# Diversity Combining in Layered Asymmetrically Clipped Optical OFDM

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***Abstract*— We describe a new receiver for layered asymmetrical clipped optical orthogonal frequency division multiplexing (LACO-OFDM) in which diversity combining is used at each layer to improve the performance. In each layer of LACO-OFDM, some subcarriers carry data and others carry only the components which result from clipping. In the new receiver at each layer, a non-linear process is used to recover this clipped signal and use it for diversity combining. Simulation results show that the new receiver outperforms a conventional receiver by up to 2 dB when two layers are used. The performance of LACO-OFDM with diversity combining is compared with other forms of optical OFDM. The results show that at high bit rates LACO-OFDM with diversity combining is the most power efficient system, while ACO-OFDM with diversity combining but without layering is the best for low bit rates. In addition, the diversity combining receiver is compared to the recently proposed receiver for LACO-OFDM based on noise cancellation. Results show that diversity combining and noise cancellation have the same performance over flat channels. However, over frequency selective channels the diversity combining receiver has better performance.**

***Index Terms*— asymmetrically clipped optical OFDM, DC-biased optical OFDM, diversity combining, intensity modulated direct-detection OFDM, layered ACO-OFDM.**

## I. INTRODUCTION

Visible light communication (VLC) systems using light emitting diodes (LEDs) as transmitters always use intensity modulation and direct detection (IM/DD). Orthogonal frequency division multiplexing (OFDM) applied to VLC has many advantages [1] but as the intensity of light cannot be negative, a number of forms of OFDM which are non-negative have been developed. DC-biased optical OFDM (DCO-OFDM) is one simple technique [2]. However, the addition of a DC bias results in transmit power inefficiency. Asymmetrically clipped optical OFDM (ACO-OFDM) is a power efficient OFDM technique. Asymmetrically clipped optical OFDM (ACO-OFDM) is a power efficient OFDM technique. It has been shown that for many cases it requires less optical power per bit for a given bit error rate (BER) than DCO-OFDM. However ACO-OFDM is not spectrally efficient because only odd frequency subcarriers are independently modulated [3]. In ACO-OFDM, the even subcarriers carry only the clipping distortion, which results from clipping the original bipolar signal. Subsequently, a number of other related forms of optical OFDM for IM/DD have been developed in which not all of the available dimensions are independently modulated. For example, in pulse-amplitude-modulated discrete multitone modulation (PAM-DMT) only the real component of each subcarrier is modulated and the imaginary component is set to zero [4].

There has been extensive recent research into ways of improving the performance of ACO-OFDM and related techniques. One approach aims to improve the signal-to-noise ratio (SNR) of conventional ACO-OFDM by using diversity-combining [5] or noise cancellation [6] in the receiver. Diversity combining can be implemented either in the time domain or in the frequency domain. Application in the frequency domain allows the combination factor to be varied for each subcarrier, depending on the SNR of that subcarrier, and results in improved performance for frequency selective channels [7].

Another approach aims to improve the spectral efficiency by transmitting modified forms of ACO-OFDM, where the even subcarriers are used to carry additional data [8-12]. In asymmetrically clipped DC-biased optical OFDM (ADO-OFDM), an ACO-OFDM signal is transmitted on the odd subcarriers and a DCO-OFDM signal on the even subcarriers [8]. A similar system has been presented in [9] where the even subcarriers are used to transmit PAM-DMT signals. Layered ACO-OFDM (LACO-OFDM) is a system which exploits the even subcarriers by transmitting multiple 'layers' of ACO-OFDM [10], with similar approaches described in [11, 12].

In the improved ACO-OFDM techniques described above [5, 6, 8-12] , the ACO-OFDM clipping signal is exploited in three distinctly different ways. In diversity combining, non-linear processing of the even subcarriers results in a reconstructed version of the original ACO-OFDM signal which is ***added*** to the signal on the odd subcarriers to give an SNR improvement of up to 3 dB. In ADO-OFDM, LACO-OFDM and other related techniques the reconstructed signal is ***subtracted*** from the signal on the even subcarriers so that the additional data carried on the even subcarriers can be recovered. While in noise cancellation, the clipping noise structure is ***exploited*** to reduce the amount of noise, like diversity combining this gives up to 3dB improvement for a flat channel. Recently, a new receiver has been described for LACO-OFDM [13] which uses noise cancellation to improve the SNR, resulting in a system which combines both improved spectral

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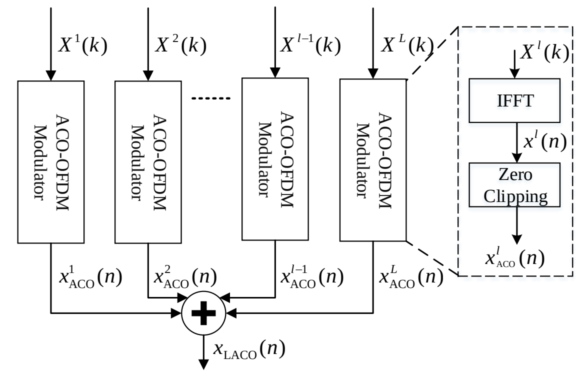


Fig. 1. Layered ACO-OFDM transmitter

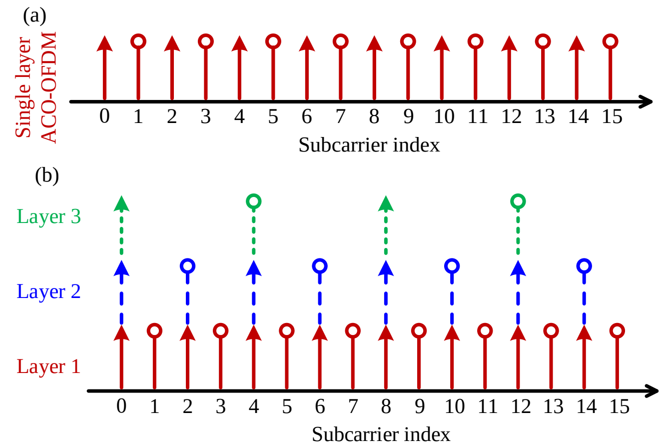


Fig. 2. Subcarriers mapping for: (a) ACO-OFDM, (b) LACO-OFDM using 3 layers

and power efficiency.

