A Parameter-Free Derivation of the Fine-Structure Constant α from Information-Theoretic Constraints in Octonionic Space

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The Central Result

$$\alpha^{-1} = 137.035999143$$

praedictum in vacuo

Zero free parameters

CODATA 2022: $137.035999177(21)$ *in situ*

Deviation: 1.62σ

This derivation does not assume quantization of spacetime; discreteness arises as a necessary condition for reversible projection from an 8-dimensional normed division algebra.

The So-called "Mystery" of α

"It has been a mystery ever since it was discovered... a magic number that comes to us with no understanding by man." — Feynman, QED (1985)

The fine-structure constant determines electromagnetic interaction strength. Its dimensionless nature suggests geometric origin.

We derive it from first principles.

The Seven Axioms

Axiom 1 (Zero/Identity) $\exists 0 \in S$

Axiom 2 (Succession) $\forall x \in S,\ \exists s(x) \in S$

Axiom 3 (Injectivity) $s(x) = s(y) \Rightarrow x = y$

Axiom 4 (Initiality) $\forall x \in S,\ s(x) \neq 0$

Axiom 5 (Induction) $\forall P \subseteq S:\ \big(0 \in P \wedge (\forall x \in P,\ s(x) \in P)\big) \Rightarrow P = S$

Axiom 6 (Triadic Closure) $\forall x, y \in S,\ \exists! z \in S:\ x \circ y = z$
All stable relations close on triples

Axiom 7 (Computability) $f : S \to S$ is total and decidable

Peano axioms (1–5) define arithmetic. Axioms 6–7 add triadic closure and total computability.

Lemma (Reversibility on the admissible subset). Let $S_{\text{real}} \subseteq S$ denote the physically admissible (information-preserving) subset. Restricting f to S_{real} and requiring injectivity on S_{real} implies $f|_{S_{\text{real}}}$ is bijective when S_{real} is finite; hence evolution is reversible on S_{real}.

Hurwitz's Theorem (1898)

Theorem. The only finite-dimensional normed division algebras over $\mathbb{R}$ are:

$$\underbrace{\mathbb{R}}_{\dim=1} \longrightarrow \underbrace{\mathbb{C}}_{\dim=2} \longrightarrow \underbrace{\mathbb{H}}_{\dim=4} \longrightarrow \underbrace{\mathbb{O}}_{\dim=8}$$

No others exist. This is theorem, not choice.

The Cayley-Dickson Construction

Each algebra is constructed from the previous by **doubling**:

$$(a, b) \cdot (c, d) = (ac - \bar{d}b,\ da + b\bar{c})$$

Algebra	Dim	Ord.	Comm.	Assoc.	Div.
$\mathbb{R}$ (Reals)	1	✓	✓	✓	✓
$\mathbb{C}$ (Complex)	2	×	✓	✓	✓
$\mathbb{H}$ (Quaternions)	4	×	×	✓	✓
$\mathbb{O}$ (Octonions)	**8**	×	×	×	✓
$\mathbb{S}$ (Sedenions)	16	×	×	×	×

Beyond $\mathbb{O}$: Sedenions have **zero divisors**—they cannot compute, but they can **index**.

Why 8D → 4D?

- We observe in **4 dimensions** (3 space + 1 time)
- A **reversible** substrate projecting to 4D must come from a higher-dimensional normed division algebra
- Hurwitz allows only: $\mathbb{R}$ (1D), $\mathbb{C}$ (2D), $\mathbb{H}$ (4D), $\mathbb{O}$ (8D)
- Only $\mathbb{O}$ is **larger** than 4D ⇒ **Octonions are mandatory for a reversible higher-dimensional normed substrate.**

Channel capacity of $8D \to 4D$ projection:

$$\alpha^{-1}_{\text{bare}} = 2\binom{8}{4} - 3 = 2 \times 70 - 3 = \mathbf{137}$$

(Subspaces × bidirectionality − closure overhead)

