Performance Analysis of ACO-OFDM and DCO-OFDM using Bit and Power Loading in Frequency Selective Optical Wireless Channels

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Abstract— **In this paper, we present a detailed comparison of the performance of asymmetrically clipped optical OFDM (ACO-OFDM), ACO-OFDM with diversity combining, and DC-biased optical OFDM (DCO-OFDM) using a range of different bias levels. Comparisons are made for both an additive white Gaussian noise (AWGN) channel and for a typical frequency selective visible light communication channel. Adaptive bit loading is used to maximize the bit rate for the frequency selective channel using a target bit error rate of 10^{-3}. Clipping noise in DCO-OFDM unlike AWGN is added at the transmitter not the receiver. New analytical results are derived which include this effect in the bit-loading calculations. It is shown that diversity-combining alters the spectral distribution of the noise after equalization. The effect of this in bit loading is also analyzed. It is shown that for both the AWGN and the frequency selective channel ACO-OFDM with diversity combining is the most energy efficient at lower bit rates/normalized bandwidths but for higher data rates DCO-OFDM requires the smallest transmit power. For example, for the frequency selective channel for bit rates/normalized bandwidths up to 6, ACO-OFDM with diversity combining is the most power efficient. For higher data rates DCO-OFDM requires the smallest transmit power, but these data rates require very large constellation sizes, for example, the constellation size of 128 for DCO-OFDM for some subcarriers and 2048 for ACO-OFDM with diversity-combining.**

Keywords —asymmetrically clipped optical OFDM, DC-biased optical OFDM, frequency selective channel, intensity modulated direct detection OFDM, VLC, achievable data rate

I. INTRODUCTION

Orthogonal frequency-division multiplexing (OFDM) is a key technology for next generation optical systems because of its high spectral efficiency and its immunity to inter-symbol interference [1], [2]. A further important advantage is that power and bit-loading can be used to maximize the data rate in a frequency selective channel. In optical wireless applications such as visible light communication (VLC) systems, intensity modulation with direct detection (IM/DD) is the only practical solution due to the non-coherent characteristics of the light emitting diodes (LEDs) and so the transmitted signal must be unipolar. The most common unipolar OFDM techniques are DC biased OFDM (DCO-OFDM) [3] and asymmetrically clipped optical OFDM (ACO-OFDM) [4]. To create a unipolar signal DCO-OFDM adds a DC bias to the overall time domain signal, while in ACO-OFDM only the odd subcarriers are used to carry data. It was later shown that the performance of ACO-OFDM can be improved by up to 3 dB at high signal-to-noise ratios (SNR) using 'diversity combining' in the receiver [5]. This uses a non-linear process to recover information from the unused even subcarriers. In an additive white Gaussian noise (AWGN) channel the relative performance of ACO-OFDM and DCO-OFDM is a trade-off between power and spectral efficiencies. For low spectral efficiency, where only small constellations are used, ACO-OFDM requires less power, but at higher spectral efficiencies, DCO-OFDM outperforms ACO-OFDM.

Much of the theoretical work on the performance of OFDM in optical wireless systems has considered only AWGN channels [6-8]. However, experimental work shows that because of the characteristics of typical components, optical wireless channels tend to have a channel frequency response with low-pass characteristics [9, 10]. A major limitation of some of the theoretical literature is that it uses metrics based on the ratio of the variance of the signal to noise, or do not take account of the different relationships between variance and optical power of different modulation techniques [11-13]. While this is a key metric for RF systems, where the power of the signal is related to its variance, it gives misleading results for IM/DD systems particularly for VLC systems where the major limitation is the transmitted optical power and the relationship between transmitted optical power and the variance of the signal is very dependent on the modulation technique used. Because ACO-OFDM typically has a much larger variance for a given optical power, comparisons based on the variance often incorrectly conclude that DCO-OFDM outperforms ACO-OFDM.