In this paper, we describe a new receiver design for LACO-OFDM based on diversity combining. The major contributions made in this paper are summarized as follows;

1. A novel LACO-OFDM receiver is developed where both diversity combining and LACO-OFDM approaches are combined to simultaneously improve the spectral efficiency and SNR of ACO-OFDM.
2. Two algorithms for diversity combining in LACO-OFDM are described. One where the combining occurs in the time domain and one where the combining occurs in the frequency domain.
3. Channel dependent combination factors for the LACO-OFDM with diversity combining receiver in the frequency domain has been derived and the advantage of applying has been described.
4. Various optical OFDM systems are compared in terms of transmitted optical power and bit rate/normalized bandwidth. The systems compared include conventional LACO-OFDM, LACO-OFDM with diversity combining, LACO-OFDM with noise cancellation, ACO-OFDM, DCO-OFDM and ADO-OFDM. This is the first paper to make such a comprehensive comparison.
5. Finally, this paper compares the diversity combining receiver against the recently described noise cancellation receiver for both frequency flat and frequency selective channels.

The rest of this paper is organized into the following sections; Section II presents the structure of the new receiver and describes it mathematically. In Section III, diversity combining in the frequency domain is described and Channel dependent combination factors are derived. Section IV presents simulation results for a range of optical OFDM systems over a flat channel. In Section V diversity combining and noise cancellation are compared for a frequency selective channel. Finally, the paper is concluded in Section VI.

## II. Layered ACO-OFDM with Diversity Combining

In this section, the new LACO-OFDM system using diversity combining is described and its performance in a flat channel analyzed.

### A. *Transmitter*

Fig. 1 shows the structure of an LACO-OFDM transmitter and the detail of one of the ACO-OFDM modulators which form the transmitter. In LACO-OFDM the first layer carries conventional ACO-OFDM in which initially data is mapped only to the odd frequency subcarriers and the even subcarriers are set to zero. The clipping process after the IFFT in the ACO-OFDM modulator results in clipping distortion which falls on the even subcarriers. Fig. 2 (a) shows the mapping of signal components to subcarriers in conventional ACO-OFDM. The circle symbols represent the odd subcarriers, which carry the data. The arrow symbols represent the clipping distortion which falls on the even subcarriers [3].

In LACO-OFDM some of the even subcarriers also carry layers of a modified form of ACO-OFDM. These even subcarriers are divided into subgroups and each subgroup of subcarriers carries a different layer.

In the following we will use the subscript $D$ to denote the data-carrying subcarriers at each layer and $C$ to denote the subcarriers which carry the clipping distortion for that layer. So the sets of Layer 1 subcarriers carrying data and clipping distortion are given by

$$K_{\mathrm{D}}^{1}=\{1,3,5,\cdots N-1\}, \tag{1}$$

$$K_{\mathrm{C}}^{1}=\{0,2,4,\cdots N-2\}. \tag{2}$$

respectively. Similarly the sets of Layer $l$ subcarriers carrying data and clipping distortion are given by

$$K_{\mathrm{D}}^{l}=\{1\times 2^{l-1},3\times 2^{l-1},5\times 2^{l-1},\cdots N-2^{l-1}\}, \tag{3}$$

$$K_{\mathrm{C}}^{l}=\{k-2^{l-1}:k\in K_{\mathrm{D}}^{l}\}. \tag{4}$$

The input to the Layer $l$ modulator, $\mathbf{X}^{l}$, is a complex vector with

$$X^{l}(k)=\begin{cases}X, & k\in K_{\mathrm{D}}^{l}\\ 0, & \text{otherwise}\end{cases} \tag{5}$$

where $X\in\Im_{\text{M-QAM}}$, $\Im_{\text{M-QAM}}$ is the M-QAM constellation space. To ensure a real-only output, $\mathbf{X}^{l}$ is constrained to have Hermitian symmetry. As shown in Fig.1 separate IFFTs are performed for each layer. Note that data on a layer is independent of the data on the other layers. The output of the IFFT for layer $l$ is

$$x^l(n)=\frac{1}{\sqrt{N}}\sum_{k=0}^{N-1}X^l(k)\exp\left(\frac{j2\pi nk}{N}\right), \quad (6)$$

and this is clipped at zero to give

$$x^l_{\mathrm{ACO}}(n)=\begin{cases}x^l(n), & x^l(n)>0\\ 0, & \text{otherwise}\end{cases}. \quad (7)$$