Postulate: Reversible Bijection

The physical universe is the **reversible projection** of three quantized flux eigenstates:

$$\mathcal{F} = \{|-1\rangle\ ,\ |0\rangle\ ,\ |+1\rangle\}$$

Under octonionic non-associativity, exactly **42 eigenvectors** preserve reversibility:

$$7\ \text{(Fano points)} \times 3\ \text{(Frobenius steps)} \times 2\ \text{(orientations)} = 42$$

Steiner Systems: Triadic Closure Forces Incidence

Theorem (STS Existence; Kirkman, 1847): A Steiner triple system $S(2, 3, n)$ exists when $n \equiv 1, 3 \pmod 6$.

Smallest nontrivial case: $n = 7 \quad \Rightarrow \quad S(2, 3, 7) \cong PG(2, 2)$ **(Fano plane)**

7 points ($M_2 = 2^3 - 1$) **7** lines **3** points per line ($M_1 = 2^2 - 1$)

$7 \times 3 \times 2 = \mathbf{42}$ oriented operations (glyphs)

"A theorem is not understood until it can be seen." — Jakob Steiner

The Fano plane is not chosen — it is **forced** by triadic closure at minimal scale.

The Fano Plane PG(2,2): Octonionic Structure and the Co-Computational Pascal's Triangle

Triangular Form

Heptagonal Form

Pascal's Triangle

An octonion is: $x = x_0 + x_1e_1 + x_2e_2 + x_3e_3 + x_4e_4 + x_5e_5 + x_6e_6 + x_7e_7$

x_0 is **real** (from $\mathbb{R}$, from Pascal's domain) · $e_1, \ldots, e_7$ are **imaginary** (from the Fano plane)

You can't have octonions without both.

The Fano plane tells you how the e_i multiply · Pascal's Triangle (as an 8 Simplex) tells you how to count and combine

The 42 glyphs are the operations that arise when both structures are present.

Computational Verification

Four independent programs generate identical 42 glyphs from Fano geometry and the Pascal 8 Simplex.

Canonical Table of the 42 Glyphs (Primordial Eigenvectors)