When OFDM is used in a frequency selective channel, the overall transmission rate can be increased by using adaptive bit and power (i.e. variance) loading[1]. Bit loading involves the use of larger constellations on subcarriers with high received SNR and smaller constellations on low SNR subcarriers. To maximize the overall transmission rate, the transmit variance allocated to each subcarrier is also optimized. Many algorithms have been proposed for bit/variance allocation [14]. In this paper the Hughes-Hartogs algorithm is used [15] as it maximizes the overall data rate of a multicarrier system for a given target total transmit variance.

Applying the algorithm correctly to ACO-OFDM and DCO-OFDM VLC systems is difficult and many factors have to be taken into account. As a result there is very little work on this

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[1] Bit-loading was originally used for ADSL and wireless applications, where the variance of the signal corresponds to the power of the signal. In an IM/DD system the power corresponds to the mean (not the variance) of the transmitted signal, so in this paper the loading is discussed in terms of variance, not power.

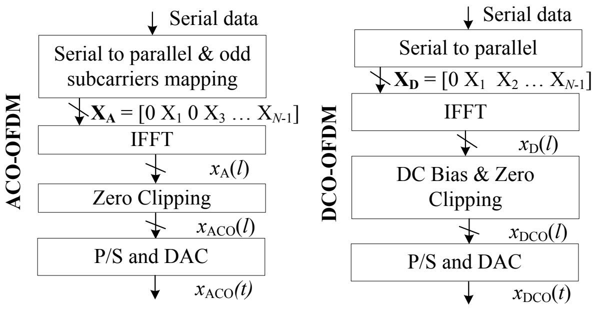

Fig. 1. ACO-OFDM, DC-ACO-OFDM, and DCO-OFDM transmitter

topic [16]. This is because VLC systems unlike RF are subject to a constraint on the average of the transmitted signal (not the variance) but the bit error rate (BER) depends on the variance of the received signal. Also, the clipping noise in a DCO-OFDM system is added at the transmitter, not the receiver so fades along with the channel [17]. Finally, when diversity combining is applied to ACO-OFDM, the nonlinear process in the receiver changes the spectrum of the noise at the decision point [18].

This paper makes a number of major contributions. It is the first paper to provide a comprehensive comparison of different IM/DD optical OFDM techniques in terms of the key parameters of transmitted optical power and bit rate/normalized bandwidth. This means that it is the first research to clearly show what the relative performance of the techniques will be in a VLC system subject to practical constraints. It is the first paper to apply adaptive bit loading to an ACO-OFDM using diversity combining. The paper also identifies and analyzes the key noise characteristics for the different systems and how they affect the bit loading algorithms. Finally because the conventional transmit power budget which is used in RF bit-loading cannot be directly applied the mathematical relationship between the bias in dB is calculated as $10\log_{10}(1+\mu)$ dB, the transmitted optical power and the signal variance is derived for each optical OFDM system.

II. Signals and Noise in ACO-OFDM and DCO-OFDM Systems

In this section we analyze in detail the relationship between the variance of the signal at the *receiver* to the *transmitted* optical power. We also analyze in detail the received noise taking into account the effect of clipping in DCO-OFDM and the effect of diversity-combining in ACO-OFDM. These results are required if bit and power loading are to be correctly applied to the IM/DD systems considered and are also required to evaluate their performance subject to a constraint on transmitted optical power.

Fig. 1 illustrates the transmitter structures for ACO-OFDM and DCO-OFDM. We consider a typical ACO-OFDM transmitter [4] where $X_A(k)$ is the value on the $X_A(k)$ k^{th} input of an N-point inverse fast Fourier transform (IFFT). The samples on the output of the IFFT are given by

$$x_A(l) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_A(k) e^{\frac{j2\pi nk}{N}}. \quad (1)$$

The continuous analog signal after clipping, serial-to-parallel (S/P) conversion and digital-to-analog (DAC) conversion is $x_{ACO}(t)$. It can be shown that the transmitted signal optical power is given by [8]

$$P_{opt,\text{ACO}} = E\{x_{\text{ACO}}(t)\} = \sqrt{E\{x_{\text{ACO}}(t)^2\}/\pi} = \sqrt{\sigma_A^2/4\pi}, \quad (2)$$

where σ_{A}^2 is the ACO-OFDM signal variance after the IFFT,

$$\sigma_{\text{A}}^2 = \frac{2}{N} \sum_{\substack{k=0 \\ k,\text{odd}}}^{N-1} E\left\{|X_A(k)|^2\right\} \quad (3)$$

and $E\left\{|X_A(k)|^2\right\}$ denotes the variance of the k^{th} subcarrier.

