paper ‘the finite element method in plane stress analysis’ (Clough, 1960). A significant contribution was carried into FEM expansion by the papers of Argyris (1955), Turner (2012) and Hrennikoff (1941). Figure 7 shows the common methods available for the solution of general field problems. The methods are divided into numerical and analytical. The Analytical solution is determined either via an exact solution such as the separation of variables method or approximate method implemented by Rayleigh-Ritz method. While, the numerical method involves either numerical solution such as the numerical integration and finite differences, or the FEM technique.

A significant use of FEA technique is done in the domain of terrain mechanics. FEA technique has been very useful in solving several problems in terrain mechanics. In 1978 Yong et al. (1978) investigated the performance of off-road pneumatic tires using FEA technique. Yong performed various tests to determine the tire stiffness and tractive forces as a function of the tire inflation pressure. In 1997, Hiroma et al. (1997) implemented FEA method to predict the tractive forces and pressure distributions beneath a rolling wheel. Hiroma compared the FEA prediction to those from measurements and found that the predictions were reasonable. Hiroma concluded that under small slip conditions, FEA methods could be used to predict traction force.

Pam-Crash, the virtual environment software, corroborated in this study uses FEA method for building structures such as tires and vehicles. FEA uses the meshing methodology for creating objects. The meshes have the advantage of establishing numerical models of a physical structure with smooth and realistic discretisation and representation of the boundary conditions. Pam-Crash also implements the principle of explicit time integration which advances the solution along the time axis. The explicit solution method expresses the equilibrium equation at time \( t_n \) (PAM System International, 2000):

\[
m \frac{d^2 x_n}{dt^2} + kx_n = f_n
\]  

(2)
where $m$ is the mass, $\text{in}$ is the position at node $n$, $k$ is the stiffness and $f_n$ is the acceleration. The advantage of the explicit method is that only the mass, $m$, appears in the denominator. On the other side, the disadvantage is that the requirement for stability puts an upper limit on the time step.

Figure 7  Classification of common methods

![Diagram of classification of common methods]

Source: Barkanov (2001)

1.3 Tire modelling techniques

Modern computers and technology enable virtual tire testing in 3D environments. FEA tire models have been developed to design progressive and safer tires. Several truck tires have been modelled and validated using a virtual environment, such as Pam-Crash. In 2006 Chae (2006) modelled the Goodyear’s off-road RHD size 295/75R22.5 drive tire for tractor semi-trailers. The off-road tire is a radial ply tire with rim diameter of 22.5 inches. The truck tire-rim assembly model includes 27 different material definitions with 4,200 solid elements, 1,680 membrane elements and 120 beam elements. The section width of the off-road tire is 315 mm and the aspect ratio is 75%. The off-road tire and components are shown in Figure 8.

Later, Slade (2009) and Slade et al. (2009) modified the off-road tire model built by Chae to represent the Goodyear’s off-road RHD size 315/80R22.5 with four grooves. Figure 9 shows the basic dimensions of the finite element tire model. The off-road tire built by Slade is a modification of the rigid ring tire model developed by Pacejka and Zegelaar. Slade model includes additional parameters to incorporate the flexibility of the soil. The cross-section was then rotated about the tire axle axis in 6-degree increments to create the full tire with 60 equal pieces. This tire model is built using 9,200 nodes, 1,680 layered membrane elements, 120 beam elements, 27 material definitions and one rigid body definition. The rim is defined as a rigid body for the simplicity of the model because the deformation of the rim is negligible.

In 2016, Marjani (2016) modelled and validated the wide base tire developed by Michelin XOne line energy T using FEA technique implemented in Pam-Crash. The Michelin XOne line energy T tire size is 445/50R22.5 with a tread width of 371 mm and loaded radius of 465.3 mm. Michelin XOne line energy T basic dimensions are shown in Figure 10.
The wide base tire consists of rubber materials and reinforced rubber composites. The tire was modelled as an assembly of three-dimensional Mooney-Rivlin hyperelastic solid elements. The wide base has 212 beam elements, 2014 shell elements, 3,604 membrane elements, 6,360 solid elements and 11,978 four tetrahedral node elements. Marjani carried out the numerical model node by node using Pam-Mesh module in Pam-Crash. A section cut of the wide base tire is preliminary created. The section contains all tire parts and the tread is then assembled on the section cut as shown in Figure 11. The sidewall methodology and modelling procedures employed in Marjani thesis are considered as 2D
layered membrane owing to the very complex rubber material compounds of tire sidewalls. This methodology was developed through research works done by Chae (2006), Slade (2009), Dhillon (2013) and Reid (2015).

Figure 10  Michelin XOne line energy T tire 445/50R22.5 basic dimensions

Source: Marjani (2016)

Figure 11  Complete FEA Michelin XOne line energy T tire model section (see online version for colours)

Source: Marjani (2016)
1.4 Tire validation methods

Several tire characteristics must be matched closely to achieve the appropriate tire response. The FEA tire model should be validated in both static and dynamic responses. The static response is verified by vertical stiffness and static footprint tests. The dynamic drum-cleat test validates the dynamic response of the tire.

1.4.1 Vertical stiffness test

The vertical stiffness test was implemented by Chae (2006), Slade (2009) and Marjani (2016). The vertical stiffness test allows for the calculation of the tire’s spring rate. During the vertical stiffness test the tire is constrained in all directions except for the vertical direction. The free motion in the vertical direction allows the tire to move on the vertical axis as shown in Figure 12. Then, the tire is subjected to a ramp load which causes the tire to deform. The resultant deflection is then recorded for the corresponding vertical loads and the relationship between vertical load and the deflection is considered.

![Figure 12](see online version for colours)

1.4.2 Static footprint test

Static footprint test is the second validation test applied to validate the tire. The contact patch of a tire is affected by the inflation pressure and vertical load. In the static footprint test, the same procedure of the vertical stiffness test is applied. However, in this case, the contact patch area is recorded instead of the deflection. The footprint of the proposed model of Michelin XOne line energy T at 120 psi inflation pressure and 9,000 lbs vertical load is demonstrated in Figure 13.

1.4.3 Drum-cleat test

The dynamic validation of the tire is done through the drum-cleat test. A significant amount of the tire mass is concentrated near the tread. The rolling tire radius is not constant due to the radial stiffness. However, the stiffness of the tire is affected by the inflation pressure and the material properties. During this test, the drum-cleat test is virtually simulated to determine the first mode of vibration as shown in Figure 14 by exciting the tire over a cleat on a rigid circular drum. Vertical forces acting on the spindle
of the tire are translated due to the vibrations. These vertical forces are measured and converted from a time domain to a frequency domain using an FFT algorithm (PAM-CRASH). Thus, obtaining the harmonics of the first mode of vibration. Tire’s Sidewall damping is calculated using equation (3) (Chae, 2006).

\[ \alpha = \varepsilon \cdot 2w \]  

(3)

where \( \alpha \) is sidewall damping, \( \varepsilon = 5\% \), is 5\% critical damping effect and \( w \) is considered as the first mode of vibration frequency.

Figure 13  Tire 3D contact patch of XOne linear energy T (see online version for colours)

Source: Marjani (2016)

Figure 14  Tire drum-cleat test (see online version for colours)

2  Soil modelling and calibration

Soil calibration techniques given in literature are heavily depending upon the laboratories and soil modelling and calibration are a preliminary step towards tire-soil interaction modelling. Soil calibration methods presented in the literature are significantly depending upon the laboratories and equipment available. Several approaches for measuring soil properties exists including the bevameter, the cone penetrometer, triaxial apparatus and the traditional civil engineering techniques (Wong, 2008). For vehicle applications, the
penetrometer and the bevameter are frequently used. Onafeko and Reece (1967) measured the radial and tangential stresses beneath a tire under driven and towed conditions over a range of longitudinal slip/skid ratios (Onafeko, 1964). Wong and Reece (1967a, 1967b) formulated the radial and tangential stress distributions underneath a rolling tire as functions of the tire sinkage and slip/skid ratio.