k	a	r	σ	C_k	Conn.	Physical Role	Basin	$\frac{Q}{e_1 \times e_2 \times e_3}$	$\frac{\text{Basin}}{e_8 \times e_7 \times e_{11}}$
Line 0: Fundamental Integers									
1	0	1	−	1	$u-1$	Identity/existence	Pascal	+1, 0, 0, 0, 0, 0, 0	+1, 0, 0, 0
2	0	1	+	2	$u+1$	Binary doubling	Pascal	−1, 0, 0, 0, 0, 0, 0	+1, 0, 0, 0
3	0	2	−	3	$u-2$	Ternary basis	Pascal	0, +1, 0, 0, 0, 0, 0	+1, 0, 0, 0
4	0	2	+	4	$u+2$	Quaternionic dimension	Pascal	0, −1, 0, 0, 0, 0, 0	+1, 0, 0, 0
5	0	4	−	7	$u-4$	Fano plane points	Pascal	0, 0, 0, +1, 0, 0, 0	+1, 0, 0, 0
6	0	4	+	8	$u+4$	Octonionic dimension	Pascal	0, 0, 0, −1, 0, 0, 0	+1, 0, 0, 0
Line 1: Transcendental Generators									
7	1	1	−	ϕ^{-1}	$u-1$	Golden conjugate	Fib	0, +1, 0, 0, 0, 0, 0	0, +1, 0, 0]
8	1	1	+	ϕ	$u+1$	Golden ratio	Fib	0, −1, 0, 0, 0, 0, 0	0, +1, 0, 0]
9	1	2	−	e^{-1}	$u-2$	Natural decay	Pascal	0, 0, +1, 0, 0, 0, 0	+1, 0, 0, 0]
10	1	2	+	e	$u+2$	Natural growth	Pascal	0, 0, −1, 0, 0, 0, 0	+1, 0, 0, 0]
11	1	4	−	π^{-1}	$u-4$	Circular inverse	Wallis	0, 0, 0, +1, 0, 0, 0	0, 0, +1, 0]
12	1	4	+	π	$u+4$	Circle constant	Wallis	0, 0, 0, −1, 0, 0, 0	0, 0, +1, 0]
Line 2: Algebraic Roots									
13	2	1	+	$\sqrt{2}^{-1}$	$u-1$	Diagonal inverse	Pascal	0, 0, +1, 0, 0, 0, 0	+1, 0, 0, 0]
14	2	1	+	$\sqrt{2}$	$u+1$	Orthogonal basis	Pascal	0, 0, −1, 0, 0, 0, 0	+1, 0, 0, 0]
15	2	2	−	$\sqrt{3}^{-1}$	$u-2$	Hexagonal inverse	Pascal	0, 0, 0, +1, 0, 0, 0	+1, 0, 0, 0]
16	2	2	+	$\sqrt{3}$	$u+2$	Hexagonal geometry	Pascal	0, 0, 0, −1, 0, 0, 0	+1, 0, 0, 0]
17	2	4	−	$\sqrt{5}^{-1}$	$u-4$	Pentagon inverse	Pascal	0, 0, 0, 0, +1, 0, 0	+1, 0, 0, 0]
18	2	4	+	$\sqrt{5}$	$u+4$	Pentagon/ϕ base	Pascal	0, 0, 0, 0, −1, 0, 0	+1, 0, 0, 0]
Line 3: Logarithmic Operations									
19	3	1	−	$\ln(2)^{-1}$	$u-1$	Binary log inverse	Pascal	0, 0, 0, +1, 0, 0, 0	+1, 0, 0, 0]
20	3	1	+	$\ln(2)$	$u+1$	Binary logarithm	Pascal	0, 0, 0, −1, 0, 0, 0	+1, 0, 0, 0]
21	3	2	−	$\ln(3)^{-1}$	$u-2$	Ternary log inverse	Pascal	0, 0, 0, 0, +1, 0, 0	+1, 0, 0, 0]
22	3	2	+	$\ln(3)$	$u+2$	Ternary logarithm	Pascal	0, 0, 0, 0, −1, 0, 0	+1, 0, 0, 0]
23	3	4	−	$\ln(\phi)^{-1}$	$u-4$	Golden log inverse	Pascal	0, 0, 0, 0, 0, +1, 0	+1, 0, 0, 0]
24	3	4	+	$\ln(\phi)$	$u+4$	Golden logarithm	Pascal	0, 0, 0, 0, 0, −1, 0	+1, 0, 0, 0]
Line 4: Trigonometric Functions									
25	4	1	−	$\sin(1)^{-1}$	$u-1$	Sine inverse	Pascal	0, 0, 0, 0, +1, 0, 0	+1, 0, 0, 0]
26	4	1	+	$\sin(1)$	$u+1$	Unit sine	Pascal	0, 0, 0, 0, −1, 0, 0	+1, 0, 0, 0]
27	4	2	−	$\cos(1)^{-1}$	$u-2$	Cosine inverse	Pascal	0, 0, 0, 0, 0, +1, 0	+1, 0, 0, 0]
28	4	2	+	$\cos(1)$	$u+2$	Unit cosine	Pascal	0, 0, 0, 0, 0, −1, 0	+1, 0, 0, 0]
29	4	4	−	$\tanh(1)^{-1}$	$u-4$	Hyperbolic inverse	Pascal	+1, 0, 0, 0, 0, 0, 0	+1, 0, 0, 0]
30	4	4	+	$\tanh(1)$	$u+4$	Hyperbolic tangent	Pascal	−1, 0, 0, 0, 0, 0, 0	+1, 0, 0, 0]
Line 5: Special Functions									
31	5	1	−	γ^{-1}	$u-1$	Euler-Mascheroni inverse	Wallis	0, 0, 0, 0, +1, 0	0, 0, +1, 0]
32	5	1	+	γ	$u+1$	Euler-Mascheroni	Wallis	0, 0, 0, 0, −1, 0	0, 0, +1, 0]
33	5	2	−	$\zeta(2)^{-1}$	$u-2$	Basel inverse	Wallis	0, 0, 0, 0, 0, +1	0, 0, +1, 0]
34	5	2	+	$\zeta(2)$	$u+2$	Basel problem	Wallis	0, 0, 0, 0, 0, −1	0, 0, +1, 0]
35	5	4	−	$\zeta(3)^{-1}$	$u-4$	Apéry inverse	Wallis	0, +1, 0, 0, 0, 0	0, 0, +1, 0]
36	5	4	+	$\zeta(3)$	$u+4$	Apéry constant	Wallis	0, −1, 0, 0, 0, 0	0, 0, +1, 0]
Line 6: Boundary Integers (Frobenius Class)									
37	6	1	−	21	$u-1$	Half of 42 (3×7)	Pascal	0, 0, 0, 0, 0, +1	0, 0, +1, 0]
38	6	1	+	42	$u+1$	Fano (6×7)	Pascal	0, 0, 0, 0, 0, −1	+1, 0, 0, 0]
39	6	2	−	23	$u-2$	Frobenius boundary prime	Pascal	+1, 0, 0, 0, 0, 0	+1, 0, 0, 0]
40	6	2	+	46	$u+2$	Composite resonance	Pascal	−1, 0, 0, 0, 0, 0	+1, 0, 0, 0]
41	6	4	−	147	$u-4$	Geometric boundary	Pascal	0, 0, +1, 0, 0, 0	+1, 0, 0, 0]
42	**6**	**4**	**+**	**137**	$u+4$	**Entropic boundary α^{-1}**	**Alpha**	**0, 0, −1, 0, 0, 0**	**0, 0, 0, +1**