It can be shown that

$$x^l_{\mathrm{ACO}}(n)=\frac{1}{2}\left(x^l(n)+\left|x^l(n)\right|\right)=x^l_{\mathrm{D}}(n)+x^l_{\mathrm{C}}(n) \quad (8)$$

where $x^l(n)$ is the $l^{\text{th}}$ layer signal before clipping, $x^l_{\mathrm{C}}(n)$ is the $l^{\text{th}}$ layer clipping distortion

$$x^l_{\mathrm{C}}(n)=\left|x^l_{\mathrm{D}}(n)\right|, \quad (9)$$

and $x^l_{\mathrm{D}}(n)$ is the $l^{\text{th}}$ layer signal after clipping

$$x^l_{\mathrm{D}}(n)=\frac{1}{2}x^l(n). \quad (10)$$

Note that each of the layers is clipped separately. It is not the sum of the signals which is clipped. All the layers are then combined to give

$$x_{\mathrm{LACO}}(n)=\sum_{l=1}^{L}x^l_{\mathrm{ACO}}(n), \quad (11)$$

which is converted to analog before transmission. In the following, we denote the FFT of $\mathbf{x}^l_{\mathrm{D}}$ by $\mathbf{X}^l_{\mathrm{D}}$ and of $\mathbf{x}^l_{\mathrm{c}}$ by $\mathbf{X}^l_{\mathrm{c}}$, where $X^l_{\mathrm{D}}(k)$ are the components for the modulated subcarriers for the $l^{\text{th}}$ layer (after clipping) and $X^l_{\mathrm{C}}(k)$ are the components of the subcarriers affected by the $l^{\text{th}}$ layer clipping distortion.

Fig. 2 (b) shows the mapping of signal components to subcarriers in an LACO-OFDM symbol with three layers and also the components of the clipping distortion. Red represents Layer 1, blue represents Layer 2 and green represents Layer 3. The red, blue, and green circles represent the values of $X^1_{\mathrm{D}}(k)$, $X^2_{\mathrm{D}}(k)$, and $X^3_{\mathrm{D}}(k)$ respectively. Similarly red, blue, and green arrows represent the values of $X^1_{\mathrm{C}}(k)$, $X^2_{\mathrm{C}}(k)$, and $X^3_{\mathrm{C}}(k)$, respectively.

### *B. Receiver*

The operation of the new receiver depends on the structure and properties of the LACO-OFDM signal [10]. It involves combining two existing techniques: diversity combining and layering. In the following we first describe these techniques separately and then show how they can be combined.

*1) Diversity Combining:* Consider the mapping of signal components to subcarriers in conventional ACO-OFDM as shown in Fig. 2 (a). To perform diversity combining separate IFFTs are first performed on the odd and even subcarriers. To simplify the initial discussion we ignore the effect of noise. In this case

$$x_{\mathrm{D}}(n)=\frac{1}{\sqrt{N}}\sum_{k=0}^{N-1}X_{\mathrm{D}}(k)\exp\left(\frac{j2\pi nk}{N}\right), \quad (12)$$

$$x_{\mathrm{C}}(n)=\frac{1}{\sqrt{N}}\sum_{k=0}^{N-1}X_{\mathrm{C}}(k)\exp\left(\frac{j2\pi nk}{N}\right). \quad (13)$$

The two signals are then combined to give

$$\widehat{x}_{\mathrm{D}}(n)=(1-\alpha)x_{\mathrm{D}}(n)+\alpha\times\mathrm{sign}\left(x_{\mathrm{D}}(n)\right)\times x_{\mathrm{C}}(n). \quad (14)$$

where $\widehat{x}_{\mathrm{D}}(n)$ is the signal after diversity combining and $\alpha$ is the combination factor. Diversity combining has been shown to improve the overall performance of ACO-OFDM by up to 3dB. This is because the noise on the odd and even subcarriers is independent [5].

*2) LACO-OFDM Receiver:* To explain how the layers are recovered in an LACO-OFDM receiver we consider the three-layer LACO-OFDM shown in Fig 2 for the case of a flat AWGN channel. The received signal in the time-domain is

$$y(n)=x_{\mathrm{LACO}}(n)+w(n), \quad (15)$$

where $w(n)$ is the noise. Transforming this to the discrete frequency-domain gives

$$\begin{aligned}Y(k)&=\frac{1}{\sqrt{N}}\sum_{n=0}^{N-1}y(n)\exp\left(\frac{-j2\pi nk}{N}\right)\\&=\sum_{l=1}^{3}X^l_{\mathrm{D}}(k)+\sum_{l=1}^{3}X^l_{\mathrm{C}}(k)+W(k).\end{aligned} \quad (16)$$

In LACO-OFDM the different layers are recovered in sequence starting with Layer 1. From Fig 2 (b) it can be seen that no clipping distortion falls on the data-carrying odd subcarriers, so the Layer 1 data can be estimated directly from these subcarriers using

$$\hat{X}^1_{\mathrm{D}}(k)=\frac{1}{2}\underset{X\in\Im_{\text{M-QAM}}}{\arg\min}\left\|2\breve{X}^1_{\mathrm{D}}(k)-X\right\|,\ k\in K^1_{\mathrm{D}}. \quad (17)$$

where

$$\breve{X}^1_{\mathrm{D}}(k)=Y(k)=X^1_{\mathrm{D}}(k)+W(k),\ k\in K^1_{\mathrm{D}}. \quad (18)$$

Fig 2 (b) also shows that the only clipping distortion affecting the data-carrying in Layer 2 comes from Layer 1, not from higher layers.

$$Y(k)=X^2_{\mathrm{D}}(k)+X^1_{\mathrm{C}}(k)+W(k),\ k\in K^2_{\mathrm{D}}, \quad (19)$$

So to recover the data symbols transmitted on the second layer, only the clipping distortion from the first layer must be estimated and removed. This is achieved by inputting the estimated Layer 1 data given by (17) to an IFFT to give

$$\hat{x}^1_{\mathrm{D}}(n)=\frac{1}{\sqrt{N}}\sum_{k\in K^1_{\mathrm{D}}}\hat{X}^1_{\mathrm{D}}(k)\exp\left(\frac{j2\pi nk}{N}\right), \quad (20)$$

then estimating the clipping distortion from Layer 1 using

$$\hat{X}^1_{\mathrm{C}}(k)=\frac{1}{\sqrt{N}}\sum_{n=0}^{N-1}\hat{x}^1_{\mathrm{C}}(n)\exp\left(\frac{-j2\pi nk}{N}\right). \quad (21)$$

And substituting (9) gives

$$\hat{X}^1_{\mathrm{C}}(k)=\frac{1}{\sqrt{N}}\sum_{n=0}^{N-1}\left|\hat{x}^1_{\mathrm{D}}(n)\right|\exp\left(\frac{-j2\pi nk}{N}\right) \quad (22)$$

A similar process is used to recover all the layers. A key feature of the mapping of data to layers in LACO-OFDM is that clipping distortion from a given layer only affects the subcarriers in higher layers not in lower layers. The algorithm for a generalized LACO-OFDM receiver is shown in Table I.