Similarly, for DCO-OFDM it can be shown that the transmitted optical power is given by

$$P_{opt,\text{DCO}} = E\{x_{\text{DCO}}(t)\} = \sqrt{\sigma_D^2}\left(\mu\left(1-Q(\mu)\right) + \frac{1}{\sqrt{2\pi}} e^{-\frac{\mu^2}{2}}\right), \quad (4)$$

where $x_{DCO}(t)$ is the signal after adding DC bias, clipping, DAC and S/P conversion, where σ_D^2 is the variance

$$\sigma_D^2 = \frac{1}{N} \sum_{k=0}^{N-1} E\{|X_D(k)|^2\} \quad (5)$$

and $E\left\{|X_D(k)|^2\right\}$ is the k^{th} subcarrier variance, $\mu = B_{DC}/\sqrt{\sigma_D^2}$ (B_{DC} is the DC bias), and $Q(\mu) = \left(1/\sqrt{2\pi}\right)\int_\mu^\infty e^{-x^2/2} dx$.

We consider the case where AWGN of single-sided spectral density N_o is added at the receiver. After low pass filtering the noise power is $\sigma_n^2 = N_0 B$, where B is the bandwidth (BW). This value can be used directly in the bit loading calculations for ACO-OFDM, the noise on ACO-OFDM is given by

$$\sigma_{n,\text{ACO}}^2(k) = \sigma_n^2 / |H(k)|^2, \quad (6)$$

where $H(k)$ is the channel response. However, when diversity combining is used the effect of the non-linear processing on the noise must be considered [18]. The noise power on each subcarrier is the sum of two components; the noise on odd subcarriers, which is colored noise due to the equalization and the noise on even subcarriers, which is evenly distributed among odd subcarriers due to the non-linear process of diversity combining [18]. Assuming zero forcing equalization followed by diversity combining with factor α, and that the noise due to the nonlinear process is negligible at high SNR [5], the noise power for ACO-OFDM with diversity combining at the receiver is given by

$$\sigma_{n,\text{Diversity}}^2(k) = \sigma_n^2 \left(\frac{1-\alpha}{|H_{odd}(k)|} + \frac{2}{N}\sum_{\substack{i=0 \\ i,even}}^{N-1} \frac{\alpha}{|H_{even}(i)|}\right)^2, \quad (7)$$

where H_{odd} and H_{even} are the channel response on the odd and even subcarriers, respectively.

For DCO-OFDM, clipping at the transmitter results in 'clipping noise' being added to the signal. In the general case where not all DCO-OFDM subcarriers have equal variance the clipping noise will be colored. However, our simulations have shown that this effect is negligible for the cases we consider. Because clipping noise is added at the transmitter, rather than the receiver and fades with the signal, its effect is different from AWGN of the same power. The noise on DCO-OFDM for each subcarrier is given by

$$\sigma_{n,\mathrm{DCO}}^2(k) = \sigma_n^2 \big/ |H(k)|^2 + \sigma_c^2, \tag{8}$$

where σ_c^2 is the clipping noise, which is defined as [8],

$$\sigma_c^2 = \sigma_D^2\left(1+\mu^2\left(Q(\mu)-Q^2(\mu)\right)-\frac{1}{2\pi}e^{-\mu^2} - \frac{\mu}{\sqrt{2\pi}}e^{-\frac{\mu^2}{2}}\left(1-2Q(\mu)\right)\right). \tag{9}$$