Structure: Octonions ($e_1 \cdot e_7$) carry Fano multiplication.
Sedenion extension ($e_8 \cdot e_{15}$) encodes/indexes basin dynamics.
$|e_1 \cdots e_7| \ |\ e_8 \cdots e_{15}| = |$**Fano address**$|\ |$ **Basin signature**$|$
$42 = C(7, 5, 1, 1) \quad \alpha^{-1} = |GL(3, \mathbb{F}_2)| - M_3 = 168 - 31 = \mathbf{137}$

Basin Encoding (Sedenionic):
e_8: **Pascal** $2^5 - 1 = 31$ glyphs
e_{11}: **Fibonacci** $2^3 = 2$ glyphs
e_{15}: **Wallis** $2^3 = 8$ glyphs
e_{11}: **Alpha** $2^0 = 1$ glyph

Highlighted: Constants in α^{-1} formula. Fano → octonions (calculation). Sedenions → basin structure (classification).

The Projection Operator Decomposition

$$\alpha^{-1} \equiv I = \langle\psi|\,\hat{P}\,|\psi\rangle$$

$|\psi\rangle$ is the Fano-walk state, $\hat{P}$ is the projection operator

$$\hat{P} = \hat{N} + \hat{C}_{\text{circ}} + \hat{C}_{\text{ent}} + \hat{C}_{\text{ren}}$$

1. A Hilbert space $\mathcal{H}_{Fano}$ of dimension 42
2. A projection operator $\hat{P}$ that is self-adjoint ($\hat{P} = \hat{P}^\dagger$)
3. A specific state $|\psi\rangle \in \mathcal{H}_{Fano}$
4. An expectation value $\langle\psi|\,\hat{P}\,|\psi\rangle = \alpha^{-1}$

Operator Decomposition

These operators form a hierarchy of **projection harmonics** induced by projection: dynamical (x), analytic (γ), and structural ($\zeta(3)$).