*3) LACO-OFDM Diversity Combining Receiver*: The new

TABLE I
LAYERED ACO-OFDM RECEIVER ALGORITHM

**Input**: received values $Y(k)$

**Output**: $\hat{X}_{\mathrm{D}}^{l}(k)$

1: **for** $l=1:L$ **do**

2: **if** $l=1$

$\breve{X}_{\mathrm{D}}^{1}(k)=Y(k),\ k\in K_{\mathrm{D}}^{1}$;

**else**

$\breve{X}_{\mathrm{D}}^{l}(k)=Y(k)-\sum_{i=1}^{l-1}\hat{X}_{\mathrm{C}}^{i}(k),\ k\in K_{\mathrm{D}}^{l}$;

**end if**

3: $\hat{X}_{\mathrm{D}}^{l}(k)=\frac{1}{2}\arg\min_{X\in\Im_{\text{M-QAM}}}\left\|2\breve{X}_{\mathrm{D}}^{l}(k)-X\right\|,\ k\in K_{\mathrm{D}}^{l}$;

4: $\hat{x}_{\mathrm{D}}^{l}(n)=\frac{1}{\sqrt{N}}\sum_{k\in K_{\mathrm{D}}^{l}}\hat{X}_{\mathrm{D}}^{l}(k)\exp\left(\frac{j2\pi nk}{N}\right)$, ;

5: $\hat{X}_{\mathrm{c}}^{l}(k)=\frac{1}{\sqrt{N}}\sum_{n=0}^{N-1}\left|\hat{x}_{\mathrm{A}}^{l}(n)\right|\exp\left(\frac{-j2\pi nk}{N}\right)$,;

6: **end for**

7: **return** $\hat{X}_{\mathrm{D}}^{l}(k)$;

TABLE II
DIVERSITY COMBINING RECEIVER ALGORITHM

**Input**: received values $Y(k)$, $\breve{X}_{\mathrm{D}}^{l}(k)$ and $\hat{X}_{\mathrm{D}}^{l}(k)$

**Output**: $\tilde{X}_{\mathrm{D}}^{l}(k)$ the symbols after diversity combining

1: **for** $l=L:1$ **do**

2: **if** $l=L$

$\tilde{X}_{\mathrm{C}}^{L}(k)=Y(k)-\sum_{i=1}^{L-1}\hat{X}_{\mathrm{C}}^{i}(k),\ k\in K_{\mathrm{C}}^{L}$,

**else**

$\tilde{X}_{\mathrm{C}}^{l}(k)=Y(k)-\left(\sum_{\substack{i=1\\ i\neq l}}^{L}\hat{X}_{\mathrm{C}}^{i}(k)+\sum_{i=l+1}^{L}\tilde{X}_{\mathrm{D}}^{i}(k)\right),\ k\in K_{\mathrm{C}}^{l}$,

**end if**

3: $\tilde{x}_{\mathrm{C}}^{l}(n)=\frac{1}{\sqrt{N}}\sum_{k\in K_{\mathrm{C}}^{l}}\tilde{X}_{\mathrm{C}}^{l}(k)\exp\left(\frac{-j2\pi nk}{N}\right)$. ;

4: $\widehat{x}_{\mathrm{D}}^{l}(n)=(1-\alpha)\breve{x}_{\mathrm{D}}^{l}(n)+\alpha\times\mathrm{sign}\left(\breve{x}_{\mathrm{D}}^{l}(n)\right)\times\tilde{x}_{\mathrm{C}}^{l}(n)$;

5: $\widehat{X}_{\mathrm{D}}^{l}(k)=\frac{1}{\sqrt{N}}\sum_{n=0}^{N-1}\widehat{x}_{\mathrm{D}}^{l}(n)\exp\left(\frac{-j2\pi nk}{N}\right)$, ;

6: $\tilde{X}_{\mathrm{D}}^{l}(k)=\frac{1}{2}\arg\min_{X\in\Im_{\text{M-QAM}}}\left\|2\widehat{X}_{\mathrm{D}}^{l}(k)-X\right\|,\ k\in K_{\mathrm{D}}^{l}$;

7: **end for**

8: **return** $\tilde{X}_{\mathrm{D}}^{l}(k)$;

diversity combining receiver uses both diversity combining and layering techniques.

The new receiver begins by performing all of the steps described above for the LACO-OFDM receiver without diversity combining, except that the values of $\hat{X}_{\mathrm{C}}^{l}(k)$ and $\hat{X}_{\mathrm{D}}^{l}(k)$ are retained for later use. It then begins applying diversity combining starting at the highest layer and working down through the layers. A key aspect of the operation of the diversity combining algorithm is that the *recovered* values of $X_{\mathrm{C}}^{l}(k)$ denoted by $\tilde{X}_{\mathrm{C}}^{l}(k)$ not the estimated values, $\hat{X}_{\mathrm{C}}^{l}(k)$ are used in the diversity combining stage. This is important as the improvement in performance achieved by diversity combining depends on the degree of correlation of the noise in the different signals that are combined. Diversity combining results in new more accurate estimates of the data on each layer given by $\tilde{X}_{\mathrm{D}}^{l}(k)$, so these rather than $\hat{X}_{\mathrm{D}}^{l}(k)$ are used in the later stages of the algorithm.