The most important parameter for bit-loading is the ratio of the subcarrier variance to the noise variance after zero-forcing equalization. For ACO-OFDM for the k^{th} subcarrier this ratio is,

$$\gamma_{\mathrm{ACO}}(k) = \frac{1}{4}E\left\{|X_{\mathrm{A}}(k)|^2\right\}\Big/\sigma_{n,\mathrm{ACO}}^2(k) \tag{10}$$

When diversity combining is used, the ratio becomes

$$\gamma_{\mathrm{ACO\ (Diversity)}}(k) = \frac{1}{4}E\left\{|X_{\mathrm{A}}(k)|^2\right\}\Big/\sigma_{n,\mathrm{Divesity}}^2(k). \tag{11}$$

In DCO-OFDM the ratio for the kth subcarrier is

$$\gamma_{\mathrm{DCO}}(k) = \beta^2 E\left\{|X_D(k)|^2\right\}\Big/\sigma_{n,\mathrm{DCO}}^2(k). \tag{12}$$

where $\beta = 1-Q(\mu)$, is the attenuation factor [8].

III. Adaptive bit and Power loading

To apply the Hughes-Hartogs algorithm, we calculate $\sigma_m^2(k)$ which is defined as the variance, $E\left\{|X_A(k)|^2\right\}$ for ACO-OFDM and $E\left\{|X_D(k)|^2\right\}$ for DCO-OFDM, which achieves the target bit error rate (BER) for modulation order, m, for the given channel, and for noise of variance σ_n^2. A table is created with the values of $\sigma_m^2(k)$ for each subcarrier and each modulation order. The values of $\sigma_m^2(k)$ are then used to calculate the difference in variance between each two consecutive modulation orders m_1 and m_2 for subcarrier k,

$$\zeta_{m_1-m_2}(k) = \sigma_{m_1}^2(k) - \sigma_{m_2}^2(k). \tag{13}$$

From (2), it can be seen that the total variance that corresponds to a unity optical power for ACO-OFDM is

$$\sigma_A^2 = \frac{2}{N}\sum_{\substack{k=0\\k,odd}}^{N-1}\sigma_{k,m}^2 = 4\pi\ . \tag{14}$$

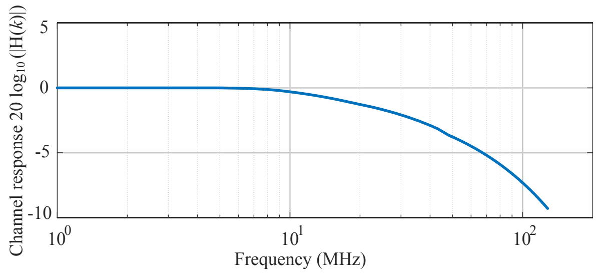

Fig. 2. Channel frequency response

While for DCO-OFDM from (5) the total variance will be

$$\sigma_D^2 = \frac{1}{N}\sum_{k=0}^{N-1}\sigma_m^2(k) = \left(\mu\left(1-Q(\mu)\right)+\frac{1}{\sqrt{2\pi}}e^{-\frac{\mu^2}{2}}\right)^{-2}. \tag{15}$$

For, unity optical power and 10 dB bias the total variance $\sigma_D^2 \approx 0.1111$, while for 13 dB bias $\sigma_D^2 \approx 0.0528$. Note that these are very small compared with the value of $4\pi = 12.5664$ for ACO-OFDM.

IV. Bit loading on frequency selective channel

The channel used for our comparison study is shown in Fig. 2. It is the response of an ideal first-order low pass filter with 35 MHz cutoff frequency, which was shown in [9] to approximate their practical optical wireless channel measurements. Without loss of generality the channel is normalized to unity gain at zero frequency so that the optical power at the input is equal to the optical power at the output.