**Figure 15** Schematic of a bevameter equipment

![Schematic of a bevameter equipment](image1)

*Source:* Wong (2008)

**Figure 16** WES cone penetrometer

![WES cone penetrometer](image2)

*Source:* Wong (2009)
Bekker developed the bevameter machine shown in Figure 15 in 1950s. The bevameter measures soil characteristics by applying pressure-sinkage and shear-strength test. The pressure-sinkage test also known as plate penetration test is performed with a plate on top of the soil and normal stress is measured. The shear-strength test is conducted with a finned plate being twisted within the soil and the shear stress is measured (Lescoe, 2010; Chu and Yin, 2005).

The penetrometer shown in Figure 16 was originally developed for military evaluations of terrain during the Second World War (Bekker, 1969). This equipment uses a 30º cone at the bottom end with a base area of 3.23 cm² and is pushed into the terrain to be evaluated (Bekker, 1969). The measurement defines the soil resistance against a penetrating cone-shaped object known as the cone index (Wismer and Luth, 1973).

Triaxial apparatus is equipment to measure the mechanical properties of deformable solids and soils. During the test, a cylindrical specimen of soil is subjected to hydrostatic pressure and axial load. The triaxial apparatus features measurement feedback control system that can simulate idealised states such as hydrostatic and triaxial compressions as well as uniaxial strain loading/unloading. This feature is essential for characterising compressibility, shear strength and unloading behaviour of soil (Bui et al., 2008).

Figure 17  WES cone penetrometer


2.1 SPH method and equations

SPH is a purely Lagrangian mesh free technique. SPH was originally invented for astrophysical application and then extended to general geotechnical problems. SPH is also applied to an enormous range of implementation such as the dynamic response of material strength free surface fluid flows turbulence flows. The need for SPH analysis triggered when the deformation of materials, modelled by FEA, becomes very high; element tingling may occur (Bui et al., 2008). In this case, FEA technique is no longer reliable. SPH incorporates significant deformation and post-failure of geomaterial in the framework. Thus, SPH is applied to simulate the large deformation of the continuum or dispersed material.
2.1.1 SPH mathematical representation

The SPH particles carry material properties such as velocity, density, stress, etc. and move with the material speed according to the governing equations. The partial differential equations for the continuum are converted into equations of motion of these particles and then solved by updated Lagrangian numerical scheme. The mass and momentum conservation equations are as follows (Bui et al., 2008):

\[
\frac{D\rho}{Dt} = -\frac{1}{\rho} \frac{\partial \rho v^x}{\partial x} \tag{4}
\]

\[
\frac{Dv^x}{Dt} = \frac{1}{\rho} \frac{\partial \sigma^{x\beta}}{\partial x} + f^x \tag{5}
\]

where \(\alpha\) and \(\beta\) denote the Cartesian components \(x, y\) and \(z\), \(\rho\) is the SPH density, \(v\) is the velocity, \(\sigma^{x\beta}\) stands for the total stress tensor of SPH and \(f^x\) is the component of acceleration caused by external force. The \(D/Dt\) is defined as:

\[
\frac{D}{Dt} = \frac{\partial}{\partial t} + v^x \frac{\partial}{\partial x} \tag{6}
\]

In traditional SPH a fluid, the total stress tensor is normally divided into two parts, the isotropic hydrostatic pressure \(p\) and a deviatoric shear stress:

\[
\sigma^{x\beta} = -p\delta^{x\beta} + \sigma^{x\beta} \tag{7}
\]

where \(\delta^{x\beta}\) Kronecker’s delta, \(\delta^{x\beta} = 1\) if \(\alpha = \beta\) and \(\delta^{x\beta} = 0\) if \(\alpha \neq \beta\). SPH is normally calculated as a function of density change by an ‘equation of state’, whereas the deviatoric shear stress is typically purely viscous and depends on the fluid models. The hydrostatic pressure of SPH soil is instead calculated directly from the soil constitutive equation by:

\[
p = -\frac{\sigma^{xy}}{3} = -\frac{1}{3} (\sigma^{xx} + \sigma^{yy} + \sigma^{zz}) \tag{8}
\]

where \(\sigma^{xx}, \sigma^{yy}, \sigma^{zz}\) are the components of the stress tensor in the \(x, y\) and \(z\) directions. \(K\) is the elastic bulk modulus, which relates to the shear modulus \(G\) and Poisson’s ratio \(\nu\) through the following equations:

\[
K = \frac{E}{3(1-2\nu)} \tag{9}
\]

\[
G = \frac{E}{2(1+\nu)} \tag{10}
\]

SPH artificial viscosity is added to the virtual environment to improve the numerical simulation stability. It is noted that shocks are always present, mostly in the first stages when initial conditions must relax. A passive term should be introduced to the governing equations to reduce the unwanted unphysical oscillations. Thus, to improve the numerical stability and to damp out such undesirable oscillations, a dissipative term \(\pi_{ij}\) (or artificial viscosity) is introduced into the pressure term of the momentum equation.
\[ \frac{Dw^\alpha}{Dt} = \sum_{j=1}^{N} m_j \left( \frac{\sigma_{ij}^{\alpha\beta}}{\rho_j} + \frac{\sigma_{ij}^{\alpha\beta}}{\rho_j^2} - \pi_{ij} \delta_{ij} \right) \frac{\partial W_{ij}}{\partial x_{ij}} + g^\alpha \] (11)

According to Monaghan the artificial viscosity can vanish the rigid-body rotation and conserve the total linear and angular momenta (Bui et al., 2008). Monaghan selected an artificial viscosity within the range of 0.01 and 0 for his first implementation of SPH for quasi-incompressible free surface flow.

### 2.2 Soil modelling techniques

Pam-Crash defines the SPH as a sphere centred on the particle centre of mass, a radius \( r \). Each SPH particle has an associated mass, velocity and stress state which evolves according to the discretised conservation equations. Each SPH also has three degrees of freedom (DOF), the centre of mass, the volume and the domain of influence (PAM System International, 2000).

**Figure 18** FEA to SPH conversion for a 100 by 100 mm square, (a) FEA mesh (b) FEA to SPH (c) SPH
SPH is composed of a finite collection of particles; these particles are created from a FEA mesh as shown in Figure 18. SPH is modelled from FEA elements which are defined as a rigid body, the centre of every FEA square is taken to be one SPH particle.

In 2010, Lescoe (2010) modelled soil using FEA and SPH techniques in Pam-Crash for dense sand. Lescoe solved the equation of state to find the pressure-volume relationship for dilatational elastic materials. Also, Lescoe classified terrain materials according to The Idaho Association of Soil Conservation Districts (1939) as shown in Figure 19. In 2013, Dhillon (2013) validated different FEA and SPH soil models through Pam-Crash using pressure sinkage and shear-strength tests for various soils including dry sand and clayey. Dhillon also indicated material properties of several terrain materials as shown in Table 1. In 2016, Marjani (2016) optimised soil models using FEA and SPH methods. Marjani also compared FEA and SPH results for the pressure-sinkage test in Pam-Crash and reported the results in her thesis. Also, Marjani developed a new modelling combination which reduces the computational time, combining hybrid FEA/SPH soil models for an optimised tire-soil interaction process. In 2017, Shahram developed soil models using LS-Dyna with material type-5 which is MAT-SOIL-AND-FOAM. Shahram developed and validated several soil models including high-density clayey sand, low-

Source: The Idaho Association of Soil Conservation Districts (1939)
density dry sand, high-density wet sand and high density flooded sand. Shahram calibrated his soils by performing two tests the pressure-sinkage test and the shear-strength test (Shokouhfar, 2017).