Operator	Eigenvalue	Physical Meaning
$\hat{N}$	**137**	Channel count (algebraic multiplicity)
$\hat{C}_{\text{circ}}$	$\frac{1}{8\pi}$	Circular projection (8D spherical geometry)
$\hat{C}_{\text{ent}}$	$\frac{\gamma}{137}$	Entropic projection resolution (discrete → continuous)
$\hat{C}_{\text{ren}}$	$\frac{\zeta(3)}{137 \cdot 20}$	Higher-order (cubic/triadic lattice harmonic)
$\hat{P}_{\text{spin}}$	$-x_n$	State memory cost (Shannon)

20 :the minimal junction degree; dodecahedral vertex count.

The Complete Formula

$$\alpha^{-1} = 137 + \frac{1}{8\pi} - \frac{\gamma}{137 + \frac{1}{8\pi} - x} + \frac{\zeta(3)}{137 \cdot 20}$$

with

$$x = \frac{\ln n}{2 \cdot 137} + \frac{\ln(\ln n)}{42 \cdot 7}$$

where

$$\ln n = 14\pi^2 + \ln 8 + \varepsilon(\Phi)$$

$\varepsilon(\Phi)$ = residual holonomy

Standard QED

$$\alpha = \frac{e^2}{4\pi\varepsilon_0 \hbar c}$$

Measured

(input to Standard Model)

KT value:
137.035999143

CODATA 2022:
137.035999177(21)

Agreement: 1.62σ

The x-Term: Shannon's State Cost

"The actual information depends not only on what is sent but also on what the receiver already knows." — Shannon (1948)

Key point: the 8D↔4D map is a stateful bijection. Here we apply the conventional mathematics used commonly in codecs. A reversible projection cannot be memoryless because it carries history.

In Shannon's formalism, channels with memory satisfy $C = \lim_{n\to\infty} \frac{1}{n} \max I(X^n; Y^n)$, making capacity explicitly state-dependent.

We encode this state/memory cost as:

$$x = \frac{\ln n}{2\,C} + \frac{\ln(\ln n)}{N_{\text{states}} N_{\text{lines}}}$$

with $C = 137$, $N_{\text{states}} = 42$, $N_{\text{lines}} = 7$,

$\ln n/(2C)$ is the depth per channel for state reconstruction; $\ln(\ln n)/(42 \cdot 7)$ is the meta-structure penalty.

Eigenstate Closure Depth n

Definition: n is the **dimensionless depth of reversible eigenstate closure** required for the 8D↔4D bijection to saturate its admissible basis.

Geometric derivation:

$$\ln n \ = \ 14\pi^2 \ + \ \ln 8 \ + \ \varepsilon(\Phi)$$

Where:

- $14\pi^2 = 2 \times 7 \times \pi^2$ (spinor double-cover × seven Fano lines)
- $\ln 8$ (octonionic normalization)
- $\varepsilon(\Phi)$ (gravitational holonomy term)

Numerical value: $n \approx 8.07 \times 10^{60}$ Consistency: $n\,t_{\text{Planck}} \approx 13.8 \times 10^9$ years.

The Gravitational Holonomy Term $\varepsilon(\Phi)$

Physical origin: Gravitational time dilation modulates local projection rate. Stronger fields increase accumulated cycles.

The coupling:

$$\varepsilon(\Phi) = \eta \cdot \frac{\Delta\Phi}{c^2}, \quad \eta = \frac{2 \cdot 42}{20} = 4.2$$

$\Delta\Phi$: potential difference (m^2/s^2)
c^2: 9×10^{16} m^2/s^2
η: (bidirectional × glyphs) / junction

For experimentalists:

$$\frac{1}{\alpha}\frac{\partial\alpha}{\partial\Phi} \approx 4.7 \times 10^{-17} \text{ per } m^2/s^2$$

$$\frac{\Delta\alpha}{\alpha} = 4.6 \times 10^{-16} \cdot \Delta h \text{ (km)}$$

