Bahman Zohuri

Scalar Wave Driven Energy Applications

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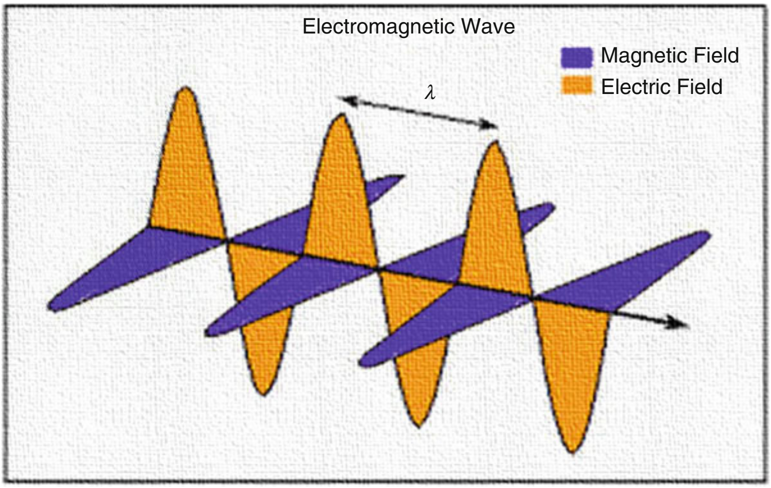

$$\vec{\nabla} \times \vec{E} = \frac{\partial \vec{B}}{\partial t}$$

$$\nabla^2 \phi - \frac{1}{c^2} \frac{\partial^2 \phi}{\partial t^2} = 0$$

$$\nabla^2 \vec{A} - \frac{1}{c^2} \frac{\partial^2 \vec{A}}{\partial t^2} = 0$$

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To my son Sasha and grandson Darius,
as well as my daughter Natalie and
Dr. Natasha Zohuri

Preface

What is a "scalar wave" exactly? A scalar wave (hereafter SW) is just another name for a "longitudinal" wave. The term *scalar* is sometimes used instead because the hypothetical source of these waves is thought to be a "scalar field" of some kind, similar to the Higgs field for example.

There is nothing particularly controversial about longitudinal waves (hereafter LWs) in general. They are a ubiquitous and well-acknowledged phenomenon in nature. Sound waves traveling through the atmosphere (or underwater) are longitudinal, as are plasma waves propagating through space (i.e., Birkeland currents). LWs moving through the Earth's interior are known as "telluric currents." They can all be thought of as pressure waves of sorts.

SWs and LWs are quite different from a "transverse" wave (TW). You can observe TWs by plucking a guitar string or watching ripples on the surface of a pond. They oscillate (i.e., vibrate, move up and down or side-to-side) perpendicular to their arrow of propagation (i.e., directional movement). As a comparison, SWs/LWs oscillate in the same direction as their arrow of propagation.

Only the well-known (transverse) Hertzian waves can be derived from Maxwell's field equations, whereas the calculation of longitudinal SWs gives zero as a result. This is a flaw of the field theory because SWs exist for all particle waves (e.g., as plasma wave, as photon- or neutrino radiation). Starting from Faraday's discovery, instead of the formulation of the law of induction according to Maxwell, an extended field theory is derived. It goes beyond the Maxwell theory with the description of potential vortices (i.e., noise vortices) and their propagation as an SW but contains the Maxwell theory as a special case. With that the extension is allowed and does not contradict textbook physics.

William Thomson, who called himself Lord Kelvin after he had been knighted, already in his lifetime was a recognized and famous theoretical physicist. To him the airship seemed too unsafe and so he went aboard a steamliner for a journey from England to America in the summer of 1897. He was on the way for a delicate mission.

Eight years before his German colleague Heinrich Hertz had detected the electromagnetic wave (EW) in experiments in Karlsruhe and scientists all over the world had rebuilt his antenna arrangements. They all not only confirmed the wave as such, but also, they could show its characteristic properties. It was a TW, for which the electric and the magnetic field pointers oscillate perpendicular to the direction of propagation. This can be seen as the reason that the velocity of propagation is displays itself as field-independent and constant. It is the speed of light c.

Because Hertz had experimentally proved the properties of this wave, previously calculated in a theoretical way by Maxwell, and at the same time proved the correctness of the Maxwellian field theory. The scientists in Europe were just saying to each other: “Well Done!” While completely other words came across from a private research laboratory in New York: “Heinrich Hertz is mistaken, it by no means is a transverse wave but a longitudinal wave!”