<table>
<thead>
<tr>
<th>Material type</th>
<th>Elastic modulus</th>
<th>Bulk modulus</th>
<th>Shear modulus</th>
<th>Yield stress</th>
<th>Density</th>
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<tr>
<td></td>
<td>E (MPa)</td>
<td>K (MPa)</td>
<td>G (MPa)</td>
<td>σ (MPa)</td>
<td>ρ (ton/mm³)</td>
</tr>
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<td>1.6E-9</td>
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<td>Loose sand</td>
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<td>11</td>
<td>7</td>
<td>0.004</td>
<td>1.44E-9</td>
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<td>23</td>
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<td>2.01E-9</td>
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<tr>
<td>Sand and gravel</td>
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<td>80</td>
<td>48</td>
<td>0.024</td>
<td>1.92E-9</td>
</tr>
</tbody>
</table>

Source: Dhillon (2013)

2.2.1 SPH part definition

SPH element part card definition implements several control parameters that influence the behaviour of the soil model apart from the material properties. Pam-Crash defines SPH restriction such that each particle is not allowed to exceed 10% of the internal energy. The neighbouring distance between each two consecutive SPH particles is specified by the smoothing length, while the minimum and maximum smoothing lengths are governed by the equation (8), where $h_0$ is the initial smoothing length.

$$HMIN \times h_0 \leq CSLH(t) \leq HMAX \times h_0$$

An option of the dynamic neighbourhood is introduced in Pam-Crash within a sphere of influence which updates automatically by the solver algorithm (Bui et al., 2008). Figure 20 shows SPH parameters definition in Pam-Crash.

SPH part card requires the definition of a ratio which is the particle smoothing length to radius ratio and it is recommended within the range 1.8–2. Hmin, which is the minimum smoothing length and is defined by default to be zero. Hmax, which is the maximum smoothing length and it is a user input with a range of 0 to 100. ETA, which is
the anti-crossing force parameter and is usually defined by default to be zero. ALPHAmg, which is the first parameter of Monaghan-Gingold artificial viscosity. BETAmg, which is the second parameter. These parameters are defined in a part definition card that may differ from a virtual performance software to another. The anti-crossing force (ETA) is defined using equation (13), where $\mu$ is the relative strength which is usually smaller than 0.5. As for the Artificial viscosity parameters (ALPHAmh, BETAmg) research indicated that for most fluid simulations, the defaults of 0.04 and 0.01 respectively should be suitable.

$$\frac{dr_i}{dt} = u_i + \sum_j m_j \frac{u_j - u_i}{2(\rho_i + \rho_j)} W_{ij}$$  (13)

Lescoe conducted a research on the sensitivity of SPH part definition car and recommended the values shown in Table 2 (Lescoe, 2010).

### Table 2 Smooth particle control properties

<table>
<thead>
<tr>
<th>Properties</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Particle smoothing length to radius ratio</td>
<td>RATIO</td>
<td>1.3 to 2.1</td>
</tr>
<tr>
<td>Minimum smoothing length</td>
<td>HMIN</td>
<td>1</td>
</tr>
<tr>
<td>Maximum smoothing length</td>
<td>HMAX</td>
<td>100</td>
</tr>
<tr>
<td>Anti-crossing force parameter</td>
<td>ETA</td>
<td>0.1</td>
</tr>
<tr>
<td>Linear artificial viscosity</td>
<td>ALPHAmg</td>
<td>0 to 0.3</td>
</tr>
<tr>
<td>Quadratic artificial viscosity</td>
<td>BETAmg</td>
<td>0</td>
</tr>
</tbody>
</table>

*Source: Lescoe (2010)*

#### 2.2.2 SPH material definition

Another SPH card that should be taken into consideration is the SPH element card. SPH element card defines the material properties of the SPH. Tangent modulus is the slope of the stress-strain curve at any specified stress or strain. The Shear modulus is defined as $G$ and the Yield Stress is defined as $SIMAy$. The initial internal energy per unit volume is designated as $Ei$. The solution of the equation of state parameters is defined as $C_0$ to $C_6$. KSI is the stiffness proportional damping ratio. Q1 is the quadratic bulk viscosity coefficient; Q2 is the linear bulk viscosity coefficient and Q3 hourglass viscosity coefficient.

In Pam-Crash for Soils and snowmaterial type-7 is used, this material is called isotro-pielastic-plastic-hydrodynamic for solid and SPH. The equation of state governs the pressure volume relation and it behaves as an elastic-plastic material at low pressure. The elasticplastic behaviour is inputted by specifying the yield stress and tangent modulus parameters. Equation (14) defines the equation of state for material type-7. Where $C_0$ to $C_6$ are material constants, $\mu = \rho/\rho_0 - 1$ is the ratio of current over initial mass density and $E_i$ denotes the internal energy (PAM System International, 2000).

$$p = c_0 + c_1\mu + c_2\mu^2 + c_3\mu^3 + (c_4 + c_5\mu + c_6\mu^2)E_i$$  (14)

On the other side, water can also be modelled using SPH technique. However, the material type is different as water is defined using material type-28 which is Murnaghan
equation of state for solid elements and SPH. The inputs required for the material definition are the bulk coefficient, the exponent gamma, the density and the viscosity parameters. The exponent Gamma is the exponent of the density ratio in the Murnaghan equation of state defined in equation (15). Where the ratio \( \rho / \rho_0 \) is the current mass density over the initial mass density ratio and gamma is equal to 7. B determines the speed of sound and can be calculated using equation (16).

\[
p = p_0 + B \left( \left( \frac{\rho}{\rho_0} \right)^\gamma - 1 \right) \tag{15}
\]

\[
B \geq 100 \rho_0 \frac{v_{\text{max}}^2}{Y} \tag{16}
\]

Material type 28 is ideal for Liquids with an artificially increased compressibility. This material is used to perform a certain class of hydrodynamic simulations where the flow velocities remain well below the physical sound speed (PAM System International, 2000).

### 2.3 Soil calibration methods

Soil calibration is performed as a validation process to optimise soil models. It is significantly important for tire-soil interaction study to have a well-calibrated soil. To calibrate soil models two tests are performed the pressure-sinkage test and the shear-strength test. These two tests are developed in Pam-Crash to duplicate the ones in reality.

#### 2.3.1 Pressure-sinkage test

The pressure-sinkage test shown in Figure 21 consists of a 600 * 600 * 600 mm box filled with SPH soil particles and topped with a 150mm radius circular rigid plate. This test can be applied for both FEA and SPH soil models. The SPH soil particles are subjected to a range of pressures from 0 KPa to 200 KPa and the sinkage of the place is measured. The relation between the pressure applied and the plate displacement is computed. The pressure-sinkage experimental results can be found in Wong’s book for terrain materials. Also, the pressure-sinkage relation shown in equation (17) is also provided by Wong (2008). Where \( p \) is the pressure in KPa, \( b \) is the smaller dimension of the contact patch, that is the width of a rectangular contact area or the radius if a circular contact area in mm, \( z \) is the sinkage of the plate in mm and \( n, k_c, \) and \( k_q \) are pressure-sinkage parameters. Finally, the process is repeated until an optimal result is computed.

\[
p = \left( \frac{k_c}{b} + k_q \right) z^n \tag{17}
\]