The performance of DCO-OFDM with 7dB, 10dB and 13dB bias levels, ACO-OFDM, and ACO-OFDM with diversity-combining were evaluated over this channel using a range of QAM constellation sizes and for an FFT/IFFT size of 256. Because of the Hermitian symmetry constraint for IM/DD systems, and because the zeroth subcarrier is not used to transmit data, this limits the number of subcarriers that can be independently modulated in DCO-OFDM to 127. For ACO-OFDM because only odd subcarriers are used, the limit is 64. The cutoff frequency of the channel corresponds to a subcarrier index of 35 in both cases.

Fig. 3 (a) to (d) show examples of optimum bit allocations for DCO-OFDM with 10 dB and 13 dB bias, ACO-OFDM, and ACO-OFDM with diversity-combining, respectively. Note that the graphs are for the same total 5 bit rate/normalized BW. To achieve the same data rate and BER for the different systems the level of AWGN, N_0, was varied. These parameters are chosen to demonstrate clearly the important properties of bit-loading for optical OFDM.

For each example, because of the low pass characteristic of the channel, low index subcarriers are allocated the largest number of bits and the number of bits decreases with increasing subcarrier index. Within a group of subcarriers with the same constellation size, the variance allocated to the low indexed subcarrier is the smallest. This is because within the group this subcarrier has the highest channel gain.

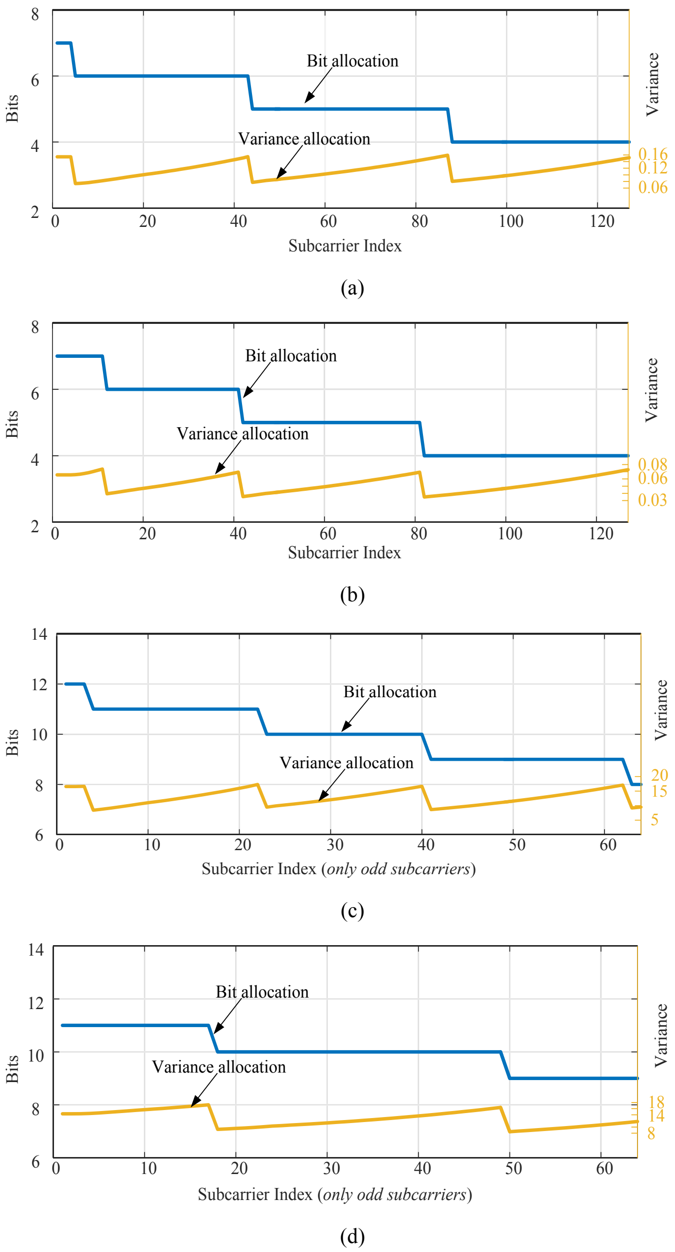

Fig. 3 Bits/variance allocation through subcarriers in: (a) DCO-OFDM with 10dB bias, (b) DCO-OFDM with 13dB bias, (c) ACO-OFDM, (d) ACO-OFDM with diversity combining at the receiver