Recipe: Measure Φ_{local} → compute $\varepsilon = 4.2 \cdot \Delta\Phi/c^2$ → update $\ln n = 140.25 + \varepsilon$ → recompute α^{-1}

Five Falsifiable Predictions

1. Vacuum Kerr Effect (Altitude α Drift)

$$\frac{\Delta\alpha}{\alpha} = (4.60 \pm 0.15) \times 10^{-16}\ \text{km}^{-1}$$

Vacuum impedance $\varepsilon(z)$ scales with gravitational potential. Testable via Sr/Yb optical clocks.

2. Structural Anisotropy (The Dipole)

$$\nabla\alpha \neq 0 \quad \text{(Directional Computation)}$$

The Webb/Keck α dipole is not error; it is the **Processing Gradient** of the Manifold. Isotropic ΛCDM is false.

3. Isotopic Impedance Shift

$$\Delta\alpha \propto \nu_{\text{local}} \quad (^{171}\text{Yb} > {}^{87}\text{Rb})$$

Mass is broadband data. Higher nucleon density saturates local bandwidth, shifting α beyond GR predictions.

4. Geometric Hubble Tension

$$\frac{H_0(\text{local})}{H_0(\text{global})} = \frac{137}{125} = 1.096$$

The tension is the invariant ratio of channel capacities: $2\binom{8}{4} - 3$ (coupling) vs. $\binom{5}{4} - 1$ (mass).

5. The Saturation Limit

No quantum experiment will ever exceed a **42-dimensional** Hilbert space realization (Channel Capacity).

Statistical Validation

Zero-parameter prediction vs. one-parameter empirical fit:

ΔAIC: ≈ 0.6 (statistically competitive)
Bayes Factor: $\gtrsim 10^{13}$ (three independent paths)

The Derivation Chain

Alphabet $\{-1, 0, +1\}$ Minimal ternary with opposition
↓ *Axiom 6*
Incidence $S(2, 3, 7) \cong \text{PG}(2, 2)$ Steiner/Kirkman
↓ *Hurwitz*
Algebra $\mathbb{O} \to \mathbb{H}$ 8D → 4D projection
↓ *Fano Walks*
Channels $2\binom{8}{4} - 3 = 137$ Bidirectional minus closure
↓ *Projection Harmonics*
Result $\alpha^{-1} = 137.035999143$ **Zero parameters**

CODATA 2022 $137.035999177(21)$ Deviation: 1.62σ

References

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Interactive Resources

GitHub
github.com/KosmoNexus

Fano Walks Demo
Interactive visualization

The KT Project
thektproject.org

Zenodo DOI: 10.5281/zenodo.17861153
Email: macedoni@umich.edu

Summary

"The important thing is not the particular representation of the operators, but the algebraic relations between them." — Dirac

The Kosmoplex is a triadic, reversible $8D \leftrightarrow 4D$ computational manifold with embedded state memory, defined by axioms of totality, triadic closure, and reversibility. These constraints force a finite operator basis and fix the channel capacity of the projection.

$$\alpha^{-1} \ = \ 2\binom{8}{4} - 3 \ + \ (\text{geometric completion}) \ = \ 137.035999143$$

Two independent rigidity derivations:

- **Combinatorial:** $2 \times 70 - 3 = 137$
  (subspace count × bidirectionality − closure overhead)
- **Group-theoretic:** $|GL(3, \mathbb{F}_2)| - M_1 = 168 - 31 = 137$

No free parameters: all numerical values arise as invariants fixed by algebraic and incidence-theoretic rigidity (Hurwitz, Steiner/Kirkman, reversibility lemma).

Five falsifiable predictions. All experimentally testable; none presently excluded by existing bounds.

Resolves ΛCDM anomalies: Webb dipole, dark matter, early massive galaxies, and Hubble tension are natural consequences of KT geometry—not paradoxes requiring ad hoc explanation.