#### 2.3.2 Shear-strength test

The shear-strength test shown in Figure 22 is the second calibration test applied to validated soil models. This test consists of a Rectangular box of 400 * 200 * 240 mm size filled with SPH soil particles. The box is made of three parts the top plate which pressure
is applied on, the upper which is the sliding plate and the lower plate which is constraint from moving in all direction. The test starts by applying the desired pressure to the top plate. Then a small ramp displacement is applied to the upper and the top plates. The SPH force is computed during this time. The stress is calculated from the force and plotted against pressure. The results are compared with Bekke’s shear-strength relation (Wong, 2008). Bekker’s equations for the shear-strength are shown in equation (18).

\[
\tau = \tau_{\text{max}} \left( 1 - e^{-r/j} \right) \\
= (c + \sigma \tan \phi) \left( 1 - e^{-r/j} \right)
\]  

(18)

(19)

Figure 21  Pressure-sinkage test with SPH soil (see online version for colours)

Figure 22  Shear-strength test with SPH soil (see online version for colours)

The maximum shear \( \tau_{\text{max}} \) can be determined through the Mohr-Coulomb failure criterion in equation (20).

\[
\tau_{\text{max}} = c + p \tan \phi
\]  

(20)
Figure 23 Pressure-sinkage and shear-strength simulation results for different terrains, (a) pressure-sinkage relationship for dry sand (b) shear-strength relationship for dry sand (c) pressure-sinkage relationship for snow (d) shear-strength relationship for snow (see online version for colours)
Figure 23  Pressure-sinkage and shear-strength simulation results for different terrains, (a) pressure-sinkage relationship for dry sand (b) shear-strength relationship for dry sand (c) pressure-sinkage relationship for snow (d) shear-strength relationship for snow (continued) (see online version for colours)
2.3.3 Modelling and calibration results

Modelling and validation of various terrain materials have been done in the course of this thesis. Several terrain models have been validated such as clayey, dry sand, dense sand and snow. Figures 23(a) and 23(b) shows the pressure-sinkage and shear-strength relations for dry sand. The results indicate simulation response to be very similar to that from experimental results.

3 Tire-soil interface

After defining the tire and soil models, it is thus time to describe the interaction. Tire soil interaction is considered a vital task towards an accurate interaction behaviour. Contact is considered very important in engineering applications as well as in human life. It is impossible to grab or hold objects for using them without frictional contact. Modelling tire and soil in a virtual environment requires virtual and numerical contact definition. Computational contact problem has been found since 1960s (Goodman et al., 1968; Wilson and Parsons, 1970; Chan and Tuba, 1971). Preliminary, the contact condition was defined using simple boundary conditions due to computational limitations. Later, solution algorithms were adopted in computational contact mechanics.

3.1 Interaction modelling

The contact algorithm includes contact search, contact pair match and two important methods for contact force calculation. Figure 24 shows the contact-impact simulation methods. The contact search algorithm can be divided into node-to-node proximity search and node-to-segment correspondence search. While the contact interface algorithm can be divided into penetration detection method and imposition of contact-impact conditions.

Figure 24 Contact-impact simulation methods

The computational contact solution algorithm defines the contact events between two objects. The algorithm explores for contact prone parts and applies the contact condition after contact has been detected. Without the contact algorithm employed in computational simulations, no contact will occur. The computational contact problem resolution is to detect penetration between two objects. After the penetration is encountered a suitable response that eliminates, minimises, or reduces the penetration is computed.

3.1.1 Contact search algorithm

Two important terms should be defined during interaction the master-slave contact and self-contact. The master-slave contacts require the definition of two surfaces: slave and master sides. Each node of the slave side is checked for penetrations to the segments/edges of the master side. The self-contact involves the definition of one slave surface only. Each node-edge of the slave side is checked for penetrations to the segments/edges of the slave side. In 1990, Benson and Hallquist (1990) recognised a search technique that subdivides the contact body into three-dimensional cubic buckets and restricts the search to the bucket that contains a slave node and to all neighbouring buckets.

The contact search algorithm evaluates which part of the structure is likely to contact a rigid wall, another part of the structure, or itself. The contact search algorithm can be subdivided into two major methods, node-to-node proximity search and node-to-segment correspondence search. The node-to-node proximity is given by their spatial distance. In this vicinity, the nodes of a structure are organised according to their distance along a given search direction for a fast and efficient search algorithm (PAM System International, 2000). The search algorithm performs pairing of the contact proximity and node-to-segment contact.

Figure 25 Search radius

Figure 25 shows the searching radius scheme for the contact search algorithm. First, the master node closest to a slave node within its contact sphere is to be located. Then the segment to which this master node is connected and which the slave node is likely to hit is to be determined. Based on equation (21), the contact segment \( s \) can be determined from neighbouring elements around the closest master node (Hallquist et al., 1985).

\[
s = g \left( g \times e_3 \right) e_3
\]

\[
e_3 = \frac{c_1 c_2}{|c_1 c_2|}
\]

\[
(c_1 \times s)(c_1 \times c_2) > 0
\]

\[
(c_1 \times s)(s \times c_2) < 0
\]

Figure 26 defines the node-to-segment correspondence search parameters implemented in equation (21). The vector, \( g \), initiates from the closest master node to the slave node above the master surface. While the vector, \( s \), is determined from the closest master node to the projection node of the slave node on the master surface. The unit vector normal to the master surface near the closest master node is called \( e_3 \). Vector \( e_3 \) can be determined using the equation (22), where \( c_1 \) and \( c_2 \) are defined in Figure 26. Vector \( s \) should also satisfy the following two constraints in equations (23) and (24) to guarantee that the slave node projects onto contact segment 1.

Figure 26  Node-to-segment correspondence search

3.1.2 Contact interface algorithm

After the contact nodes and contact, segments have been matched using the contact search algorithm. The conditions of contact are implemented whenever the contacting slave node penetrates the surface of the contact segment. The contact interface algorithm is then processed; the interface algorithms include the Lagrange multiplier method and the Penalty method.

3.2 Lagrange multiplier method

The multiplication terms of the Lagrange multiplier usually denoted as $\lambda$ and a gap function is added to the total energy equation of the system. In 1982, Oden et al. (1982) introduced several basic finite element methods and new non-classical friction laws for the study of nonlinear problems in rubber elasticity. The new friction laws involved non-local effects in approximating the deformed asperities, the adhesion effects and the damage model on the contact surface. On year later, Guerra and Browning (1983) performed several contact simulations by applying two primary contact treatments algorithms. The algorithms used are the Lagrange multiplier method and the penalty method. These methods were applied to an FEA software to compare the methods from the viewpoint of efficiency, reliability and ease of use. It was also stated that if a very high stiffness has bee used, the solution might converge gradually or even diverge.

In 1985, Bathe and Chaudhary (1985) established a solution algorithm for general two-dimensional contact problems between flexible-flexible and flexible-rigid bodies at sticking or sliding conditions. The contact forces were assessed from distributed tractions that could be calculated from the nodal forces and the frictional conditions. Later in 1990, Marques and Martins (1990) developed a three-dimensional plastic finite element formulation including frictional contact algorithm based on Lagrange multiplier method. The predicted results showed agreement with theoretical and experimental findings.

3.3 FEA-SPH interaction

In 1985, Hallquist et al. (1985) established comprehensive two and three-dimensional contact algorithms to computationally solve static and dynamic impact problems. Later in 1990, Benson and Hallquist (1990) examined the behaviour of a shell structure after it buckled. They specified that when a structure collapsed completely, a single surface might buckle enough to encounter itself. In 2001, Koishi et al. (1998) performed computational contact simulations of elastic solids using an implicit FEM approach. Hirato et al. developed a new penalty FE formulation based on the concept of material depth. This penalty represented the distance between a particle inside an object and the object’s boundary. The new algorithm was implemented in their in-house implicit FE program for static and quasi-static analysis of nonlinear viscoelastic solids.