Comparing the two DCO-OFDM examples with the two ACO-OFDM examples, much larger constellations are used for ACO-OFDM. This is because there are fewer subcarriers that can be independently modulated so each subcarrier has to carry more data. In contrast the variance values for ACO-OFDM are much larger than for DCO-OFDM. This is because for a given optical power the total variance is much greater for ACO-OFDM than DCO-OFDM see (14) and (15).

Fig. 3 (a) and (b) show the results for DCO-OFDM with a 10 dB bias and 13 dB bias, respectively. Increasing the bias decreases the clipping noise. Therefore, for 13 dB bias, as the clipping noise is less, from (5) the total noise power will depend more on channel frequency response than for 10 dB bias. As a result for 10 dB bias, the bit allocation changes from 6 bits to 4 bits over 83 subcarriers compared to 71 subcarriers with 13 dB bias. However, both masks have the same range of allocated bits, i.e. from 7 to 4 bits per subcarrier.

A similar effect can be seen for ACO-OFDM where the total noise power depends less on the channel response for ACO-OFDM with diversity combining than for ACO-OFDM.

V. Overall Performance Comparison

In the previous section we used variable noise levels for different optical OFDM systems to show more clearly aspects of bit-loading. In practice, the AWGN at the receiver and the transmitted optical power will be fixed and this is what is simulated in this section. In this case, bit and variance loading are used to maximize the data rate for a given channel. DCO-OFDM with 7dB, 10dB and 13dB biases, ACO-OFDM and ACO-OFDM with diversity combining are compared for an AWGN channel and for a frequency selective channel.

To compare the different modulation schemes in a way that takes into account the practical limitations in a VLC system we use the metric used in [7]which is a development of the work of [19]. Fig. 4 and Fig. 5 plot $E_{b(\text{opt})}/N_o$ at BER = 10^{-3} against different bit rate/normalized BW, where $E_{b(\text{opt})} = P_{\text{opt}}/r_b$, P_{opt} is the transmitted optical power and r_b is the total bit rate of the system. The use of $E_{b(\text{opt})}/N_o$ takes account of the differing relationships between signal variance and optical power for the different modulations, while the use of bit rate/normalized bandwidth allows the varying spectral efficiency to be treated in a comprehensive way.

Fig. 4 shows that over an AWGN channel, ACO-OFDM with diversity combining requires the lowest $E_{b(\text{opt})}/N_o$ for low bit rates, while DCO-OFDM with 13 dB bias is best for high bit rates. ACO-OFDM with diversity combining outperforms conventional ACO-OFDM for all data rates. At high data rates diversity combining gives an improvement of 2.5 dB, for lower data rates this falls to 1.5 dB. For DCO-OFDM the maximum bit rate that can be achieved for a given BER depends on the bias [7]. This is because clipping noise introduces a noise floor which limits the maximum size of constellation that can be supported for a given target BER.

For the results for the frequency selective channel given in Fig. 5, it is clear that the required $E_{b(\text{opt})}/N_o$ level to produce a certain bit rate for any system is higher than the level in AWGN channel. This is because of the higher attenuation at high frequencies. Up to 4.8 bit rate/normalized BW, ACO-OFDM with diversity combining gives the best performance. For higher bit rates, DCO-OFDM with a 10 dB or 13 dB bias is better. At this point, the maximum constellation sizes used is 128-QAM in DCO-OFDM and 2048-QAM in ACO-OFDM with diversity combining. The difference between ACO-OFDM and ACO-OFDM with diversity combining is consistent with the results obtained from AWGN channel, i.e. 2.5 dB improvement for high bit rates. When different DC bias levels are used for DCO-OFDM, small bias levels give better performance but with limited maximum bit rates. The maximum achievable bit rate for