To explain the algorithm we will use the example of the three layer LACO-OFDM signal shown in Fig. 2 (b). To apply diversity combining to Layer 3, $X_{\mathrm{C}}^{3}(k)$ must be recovered for $k\in K_{\mathrm{C}}^{3}$ using

$$\tilde{X}_{\mathrm{C}}^{3}(k)=Y(k)-\sum_{i=1}^{2}\hat{X}_{\mathrm{C}}^{i}(k),\ \ k\in K_{\mathrm{C}}^{3}, \tag{23}$$

This is then used to derive the more accurate estimate of the data $\tilde{X}_{\mathrm{D}}^{3}(k)$ using the following steps. First, the corresponding time-domain signal for the third layer recovered clipping distortion is

$$\tilde{x}_{\mathrm{C}}^{3}(n)=\frac{1}{\sqrt{N}}\sum_{k\in K_{\mathrm{C}}^{3}}\tilde{X}_{\mathrm{C}}^{3}(k)\exp\left(\frac{-j2\pi nk}{N}\right). \tag{24}$$

Then, the recovered clipping distortion is combined with the recovered data symbols signal $\breve{x}_{\mathrm{D}}^{3}(n)$, the combining is given by

$$\widehat{x}_{\mathrm{D}}^{3}(n)=(1-\alpha)\breve{x}_{\mathrm{D}}^{3}(n)+\alpha\times\mathrm{sign}\left(\breve{x}_{\mathrm{D}}^{3}(n)\right)\times\tilde{x}_{\mathrm{C}}^{3}(n). \tag{25}$$

The symbols after diversity combining are given by

$$\widehat{X}_{\mathrm{D}}^{3}(k)=\frac{1}{\sqrt{N}}\sum_{n=0}^{N-1}\widehat{x}_{\mathrm{D}}^{3}(n)\exp\left(\frac{-j2\pi nk}{N}\right), \tag{26}$$

Finally, a decision-directed process is then applied to $\widehat{X}_{\mathrm{D}}^{3}(k)$ to generate

$$\tilde{X}_{\mathrm{D}}^{3}(k)=\frac{1}{2}\arg\min_{X\in\Im_{\text{M-QAM}}}\left\|2\widehat{X}_{\mathrm{D}}^{3}(k)-X\right\|,\ \ k\in K_{\mathrm{D}}^{3} \tag{27}$$

and $\tilde{X}_{\mathrm{D}}^{l}(k)$ is finally used to retrieve the transmitted data.

This algorithm can be extended to more layers using the generalized algorithm described in Table II.

## III. DIVERSITY COMBINING IN THE FREQUENCY DOMAIN

The previous sections considered the application of diversity combining in the time domain. We now describe ACO-OFDM with diversity combining in the frequency domain [7] and show how its use can be extended to LACO-OFDM. We then derive the optimum combination factor for each subcarrier as a function of SNR.

### A. *Frequency domain Receiver design*

Applying diversity combining in the time domain results in the same combination factor, $\alpha$, being used for each subcarrier **Error! Reference source not found.**, but this is only optimum if all subcarriers have the same received SNR, for example in a flat AWGN channel. However, to take full advantage of diversity combining in a frequency selective channel a different combination factor should be used for each subcarrier [7].

When different combination factors are used for different subcarriers and different layers in LACO-OFDM the combined signal in the frequency domain is given by

$$\hat{X}_{\mathrm{D}}^{l}(k)=\left(1-\alpha^{l}(k)\right)\breve{X}_{\mathrm{D}}^{l}(k)+\alpha^{l}(k)\frac{1}{\sqrt{N}}\sum_{n=0}^{N-1}\mathrm{sign}\left(\breve{x}_{\mathrm{D}}^{l}(n)\right)\times\tilde{x}_{\mathrm{C}}^{l}(n)\exp\left(\frac{-j2\pi nk}{N}\right) \tag{28}$$

where $\alpha^{l}(k)$ is the combination factor for the $k^{\mathrm{th}}$ subcarrier at the $l^{\mathrm{th}}$ layer.

### B. *Channel dependent combination factor*

We now derive channel-dependent combination factors $\alpha^{l}(k)$. The simulation results in Section V show that these give significantly better results than the use of a fixed combination factor. An analytical derivation of the optimum combination factors would be intractable due to the multiple non-linear operations involved in a LACO-OFDM diversity combining receiver. In this analysis we ignore the effect of the 'decision noise' which results from errors in data detection at the earlier stages of the algorithm. As the high SNR region is the region of interest this is a reasonable approximation. With these assumptions and for a frequency selective channel, the received signal in the frequency domain is given by

$$Y(k)=H(k)X_{\mathrm{LACO}}(k)+W(k), \tag{29}$$

where $H(k)$ is the channel frequency response for the $k^{\mathrm{th}}$ subcarrier. The recovered data and clipping distortion can be expressed as,

$$\breve{X}_{\mathrm{D}}^{l}(k)=H_{\mathrm{D}}^{l}(k)X_{\mathrm{D}}^{l}(k)+W_{\mathrm{D}}^{l}(k) \tag{30}$$

$$\tilde{X}_{\mathrm{C}}^{l}(k)=H_{\mathrm{C}}^{l}(k)X_{\mathrm{C}}^{l}(k)+W_{\mathrm{C}}^{l}(k) \tag{31}$$

where $H_{\mathrm{D}}^{l}(k)=H(k),\ k\in K_{\mathrm{D}}^{l},\ H_{\mathrm{C}}^{l}(k)=H(k),\ k\in K_{\mathrm{C}}^{l}$, $W_{\mathrm{D}}^{l}(k)=W(k),\ k\in K_{\mathrm{D}}^{l}$, and $W_{\mathrm{C}}^{l}(k)=W(k),\ k\in K_{\mathrm{C}}^{l}$.