The zero-gap condition between the slave node and the master contact surface is satisfied by Lagrange multiplier method while the penalty method does not. On the other side, contact conditions are further relaxed and penetration of the impinging slave node into the contact segment is allowed. The forces are computed based on penetration depth.

For this research contact type 34 is used for tire-road and tire-soil interaction. Contact type 34 is defined as node-symmetric node-to-segment contact with edge treatment. This contact type requires a definition of master-slave definition. The input of this contact card
requires the definition of $H_{cont}$ which is the thickness of master and slave entities. FRICT which is the constant friction coefficient, it is defined to be 0.8 between road and tire, 0.4 between road and water. Other friction coefficients are available for different soils. XDMPI which is defined as the stiffness proportional damping ratio and it is set to 0.5 normally. SLFACM which is the scale factor for sliding interface penalties and it is set to 0.1 by default.

4 Rigid ring tire model parameters

In the 1980s and 1990s, many researchers adopted rigid ring tire models to efficiently predict dynamic tire responses during tire running on irregular road surfaces. The output of the rigid ring tire model allows for the calculation of the In-plane and out-of-plane parameters. During research, all the In-plane and out-of-plane parameters have been determined by performing appropriate experimental tire tests.

For the in-plane tire motions, the elastic sidewall is modelled using translational stiffness and damping. The residual vertical stiffness is introduced due to the contribution of the tread band and belts to the vertical tire deformation at the contact area. For the out-of-plane motions, the lateral stiffness and damping are implemented to model the relative translational movement of the elastic sidewall to the wheel plane.

4.1 In-plane rigid ring tire model parameter

Figure 27 shows the required in-plane characteristic parameters for the rigid ring tire model. The FEA tire model is inflated at a several pressures to simulate the under inflation, nominal inflation and over inflation conditions. To determine the in-plane parameters at various tire vertical loads, the tire vertical load ranges from 3,000 lbs to 9,000 lbs.

The effective rolling radius $R_e$ is the shortest distance from O to the line T-T’. $R_e$ can be calculated using the angular velocity $w$ and the longitudinal velocity $v$ in equation (25).

$$R_e = \frac{v}{w} = \frac{R_0}{\tan \theta}$$  \hspace{1cm} (25)

4.1.1 In-plane transnational stiffness of the sidewall and residual vertical stiffness

The in-plane translational stiffness of the sidewall and the residual stiffness at the contact area can be calculated by using equation (26).

$$w_n = \sqrt{\frac{K_{sR} + K_{Rv}}{m_b}}$$  \hspace{1cm} (26)

where $w_n$ is the natural frequency (rad/s), $K_{sR}$ is the in-plane vertical stiffness of the sidewall, $K_{Rv}$ is the residual vertical stiffness at the contact area, $m_b$ is the mass of the tire belt. Due to the symmetry about the tire spindle, the vertical stiffness ($K_{sR}$) is considered the same as the longitudinal stiffness ($K_{Rv}$). The connection between this two stiffness and
the total stiffness $K_{tot}$ is considered series connection and thus can be equated as equation (27).

$$\frac{1}{K_{hc}} + \frac{1}{K_{br}} = \frac{1}{K_{tot}}$$  \hspace{1cm} (27)

Equations (26) and (27) can be used together as a system of equations to determine the in-plane translational stiffness ($K_{hc}$) and the residual vertical stiffness ($K_{br}$).

**Figure 27** In-plane rigid ring tire model

4.1.2 In-plane longitudinal and vertical damping constants of sidewall and residual damping constant

The sum of the vertical and residual damping is calculated using equation (28). Where $C_{hc}$ is the in-plane vertical damping constant of the sidewall, $C_{br}$ the residual damping constant and $\varepsilon$ is the damping ratio which is 5%.

$$C_{hc} + C_{br} = 2 \varepsilon \sqrt{(K_{hc} + K_{br}) m_b}$$  \hspace{1cm} (28)

While the residual damping constant can be calculated using equation (29). Where, $m_s$ is the rim mass. These two damping constants can be calculated at various inflation pressures and vertical loads.

$$C_{br} = 2 \varepsilon \sqrt{K_{br} (m_s + m_b)}$$  \hspace{1cm} (29)

*Source: Zegelaar and Pacejka (1996)*
4.1.3 In-plane rotational stiffness and damping constant of the sidewall

To virtually predict the in-plane sidewall rotational stiffness and damping constant, the rim is defined as a rigid body and is constrained not to be translated and rotated. A tangential force of 5,620 lb is then applied to the node on the rigid tread to rotate the rigid tread band as shown in Figure 28.

Figure 28  Rotational excitation on FEA truck tire (see online version for colours)

![Figure 28](image)

Source: Chae 2006

Figure 29  Angular displacement of the tread and damping response of the sidewall (see online version for colours)

![Figure 29](image)

Source: Chae 2006

4.1.4 In-plane rotational stiffness of the sidewall

Using the angular displacement shown in Figure 29 and the applied moment, the in-plane rotational stiffness \( K_{\theta} \) of the sidewall can be calculated based on equation (30).
4.1.5 In-plane rotational damping constant of the sidewall

When the applied tangential force is quickly removed, the rigid tread band undergoes rotational vibrations. Thus, the magnitudes of the vibrations decrease with time due to the damping nature of the sidewall as shown in Figure 29. The logarithmic decrement $\delta$ can be calculated using equation (31), where $\theta_1$ and $\theta_2$ are the angular displacements at time $t_1$ and $t_2$. Then, the dimensionless damping ratio ($\varepsilon$) can be found using equation (32). The damped period of vibration can be expressed as $t_d = t_2 - t_1$.

$$\delta = \ln\frac{\theta_1}{\theta_2} \quad (31)$$

$$\delta = \ln\frac{2\pi \varepsilon}{\sqrt{1-\varepsilon^2}} \quad (32)$$

$$t_d = \frac{2\pi}{w_d} = \frac{2\pi}{w_n \sqrt{1-\varepsilon^2}} \quad (33)$$

The undamped natural frequency $w_n$ can be calculated using equation (33). The critical damping can be expressed as $C_c = 2lbyw_n$, where $(lby)$ is moment of inertia of the tire belt. Finally, the in-plane rotational damping constant $C_{\theta \theta}$ can be calculated as $C_{\theta \theta} = \varepsilon C_c$.

**Figure 30** Longitudinal force versus slip ratio (see online version for colours)
4.1.6 Longitudinal tread stiffness and longitudinal slip stiffness

The FEA truck tire model is several vertical loads on a 3.4 m-diameter smooth drum. A linear speed from 0 to 10 km/h is applied to the tire spindle during a very brief period as a simulation input. In the case of a tire longitudinal slip test on a drum, the slip ratio during tire acceleration is calculated by using equation (34). Where $v_{tire}$ is the tire velocity (m/s), $v_{drum}$ is the drum velocity (m/s), $R_{drum}$ is the drum radius (m) and $w_{drum}$ is the drum angular velocity (rad/sec). As shown in Figure 30, a maximum longitudinal friction force is achieved when the tire undergoes a certain amount of slip.

\[
\text{SlipRatio} = \frac{v_{tire} - v_{drum}}{v_{tire}}
\]  

(34)

4.1.7 Longitudinal slip stiffness

The longitudinal slip stiffness is defined as the slope at zero slip ratio. Longitudinal slip stiffness ($K_k$) is determined for various inflations pressures and loads.

\[\text{Figure 31} \quad \text{Out-of-plane parameters for the rigid ring tire model}\]


4.1.8 Longitudinal tread stiffness

The longitudinal tread stiffness ($K_{cx}$) can be calculated by using equation (35) at several inflation pressures and loads (Zegelaar and Pacejka, 1996). Where $a$ is half the contact length in m.
\[ K_{cx} = \frac{K_1}{a} \] (35)

**Figure 32** Out-of-plane translational excitation on sidewall (see online version for colours)

Source: Chae (2006)

**Figure 33** Lateral displacement and damping response of the sidewall (see online version for colours)

Source: Chae (2006)

### 4.2 Out-plane rigid ring tire model parameter

The out-of-plane parameters of the rigid ring tire model are virtually predicted by applying and releasing several tire loadings on the FEA truck tire model. Figure 31 shows the out-of-plan parameters of the rigid ring tire model.
The out-of-plane sidewall translational stiffness \( K_{by} \) and its damping constant \( C_{by} \) are calculated using a virtual test. The rim is set to an undeformable rigid body and it is constrained not to be translated and rotated, but it is free only in lateral direction. The tire is first inflated to the desired inflation pressure. Then a lateral force of 3,370 lbs is applied to the selected nodes. And a total lateral force of 6,740 lbs is applied to the sidewall as shown in Figure 32.