Scalar waves also are called “electromagnetic longitudinal waves,” “Maxwellian waves,” or “Teslawellen” (i.e., Tesla waves). Variants of the theory claim that scalar electromagnetics, also known as scalar energy, is background quantum mechanical fluctuations and associated zero-point energies.

In modern-day electrodynamics (both classical and quantum), electromagnetic waves (EMW) traveling in “free space” (e.g., photons in the “vacuum”) are generally considered to be TW. But then again, this was not always the case. When the preeminent mathematician James Clerk Maxwell first modeled and formalized his unified theory of electromagnetism in the late nineteenth-century, neither the EM SW/LW nor the EM TW had been experimentally proved, but he had postulated and calculated the existence of both.

After Hertz demonstrated experimentally the existence of transverse radio waves in 1887, theoreticians (e.g., Heaviside, Gibbs, and others) went about revising Maxwell’s original equations; at this time, he was deceased and could not object. They wrote out the SW/LW component from the original equations because they felt that the mathematical framework and theory should be made to agree only with experiments. Obviously, the simplified equations worked—they helped make the AC/DC electrical age engineerable.

Then in the 1889 Nikola Tesla—a prolific experimental physicist and inventor of alternating current (AC)—threw a proverbial wrench into the works when he discovered experimental proof for the elusive electric SW. This seemed to suggest that SW/LW, as opposed to TW, could propagate as pure electric waves or as pure magnetic waves. Tesla also believed these waves carried a hitherto unknown form of excess energy he referred to as “radiant.” This intriguing and unexpected result was said to have been verified by Lord Kelvin and others soon after.

Instead of merging their experimental results into a unified proof for Maxwell’s original equations, however, Tesla, Hertz, and others decided to bicker and squabble over who was more correct because they all derived correct results. Nonetheless, because humans (even “rational” scientists) are fallible and prone to fits of vanity and self-aggrandizement, each side insisted dogmatically that they were right, and the other side was mistaken. The issue was allegedly settled after the dawn of the twentieth century when (1) the concept of the mechanical (i.e., passive/viscous)

Ether was purportedly disproved by Michelson-Morley and replaced by Einstein's Relativistic Space-Time Manifold, and (2) detection of SW/LWs proved much more difficult than initially thought; this was mostly because of the wave's subtle densities, fluctuating frequencies, and orthogonal directional flow. As a result, the truncation of Maxwell's equations was upheld. Nevertheless, SW/LW in free space are quite real.

Besides Tesla, empirical work carried out by electrical engineers (e.g., Eric Dollard, Konstantin Meyl, Thomas Imlauer, and Jean-Louis Naudin, to name only some) has clearly demonstrated SW/LWs' existence experimentally. These waves seem able to exceed the speed of light, pass through EM shielding (i.e., Faraday cages), and produce overunity—more energy out than in—effects. They seem to propagate in a yet unacknowledged counterspatial dimension (i.e., hyper-space, pre-space, false-vacuum, Aether, implicit order, etc.).

In addition to the mathematical calculation of SWs, this book contains a voluminous collection of material concerning the information's technical use of SWs; for example, if the useful signal and the usually interfering noise signal change their places, if a separate modulation of frequency and wavelength makes a parallel image transmission possible, if it concerns questions of the environmental compatibility for the sake of humanity (e.g., bioresonance, among others) or to harm humanity (e.g., electro-smog) or to be used as high-energy directed weapons—also known as Star Wars or the Strategic Defense Initiative (SDI)—as tomorrow's battlefield weapons.

Albuquerque, NM, USA B. Zohuri
2018

Acknowledgments

I am indebted to the many people who aided me, encouraged me, and supported me beyond my expectations. Some are not around to see the results of their encouragement in the production of this book, yet I hope they know of my deepest appreciation. I especially want to thank my friends, to whom I am deeply indebted, and have continuously given support without hesitation. They have always kept me going in the right direction.

My most gratitude goes to Dr. Edl Schamiloglu, the Associate Dean of Engineering and Distinguished Professor in Department of Electrical Engineering and Computer Science at the University of New Mexico, who first of all gave me the opportunity and funding for this research and guided me in right direction as well.

Above all, I offer very special thanks to my late mother and father and to my children, particularly, my son Sasha and grandson Darius. They have provided constant interest and encouragement without which this book would not have been written. Their patience with my many absences from home and long hours in front of the computer to prepare the manuscript are especially appreciated.

I would like to extend my gratitude to Dr. Horst Eckardt of A.I.A.S. and his valuable write up on the subject of scalar waves, which I found very helpful and useful for me to write the chapter in this book on the subject. My many thanks also go to the pioneer of this new subject area, Dr. Konstantin Meyl, professor of Computer and Electrical Engineering at Furtwangen University in Germany; he has written a few excellent books on the subject of scalar waves.