Note that $W_{\mathrm{D}}^{l}(k)$ and $W_{\mathrm{C}}^{l}(k)$ are independent Gaussian noise components because they are the FFT of the AWGN noise components $w_{\mathrm{D}}^{l}(k)$ and $w_{\mathrm{C}}^{l}(k)$, respectively. A single tap equalizer is used at the receiver. After the equalizer the noise components on the data-carrying subcarriers and the subcarriers carrying clipping distortion are given by

$$Z_{\mathrm{D}}^{l}(k)=W_{\mathrm{D}}^{l}(k)/H_{\mathrm{D}}^{l}(k),\ k\in K_{\mathrm{D}}^{l}, \tag{32}$$

$$Z_{\mathrm{C}}^{l}(k)=W_{\mathrm{C}}^{l}(k)/H_{\mathrm{C}}^{l}(k),\ k\in K_{\mathrm{C}}^{l}, \tag{33}$$

respectively. By substituting (30) and (31) into (28)

$$\begin{aligned}\hat{X}_{\mathrm{D}}^{l}(k)&=\left(1-\alpha^{l}(k)\right)\left(X_{\mathrm{D}}^{l}(k)+Z_{\mathrm{D}}^{l}(k)\right)\\&\quad+\alpha^{l}(k)\left(X_{\mathrm{D}}^{l}(k)+\bar{Z}_{\mathrm{C}}^{l}(k)\right),\\&=X_{\mathrm{D}}^{l}(k)+\left(1-\alpha^{l}(k)\right)Z_{\mathrm{D}}^{l}(k)+\alpha^{l}(k)\bar{Z}_{\mathrm{C}}^{l}(k).\end{aligned} \tag{34}$$

where

$$\bar{Z}_{\mathrm{C}}^{l}(k)=\frac{1}{\sqrt{N}}\sum_{n=0}^{N-1}\mathrm{sign}\left(\breve{x}_{\mathrm{D}}^{l}(n)\right)\times z_{\mathrm{C}}^{l}(n)\exp\left(\frac{-j2\pi nk}{N}\right). \tag{35}$$

After the equalizer, the noise variance on the data-carrying subcarriers for the $l^{\mathrm{th}}$ layer is colored and is given by

$$\sigma_{l,\mathrm{D}}^{2}(k)=\frac{\sigma_{n}^{2}}{\left|H_{\mathrm{D}}^{l}(k)\right|^{2}} \tag{36}$$

The noise on subcarriers carrying the clipping distortion is also colored, However, because of the nonlinear operation used in diversity combining (time domain multiplication by the sign signal), the noise component resulting from the unused subcarriers for the $l^{\mathrm{th}}$ layer is white with variance given by [14]

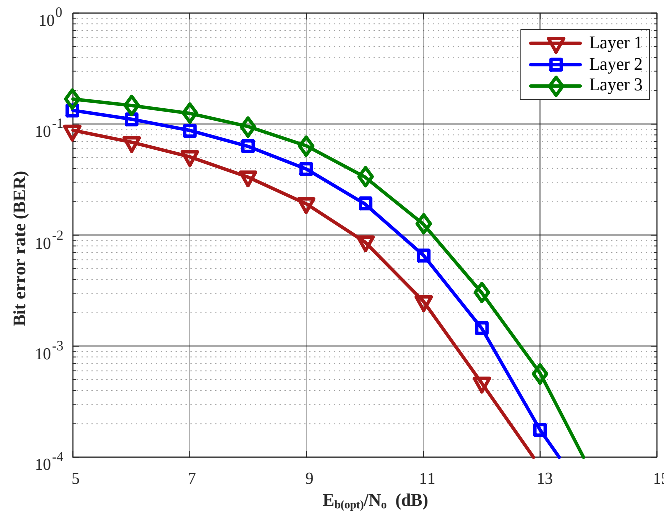


Fig. 5 $E_{b(opt)}/N_o$ versus BER on each layer of the LACO-OFDM with diversity combining with three layers over an AWGN channel using 16-QAM

$$\sigma_{l,\mathrm{C}}^{2}(k)=\sigma_{n}^{2}\left(\frac{2^{l}}{N}\sum_{i=0}^{N/2^{l}-1}\frac{1}{\left|H_{\mathrm{C}}^{l}(i)\right|}\right)^{2}, \tag{37}$$

where

$\sigma_{n}^{2}$ is the noise variance of $Z(k)$, $\sigma_{l,\mathrm{D}}^{2}(k)$ and $\sigma_{l,\mathrm{C}}^{2}(k)$ are noise variances for $Z_{\mathrm{D}}^{l}(k)$ and $\bar{Z}_{\mathrm{C}}^{l}(k)$, respectively. The channel-dependent combination factor $\alpha$ for the $k^{\mathrm{th}}$ subcarrier at the $l^{\mathrm{th}}$ layer is chosen such that it maximize the SNR for $X_{\mathrm{D}}^{l}(k)$ [7].

$$\begin{aligned}\alpha^{l}(k)&=\arg\max_{\alpha\in[0-1]}\left(\frac{E\left\{\left|X_{\mathrm{D}}^{l}(k)\right|^{2}\right\}}{(1-\alpha)\sigma_{l,\mathrm{D}}^{2}(k)+\alpha\sigma_{l,\mathrm{C}}^{2}(k)}\right),\\\alpha^{l}(k)&=\frac{\sigma_{l,\mathrm{D}}^{2}(k)}{\sigma_{l,\mathrm{D}}^{2}(k)+\sigma_{l,\mathrm{C}}^{2}(k)}.\end{aligned} \tag{38}$$