Using equation (36) the out-of-plane translational stiffness \( K_{by} \) can be calculated. When the applied lateral forces are quickly removed, the rigid tread band undergoes out-of-plane translational vibrations and response amplitude decays exponentially as seen in Figure 37.

\[
K_{by} = \frac{\text{Lateral Force}}{\text{Lateral Displacement}} \quad (36)
\]

4.2.1 Out-of-plane rotational stiffness and damping constant of the sidewall

The out-of-plane rotational stiffness \( K_{b\gamma} \) and damping constant \( C_{b\gamma} \) of the sidewall are virtually predicted. The rim of the tire model is set to a rigid body and concentrated in all direction except the out-of-plane rotational direction. The tire model is inflated to the desired pressure. Then, a lateral force of 15.0 kN is applied to two nodes on the rigid tread in the opposite direction to rotate the rigid tread band about the longitudinal axis as shown in Figure 34. The applied lateral loads are rapidly removed to observe the damping response of the sidewall.

\[
K_{b\gamma} = \frac{\text{Moment}}{\text{Angular Displacement}} \quad (37)
\]

Using the applied moment and the displacement, the out-of-plane rotational stiffness \( K_{b\gamma} \) can be calculated using equating 37. From Figure 35 one can measure the time and angular rotation for two neighbouring peak points \( t_1, t_2 \). The out-of-plane rotational damping constant \( C_{b\gamma} \) is then calculated using

\[
C_{b\gamma} = \epsilon C_c.
\]

4.2.2 Lateral free vibration tests

The lateral slip stiffness \( K_1 \) and damping constant \( C_1 \) of the tire at the contact area can be predicted by the lateral free vibration test at various tire inflation pressure and vertical loads. A lateral load of 5.0 kN is applied as shown in Figure 36.

By using the applied force and the displacement, the out-of-plane slip stiffness \( K_1 \) can be calculated using equation (38). Similar to the previous section, one can measure the time and angular rotation for two neighbouring peak points \( t_1, t_2 \). And the out-of-plane slip damping constant at the contact area \( C_1 \) is calculated using the expression

\[
C_1 = \epsilon C_c.
\]

\[
K_1 = \frac{\text{Lateral Force}}{\text{Lateral Displacement}} \quad (38)
\]
4.2.3 Cornering stiffness

Cornering stiffness \((K_f)\) is defined as the derivative of the cornering force \((F_y)\) with respect to slip angle \((\alpha)\) evaluated at zero slip angle

\[
K_f = \frac{\partial F_y}{\partial \alpha} \bigg|_{\alpha = 0} \tag{39}
\]
4.2.4 Self-aligning stiffness

The self-aligning stiffness is defined as the derivative of the self-aligning moment ($M_y$) with respect to slip angle ($\alpha$) evaluated at zero slip angle

$$K_M = \left. \frac{\partial M_y}{\partial \alpha} \right|_{\alpha = 0}$$

(40)
4.2.5 Relaxation length

The relaxation length ($\sigma$) can be calculated using equation (41). Where $K_f$ is the cornering stiffness and $K_1$ is the lateral slip stiffness of the tire.

$$\sigma = \frac{K_f}{K_1}$$

(41)

4.3 Hydroplaning prediction

Since the 1960s many researchers tend to employ an analytical or numerical method to investigate the hydroplaning problem. In 1960, Martin (1966) considered the total dynamic hydroplaning problem by applying the potential flow theory and conformal mapping techniques. In 1967, Eshel (1967) divided the tire water contact area into three zones based on the amount of the inertial and viscous effects and utilised a different method for each zone. Later in 2000, Seta et al. (2000) used a finite element model (FEM) for tire and finite volume model (FVM) for water to simulate tire hydroplaning. In 2005 Fwa established a numerical simulation model for hydroplaning prediction using CFD techniques implemented by Fluent to investigate the effect of several factors such as groove width, depth and spacing of pavement on hydroplaning speed of smooth passage car tire (Chu and Fwa, 2016).

Furthermore, Oh et al. (2008) adopted two separate mathematical models to simulate hydroplaning. One year later Jenq and Chiu (2009) implemented a hydroplaning model for a tire using LS-DYNA, the model accounted for the water viscous effect. In 2017, Zeinab et al. adopted FEM for a truck tire and SPH to model tire-water hydroplaning in Pam-Crash. Zeinab reported the variation of the hydroplaning speed as a function of tire inflation pressure, vertical load and water thickness.

4.3.1 Three-zone concept

To describe the hydroplaning phenomena, researchers developed a ‘three-zone’ concept. This concept was first applied by Gough in 1954 and then developed further to cover the
rolling tire case by Moore in 1966. Figure 38 shows a schematic of the three-zone concept. Zone A is the squeeze-film zone which is governed by the elastohydrodynamic lubrication (EHL), in this district water wedge penetrates in the backward direction. Zone B is the transition zone where tire elements penetrate the squeeze-film commence to drape about the asperities of the road surface. Zone C is the traction zone; this is the rear part of the contact area, it starts at the beginning of the end of the transition zone. In this zone, the lubricated water film is considerably removed and the vertical equilibrium of the tread elements are attained (Jenq and Chiu, 2009).

4.3.2 Hydroplaning equations

Different equations have been developed to predict the hydroplaning speed. In 1965 Horne and Joyner proposed the NASA hydroplaning equation according to aircraft tire experiments at Langley Centre of Nassa. NASA equation is shown in equation (42), where \( P \) is the tire inflation pressure in KPa and \( V \) is the minimum hydroplaning velocity km/h (Horne and Joyner, 1965).

\[
\nu = 6.36 \sqrt{P}
\]  

(42)

On the other side, Horne developed an equation that predicts hydroplaning speed for truck tires. Horne developed equation (43) in 1986; the equations relate the hydroplaning speed with the tire footprint aspect ratio (FAR) and the inflation pressure (Horne et al., 1986).

\[
\nu = 23.3 P^{0.21} \left( \frac{1.4}{\text{FAR}} \right)^{0.5}
\]  

(43)

Later in 1979, Gallaway et al. (1979) developed another equation to predict the tire hydroplaning speed. Gallaway equation involves the spin down %, tire inflation pressure, tread depth, water film thickness, mean texture depth of pavement surface. Equation (44) presents Gallaway equations and equation (45) shows the parameter \( A \). Where \( SD \) is the spin down %, \( T_w \) is the water film thickness (in), \( MTD \) is the mean texture depth (in), \( TRD \) is the tire tread depth in 1/32 in.

\[
\nu = SD^{0.44} P^{0.3} (TRD + 1)^{0.06} A
\]  

(44)

where \( A \) is:

\[
A = \max \left[ \left( \frac{10.409}{T_w^{0.06}} + 3.507 \right) , \left( \frac{28.952}{T_w^{0.06}} - 7.819 \right) MTD^{0.04} \right]
\]  