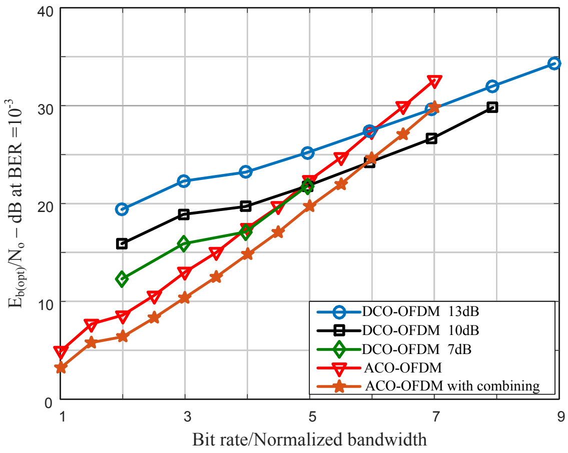

Fig. 4. $E_{b(opt)}/N_o$ at BER = 10^{-3} versus bit rate/normalized bandwidth for ACO-OFDM, ACO-OFDM with diversity combining at the receiver, and DCO-OFDM with 7dB, 10 dB and 13 dB bias over an AWGN channel

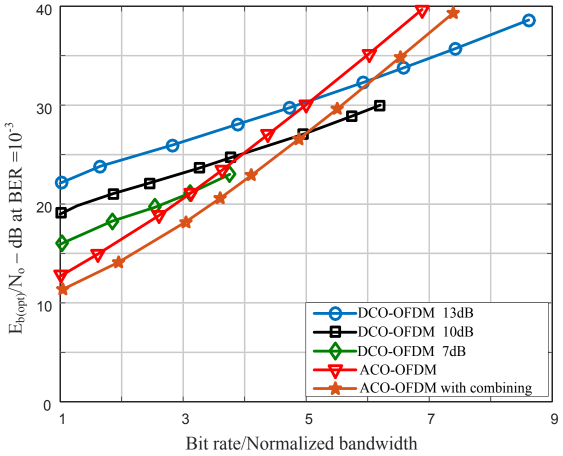

Fig. 5. $E_{b(opt)}/N_o$ at BER = 10^{-3} versus bit rate/normalized bandwidth for ACO-OFDM, ACO-OFDM with diversity combining at the receiver, and DCO-OFDM with 7dB, 10 dB and 13 dB bias over the frequency selective channel

7dB DCO-OFDM and 10 dB DCO-OFDM are 3.8 and 6.2 bit rate/normalized BW, respectively.

Simulations were also performed for reduced FFT/IFFT sizes down to 32, where the performance started to deteriorate and for higher FFT/IFFT size of 512, where the results were consistent.

VI. Conclusion

New theoretical and simulation results have been presented for ACO-OFDM, ACO-OFDM with diversity combining, and for DCO-OFDM with a range of bias levels for both flat and a frequency selective AWGN channels. For the flat channel, it is shown that applying diversity combining to ACO-OFDM gives an improvement of between 1.5dB and 2.5dB depending on the size of the constellations used. For DCO-OFDM, the results are consistent with earlier research which shows that ACO-OFDM outperforms DCO-OFDM at low values of bit rate/normalized bandwidth. For the frequency selective channel, adaptive bit and variance loading are used to maximize the data rate for a given average transmitted optical power and AWGN. To apply bit and variance loading correctly to these cases, the spectral properties of the noise after equalization and non-linear processing must be considered. Theoretical results are presented which show that applying diversity combining in an ACO-OFDM receiver changes the spectral properties of the noise. In DCO-OFDM systems because clipping noise is added at the transmitter rather than the receiver its contribution in a frequency selective channel is different from simple AWGN. As expected the performance of all systems was worse in a frequency selective channel than the flat channel. At low values of bit rate/normalized bandwidth ACO-OFDM with diversity combining had the best performance, while for higher values DCO-OFDM was better.

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