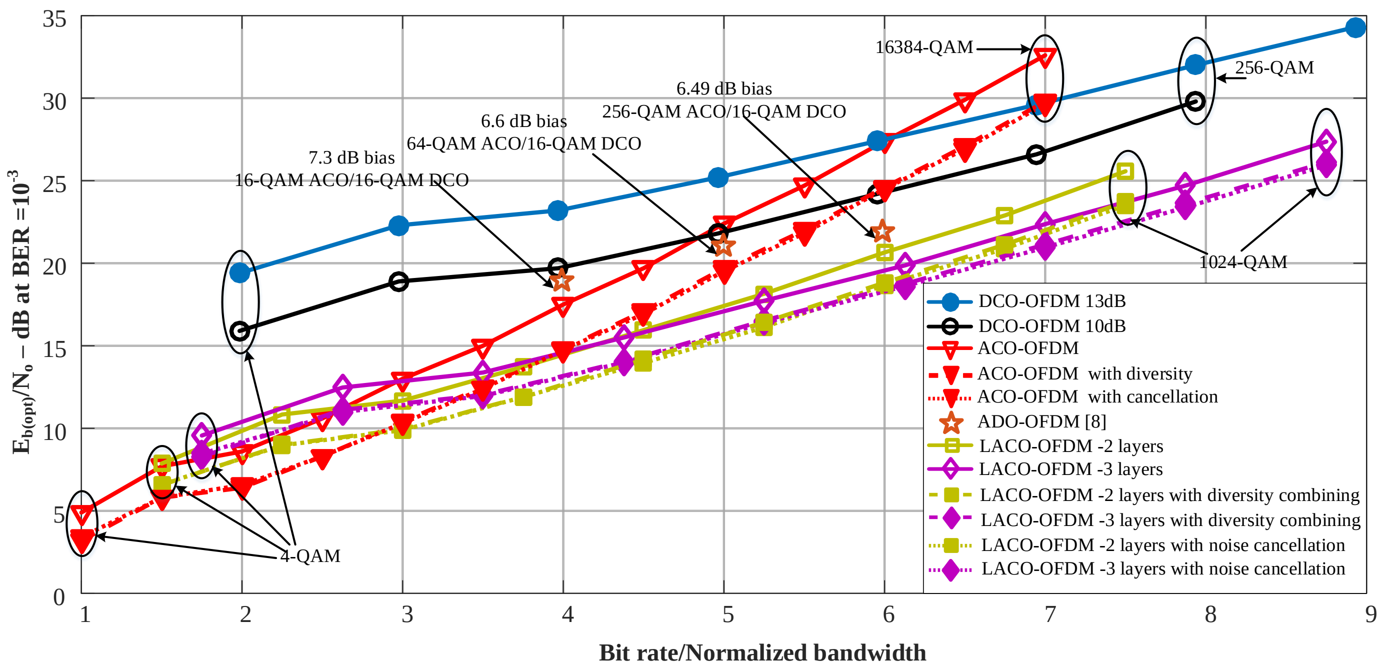


Fig. 6 $E_{b(\text{opt})}/N_o$ at BER = $10^{-3}$ versus bit rate/normalized bandwidth for DCO-OFDM with 10 dB and 13 dB bias, ACO-OFDM, ACO-OFDM with diversity combining, ACO-OFDM with noise cancellation, ADO-OFDM (only the best three points), LACO-OFDM with two and three layers, LACO- OFDM with diversity combining with two and three layers, LACO- OFDM with noise cancellation with two and three layers over an AWGN channel

$E\left\{\left|X_{\mathrm{D}}^{l}(k)\right|^{2}\right\}$ is the variance on the $k^{\text{th}}$ subcarrier. By substituting (36) and (37) into (38)

$$\alpha^{l}(k)=\frac{1}{1+\left|H_{\mathrm{D}}^{l}(k)\right|^{2}\left(\frac{2^{l}}{N}\sum_{i=0}^{N/2^{l}-1}\frac{1}{\left|H_{\mathrm{C}}^{l}(i)\right|}\right)^{2}}. \quad (39)$$

Although the complexity of the diversity-combining receiver is increased compared to the conventional receiver, the system performance is significantly increased and there are no changes required to be made at the transmitter.

## IV. System Performance for a Flat Channel

Simulations were performed for ACO-OFDM, LACO-OFDM, and LACO-OFDM with diversity combining with two and three layers for an AWGN channel and a range of constellation sizes. An FFT/IFFT size of 256 was used.

Fig. 4 shows $E_{b(\text{opt})}/N_o$ versus bit error rate (BER) for 16-QAM, where $E_{b(\text{opt})}=P_{\text{opt}}/b$, $P_{\text{opt}}$ is the optical power of the transmitted signal and $b$ is the bit rate. $P_{\text{opt}}$ is set to unity for all systems. The new diversity-combining LACO-OFDM receiver always outperforms conventional LACO-OFDM, with the greatest improvement occurring for fewer layers. Note that the data rate depends on the number of layers. Taking into account the requirement for Hermitian symmetry, in ACO-OFDM the number of independently modulated subcarriers is 64 while for LACO-OFDM with two and three layers, the values are 96 and 112 respectively. To explore the reasons for the change in performance with the number of layers, the BER for each of the layers was calculated. See Fig.5.

Fig. 5. shows the BER on each layer for three-layer LACO-OFDM with diversity combining. Layer 1 has the best BER for high SNRs and Layer 3 has the worst performance as it is, more affected by the accumulated estimation error from other layers.