(45)

In 1984, Wambold et al. (1984) developed an equation that predicts hydroplaning for Low-pressure tires based 10% SD a 165 KPa tire pressure. Equation (46) shows Wambold equation, \( WT \) is water film thickness in mm, \( MTD \) mean texture depth mm, \( TD \) tire tread mm and \( Ks \) are empirical coefficients.

\[
\nu = 3.5 k_1 \left( \frac{TD}{25.4} + 1 \right)^{k_2} MTD^{k_3} \left( \frac{k_4}{WT^{k_5}} + 1 \right)
\]  

(46)
Recently in 2017, Zeinab et al. developed a hydroplaning empirical equation for truck tires. The relation shown in equation (47) relates the tire speed to the inflation pressure ($P$) in psi, vertical load ($L$) in lbs, water thickness ($TW$) in mm, tire tread depth ($TTD$) in mm. The parameter $A$ is defined in equation (48).

\[
V = 67.16 + 0.018L + 0.0004TTD \times TW^2 + \frac{A}{P} - 1.13TW - 7.769e^7L^2 \\
A = 19.94TW - 436.64 - 0.029TTD - 0.185L
\]

Figure 39  Simple representation of the PAC2002-model

Source:  Brantin and Grundén (2016)

5  Rigid ring tire model in MATLAB-simulink

The purpose of this MATLAB/simulink code is to validate the virtual simulations using a rigid ring model. Computation of the tire-soil interaction is time-consuming and expensive. Thus, it would be economical to model the tire-soil interaction using mathematical equations in MATLAB. This work has been done in 2007 by Allen (2007) for a quarter vehicle model for durability and ride comfort prediction (Allen et al., 2008).

5.1  Rigid ring tire models

The Pacejka PAC2002 tire model is developed by MSC Software. PAC2002 is widely used in vehicle dynamics to study and calculate tire forces and moments. Figure 39 shows the simple representation of the PAC2002 model.
5.1.1 Rigid ring tire model with the ground

The rigid ring tire model presented in Figure 40 shows the In-plane and out-of-plane model for rigid ground. The model consists of two rigid rings connected with springs and dampers.

The inner ring consists of the mass and inertia of the rim and inner part of the sidewall. While the outer ring consists of the mass and inertia of tire belt and outer part of the sidewall.

**Figure 40** Representation of the wheel plane (left) and out of plane (right) rigid ring model

**Figure 41** Representation of the wheel plane (left) and out of plane (right) rigid ring model
5.1.2 Rigid ring tire model, soft soil ground

The rigid ring tire model presented in Figure 41 shows the in-plane and out-of-plane parameters for soft soil ground. The ground and the tire interact at the contact point and this is where the model needs to be expanded.

5.2 Rigid ring tire model test cases

Various equations have been developed for every test case to validate the simulation. The results of the MATLAB code have been compared with those of the Pam-Crash output. For MATLAB code validation, several test cases are required to validate the rigid ring tire model, these test cases are mentioned below for the rigid road:

5.2.1 Vertical stiffness and damping

This test procedure has been explained in Section 4.1.1. Figure 42 shows the representation of the vertical part of the rigid ring model on rigid ground. In this set up there are two sets of springs and dampers in parallel which are connected in series by solving the equations of motions of the diagram equation (49) can be conceded.

\[
\begin{bmatrix}
0 \\
0 \\
0
\end{bmatrix}
= 
\begin{bmatrix}
K_{bz} & -K_{bz} & 0 \\
-K_{bz} & K_{bz} + K_{vr} & -K_{vr} \\
0 & -K_{vr} & K_{vr}
\end{bmatrix}
\begin{bmatrix}
z_1 \\
z_2 \\
z_r
\end{bmatrix}
+ 
\begin{bmatrix}
0 & -C_{bz} & 0 \\
-C_{bz} & C_{bz} + C_{vr} & -C_{vr} \\
0 & -C_{vr} & C_{vr}
\end{bmatrix}
\begin{bmatrix}
z_1 \\
z_2 \\
z_r
\end{bmatrix}
\]

(49)

Figure 42  Representation of the vertical part of the rigid ring model on rigid ground
5.2.2 Wheel plane rotational stiffness and damping

The wheel plane rotational stiffness and damping test has been used to validate the MATLAB code. Figure 43 shows the representation of the rigid ring wheel plane rotational properties. \( \theta_1 \) and \( \theta_2 \) are the two DOF in the rotational direction. Based on the equations of motions equation (50) is derived. Similar test was done in Pam-Crash and was reported in Section 4.1.3

\[
\begin{bmatrix}
\text{Min} \\
-F_x R_e
\end{bmatrix}
= 
\begin{bmatrix}
K_{\theta_1} & -K_{\theta_2} \\
-K_{\theta_1} & K_{\theta_2}
\end{bmatrix}
\begin{bmatrix}
\theta_1 \\
\theta_2
\end{bmatrix}
+ 
\begin{bmatrix}
C_{\theta_1} & -C_{\theta_2}
\end{bmatrix}
\begin{bmatrix}
\dot{\theta}_1 \\
\dot{\theta}_2
\end{bmatrix}
+ 
\begin{bmatrix}
I_{1y} & 0 \\
0 & I_{2y}
\end{bmatrix}
\begin{bmatrix}
\ddot{\theta}_1 \\
\ddot{\theta}_2
\end{bmatrix}
\]

(50)

Figure 43  Representation of the rigid rings wheel plane rotational properties

\[
\begin{bmatrix}
\theta_1 \\
\theta_2 \\
\end{bmatrix}
\]

Figure 44  Representation of the longitudinal slip model belonging to the rigid ring model
5.2.3 Longitudinal tread and tire stiffness

The longitudinal tread and tire stiffness test are performed on the rigid ground only. Figure 44 shows the representation of the test case. Solving the equations of motion equation (51) can be derived. The calculation of the longitudinal reaction force requires two steps to be implemented. The first step is to calculate the force generated by the longitudinal slip model. The second step is to calculate the force generated in the sidewall which acts on the rim.

\[
\begin{bmatrix}
0 \\
-F_{ax}
\end{bmatrix} = 
\begin{bmatrix}
K_{bs} & -K_{bs} \\
-K_{bs} & K_{bs}
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2
\end{bmatrix} + 
\begin{bmatrix}
C_{bs} & -C_{bs} \\
-C_{bs} & C_{bs}
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2
\end{bmatrix} + 
\begin{bmatrix}
m_1 & 0 \\
0 & m_2
\end{bmatrix}
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2
\end{bmatrix}
\]

(51)

5.2.4 Lateral translational stiffness and damping

The lateral translational stiffness and damping test is presented in Figure 45. The force produced in the lateral direction is divided into two parts. The force generated in the tire belt in the lateral direction is calculated in the same fashion as in the longitudinal direction, as is the update of the states. While, the force generated in slip model, \( F_{sy} \), is calculated directly from the lateral slip angle, \( \alpha_y \), by multiplying with the cornering stiffness, \( k_f \). Equation (52) presents the solution for the equations of motion.

\[
\begin{bmatrix}
0 \\
-F_{ay}
\end{bmatrix} = 
\begin{bmatrix}
K_{by} & -K_{by} \\
-K_{by} & K_{by}
\end{bmatrix}
\begin{bmatrix}
y_1 \\
y_2
\end{bmatrix} + 
\begin{bmatrix}
C_{by} & -C_{by} \\
-C_{by} & C_{by}
\end{bmatrix}
\begin{bmatrix}
y_1 \\
y_2
\end{bmatrix} + 
\begin{bmatrix}
m_1 & 0 \\
0 & m_2
\end{bmatrix}
\begin{bmatrix}
\dot{y}_1 \\
\dot{y}_2
\end{bmatrix}
\]

(52)

Figure 45 Representation of the two rigid rings in lateral direction
5.2.5 Out of wheel plane rotational stiffness and damping

The out of plane rotational reaction moment consists of the same elements as for wheel plane with a parallel spring and damper. Figure 46 shows the representation of the rigid ring out of wheel plane rotational properties.