To compare modulation schemes for IM/DD systems it is important to consider both the energy and spectral efficiency. Fig. 6 plots $E_{b(\text{opt})}/N_o$ at BER = $10^{-3}$ against different bit rate/normalized BW, $r_b$, for an AWGN channel and a range of modulation schemes. The systems considered are DCO-OFDM with 10 dB and 13 dB bias, ACO-OFDM with a conventional receiver, ACO-OFDM with diversity combining, ACO-OFDM with noise cancellation, ADO-OFDM, LACO-OFDM with conventional receiver, LACO-OFDM with diversity combining, and LACO-OFDM with noise cancellation. Fig. 6 shows that for small values of $r_b$ (less than three), ACO-OFDM with diversity combining/noise cancellation requires the lowest energy per bit. For large $r_b$ values (above six), three-layer LACO-OFDM with diversity combining/noise cancellation is the most power efficient system. For $r_b$ values from three to six, two-layer LACO-OFDM with diversity combining/noise cancellation is the best system. Diversity combining and noise cancellation have similar performance for both ACO-OFDM and LACO-OFDM systems for an AWGN channel. Comparing LACO-OFDM to ADO-OFDM we find that LACO-OFDM with any receiver type and any number of layers is always better for all bit rates, e.g. for $r_b=6$, two-layer LACO-OFDM with diversity combining/noise cancellation is 3 dB better than ADO-OFDM. When LACO-OFDM is compared with ACO-OFDM and DCO-OFDM we can see that ACO-OFDM with diversity combining requires the minimum $E_{b(\mathrm{opt})}/N_o$ for low bit rates up to $r_b = 3$. For high bit rates, LACO-OFDM with diversity combining is the best, even better than DCO-OFDM. Note that LACO-OFDM always requires higher constellation sizes to achieve the same bit rate as DCO-OFDM. For example, to achieve $r_b$ of 8, layered ACO-OFDM uses 512-QAM and DCO-OFDM uses 128-QAM. Noise cancellation and diversity combining give similar improvement for all forms of ACO-OFDM for AWGN channels.

## V. Diversity Combining and Noise Cancellation Receivers over Frequency selective channel

In this section, we show the advantage of LACO-OFDM with diversity combining over the recently described noise cancellation LACO-OFDM for a frequency selective channel. In VLC the channel response is often dominated by the low pass characteristics of the LED at the transmitter or the photodiode ant the receiver. In the comparison we use the channel transfer function [6]

$$H(k) = \exp\left(-\frac{(k - N/2)^2}{2bN^2}\right); \quad b = 0.1803 \tag{40}$$

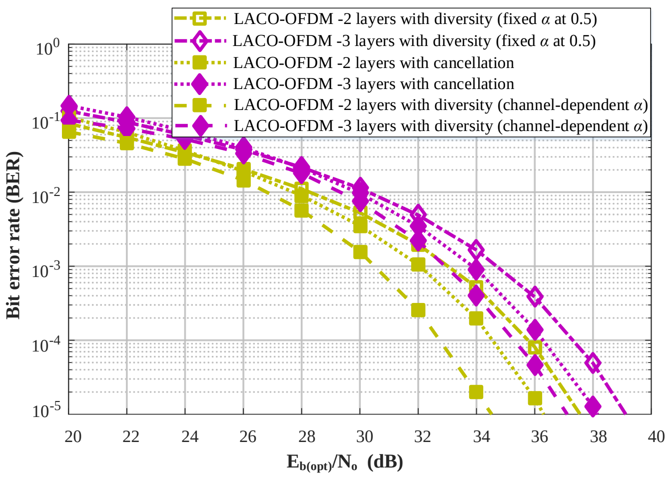


Fig. 7 $E_{b(\mathrm{opt})}/N_o$ versus BER for LACO-OFDM with diversity combining with fixed $\alpha$ for all subcarriers and with channel-dependent $\alpha$ and LACO-OFDM with noise cancellation with two and three layers over a frequency selective channel using 4-QAM

which has been experimentally shown in as an approximnation for a VLC channel.

Fig. 7 shows the performance of diversity combining and noise cancellation with two and three layers over a frequency selective channel. It can be seen that diversity combining outperforms noise cancellation by 2dB with two layers and by 1dB with three layers at the BER = $10^{-3}$. Note that the results obtained in Fig.7 are for a specific channel frequency response and the results will change for a different channel and the amount of improvement provided by diversity combining over noise cancellation will subsequently change. For severe channel conditions, the amount of improvement will increase, however, if the channel is close to a flat channel, a relatively small improvement will be achieved.

## VI. Conclusion

A new receiver which applies diversity combining to LACO-OFDM has been described. Simulation results show that for a flat AWGN channel the new receiver can improve the performance of two-layer LACO-OFDM by up to 2 dB and of three-layer LACO-OFDM by up to 1.5 dB. Detailed comparisons are made with many other forms of OFDM for a flat AWGN. These show that for all bit rates some form of ACO-OFDM with diversity combining is the most power efficient. For low bit rates conventional ACO-OFDM with diversity combining gives the best performance, while at higher bit-rates LACO-OFDM with diversity combining is better. It is shown that LACO-OFDM can be used with either diversity combining applied in the time domain, or with diversity combining applied in the frequency domain. For flat channels the two methods are equivalent, but for a frequency-selective channel, it is shown that application in the frequency domain gives better performance because the weighting factor used in diversity combining can be varied for each subcarrier. Expressions for channel-dependent weighting factors are derived. It has been also shown that diversity combining and noise cancellation receivers have the same performance over an AWGN channel but as noise cancellation cannot be applied in the frequency domain, diversity combining is preferred for a frequency selective channel.

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