**Figure 46** Representation of the rigid rings out of wheel plane rotational properties

5.3 Simulink modelling

The simulink model has three different solving methodologies. The variation of a single tire model, this method the tire is disconnected from the complete vehicle. The rigid ground model and in this model applies to the entire vehicle. The soft soil ground model and this model is compatible with the full vehicle.

In the code function, the tires are implemented as state-space functions using Simulink’s S-Function API. The general nonlinear state space formulation is shown in equations (53)–(54). Where \( x \) represents the state vector, \( y \) represents the output vector, \( u \) represents the input vector, \( f \) represents the state equation, \( h \) represents the output equation.

\[
x(t) = f(t; x(t); u(t)) \tag{53}
\]

\[
y(t) = h(t; x(t); u(t)) \tag{54}
\]
6 Soil mixing/layering

In 2001, Kenneth used soil mixing to provide stabilisation of soft or loose soils. Kenneth indicated that the use of soil mixing is considered a modern technology in the USA. Soil mixing was first introduced by Intrusion-Prepakt, Inc. of Cleveland Ohio in the 1950s as ‘intrusion grout mixed-in-place piles’ (Norman et al., 1954). The Swedes used a mixed in-place lime stabilisation process in the late 1960s and early 1970s (Christopher and Jasperse, 1989). Since 1970s, the Japanese and Scandinavians continue to refine the soil mixing technology in various foundation applications. Xia (2011) layered FEA soil to predict soil characteristics better. The top layer of soil is assumed to exhibit elastoplastic mechanical behaviour and modelled using Drucker-Prager/Cap design. The bottom layer soil is relatively stiffer and is assumed to deform elastically. Xia simulated the footprint and predicted the soil compaction on agricultural soil. Also, Xia computed the frictional coefficient of the tire-terrain interface.

Soil mixing is used in several applications as a more economical or enhanced performance alternative to some traditional and other geosystem methods. Besides, soil mixing is used in settlement control of soft soils supporting embankments. In-situ soil mixing is achieved using either single shaft or various shaft drilling equipment.

For off-road vehicle design, excellent tire manoeuvrability and little compaction on terrain are always strongly desired. It is reported that simulations to demonstrate how the tire-terrain interaction model can be used to predict soil compaction and tire manoeuvrability in the field of terramechanics (Xia, 2011). The majority of research effort was focused on field tests (Eguchi and Muro, 2007; Nguyen et al., 2008). Fervers (2004) recognised a flexible tire model and applied it to study tire/soil interaction using finite element. Nakashima et al. (2010) established soil-tire contact model based on dynamic finite element-DE method.

6.1 Purpose of soil mixing/layering

The performance of tire mobility is directly related to the inflation pressure, tire contact area, soil properties, tire/terrain interface properties and vehicle load. Operating off-road vehicle on natural terrain generates soil compaction. Soil compaction is a mechanical mechanism by which soil particles are pressed together by the momentary application of loads through rolling tires or wheels and eventually increases the bulk density of soils. Soil properties are a critical parameter in predicting tire-soil interaction. In off-road application, terrain may be a mixture of several soils, on a rainy day the off-road terrain could be wet sand or wet clay. It is thus important to accurately model mixed/layered soil situations. SPH soil mixing is new research that has not been done before for the purpose of tire-terrain interaction.

6.2 Methodology and techniques

During this research, different SPH material will be layered on top of each other to model several conditions. A similar analysis method as mentioned in Section 2.2 for soil calibration will be implemented. The thickness of each soil layer will be determined depending on the application and assumed weather conditions. The SPH layering will include rainy weathers, clayey with sand and sand with gravel. Figure 47(a) shows a pressure-sinkage test setup for a three-layered soil of water, sand and clayey. After
running the simulation, the results are reported in Figure 47(b). The counter used is just a colouring scheme to distinguish between soil material particles, more details about this test is presented in the next section.

Figure 47  Pressure-sinkage test for three-layered soil mixture, (a) pressure-sinkage test setup (b) pressure-sinkage test at 200 KPa pressure (see online version for colours)
Figure 48  Pressure-sinkage test for three-layered soil mixture with SPH density counter at different pressures, (a) pressure-sinkage initial test setup (b) pressure-sinkage at 100 KPa plate pressure (c) pressure-sinkage at 300 KPa plate pressure (see online version for colours)
Figure 48  Pressure-sinkage test for three-layered soil mixture with SPH density counter at different pressures, (a) pressure-sinkage initial test setup (b) pressure-sinkage at 100 KPa plate pressure (c) pressure-sinkage at 300 KPa plate pressure (continued) (see online version for colours)

6.2.1 Validation and calibration

Validation of each soil is done separately using pressure-sinkage and shear-strength tests. The validated soils are layered on top of each other and thus does not need validation. The terrain properties of mixed soil will be determined using pressure-sinkage and shear strength tests. The fitting of layers soils among validated soil will be done. Figure 48(a) shows the initial pressure-sinkage test before applying pressure. The counter used here is the density counter, it is noticed that the water has the least density than the sand and the clayey has the highest density. Before starting the test, the soil layers are not mixed. After starting the test, the pressure is applied to the top plate as shown in Figure 48(b). The density of the SPH particles starts changing where all the terrains start increasing in density. Some particles on the top pf the plate loses density after being compressed and released. Figure 48(c) shows the plate after reaching the sand layer. The density in the water layer returns to its normal with few inner changes. The density in the sand increase and the density of the clayey layer decrease due to the mixing of clayey with sand particles.
6.2.2 Output

The expected output of this research is the prediction of the rolling resistance and traction characteristics over mixed/layered terrains. This will allow for a better understanding of realistic terrains. The terrain properties can also be predicted at different wetness percentages. This will allow for the addition of new soil properties to the library already created from Wong’s (2008) terrain materials. A sensitivity analysis for the rolling resistance will be performed. The analysis will include different soil models and soil mixed models to identify the effect of rain and snow on off-road terrains such as clayey and sand.

7 Conclusions

The tire-terrain interaction performed in this study is done using a virtual environment simulation software Pam-Crash. Several researches developed tire models using different virtual performance software. Generally, the tire is modelled and validated using FEA technique and several materials. Soil models are modelled and calibrated using SPH technique. Several terrains are used in this research such as road, sand, clayey, water and snow. Tire-terrain interaction is defined and several contact parameters should be set. The tire-terrain model is then ready to be used.

The previous research done in the domain of tire-terrain interaction is presented. The previous work done to predict tire characteristics is investigated. Furthermore, the tire terrain models are used to compute interaction features such as rolling resistance, traction and steering characteristics. The interaction features are computed for different scenarios including the inflation pressure, vertical load and different road conditions. The rolling resistance is investigated for dry and wet surfaces and the results are presented in a conference paper. The cornering characteristics for different slip angles will be obtained and published in future work.

Previous work in computing the hydroplaning speed is presented. The equations developed by previous researches are indicated. The hydroplaning speed for the truck tire developed is also computed using the tire-terrain model developed. The hydroplaning test includes different conditions (inflation pressure, vertical load, water depth). Empirical equation was developed and presented in the course of work. A complete journal paper on the hydroplaning speed sensitivity analysis is submitted and waiting for publishing.

The soil mixing/layering models are still under development and investigation as this technique is considered as a first attempt and has not been done before. The soil models modelled and calibrated in previous work is used as layers to predicted the characteristics of soil mixture. The results of this work will be presented in future papers.

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References