I also would like to take this opportunity to express my great appreciation and gratitude to Ms Cheyenne Stradinger, the Senior Librarian, and Ms Anne D. Schultz, the manager of Library Operation at the Engineering Library of the University of New Mexico–Albuquerque,. They constantly supported my research throughout by obtaining all the resource books and journals for me. Without their help this book could not have come to its final form as presented here.

Contents

About the Author

Bahman Zohuri currently works for Galaxy Advanced Engineering, Inc., a consulting firm that he started in 1991 when he left both the semiconductor and defense industries after many years working as a chief scientist. After graduating from the University of Illinois in the field of physics and applied mathematics, he then went to the University of New Mexico, where he studied nuclear and mechanical engineering. He joined Westinghouse Electric Corporation after graduating; there he performed thermal hydraulic analysis and studied natural circulation in an inherent shutdown heat removal system (ISHRS) in the core of a liquid metal fast breeder reactor (LMFBR) as a secondary fully inherent shutdown system for secondary loop heat exchange. All these designs were used in nuclear safety and reliability engineering for a self-actuated shutdown system. Dr. Zohuri designed a mercury heat pipe and electromagnetic pumps for large pool concepts of a LMFBR for heat rejection purposes for this reactor during 1978 and received a patent for it.

Subsequently, he was transferred to the defense division of Westinghouse, where he oversaw dynamic analysis and methods of launching and controlling MX missiles from canisters. The results were applied to MX launch seal performance and muzzle blast phenomena analysis (i.e., missile vibration and hydrodynamic shock formation). Dr. Zohuri also was involved in analytical calculations and computations in the study of non-linear ion waves in rarefying plasma. The results were applied to the propagation of so-called soliton waves and the resulting charge collector traces in the rarefaction characterization of the corona of laser-irradiated target pellets.

As part of his graduate research work at Argonne National Laboratory, he performed computations and programming of multi-exchange integrals in surface and solid-state physics. He earned various patents in areas, such as diffusion processes and diffusion furnace design, while working as a Senior Process Engineer at various semiconductor companies (e.g, Intel Corp., Varian Medical Systems, and National Semiconductor Corporation). He later joined Lockheed Martin Missile and Aerospace Corporation as Senior Chief Scientist and oversaw research and development (R&D) and the study of the vulnerability, survivability, and both radiation and laser hardening of various components of the Strategic Defense Initiative, known as Star Wars.

This work included payloads (i.e., IR sensor) for the Defense Support Program, the Boost Surveillance and Tracking System, and the Space Surveillance and Tracking Satellite against laser and nuclear threats. While at Lockheed Martin, he also performed analyses of laser beam characteristics and nuclear radiation interactions with materials, transient radiation effects in electronics, electromagnetic pulses, system-generated electromagnetic pulses, single-event upset, blast, thermo-mechanical, hardness assurance, maintenance, and device technology.

He spent several years as a consultant at Galaxy Advanced Engineering serving Sandia National Laboratories, where he supported the development of operational hazard assessments for the Air Force Safety Center in collaboration with other researchers and third parties. Ultimately, the results were included in Air Force Instructions issued specifically for directed energy weapons operational safety. He completed the first version of a comprehensive library of detailed laser tools for airborne lasers, advanced tactical lasers, tactical high-energy lasers, and mobile/tactical high-energy lasers, for example.

Dr. Zohuri also oversaw SDI computer programs in connection with Battle Management C^3I and artificial intelligence and autonomous systems. He is the author of several publications and holds several patents, such as for a laser-activated radioactive decay and results of a through-bulkhead initiator. He has published the following works: *Heat Pipe Design and Technology: A Practical Approach* (CRC Press); *Dimensional Analysis and Self-Similarity Methods for Engineering and Scientists* (Springer); *High Energy Laser (HEL): Tomorrow's Weapon in Directed Energy Weapons, Volume I* (Trafford Publishing Company); and recently the book on the subject of *Directed-Energy Weapons and Physics of High-Energy Lasers* with Springer. He has published two other books with Springer Publishing Company: *Thermodynamics in Nuclear Power Plant Systems* and *Thermal-Hydraulic Analysis of Nuclear Reactors.* Many of them can be found in most universities' technical library, can be seen on the Internet, or ordered from Amazon.com.

Presently, he holds the position of Research Associate Professor in the Department of Electrical Engineering and Computer Science at the University of New Mexico–Albuquerque, and continues his research on neural science technology and its application in super artificial intelligence. Dr. Zohuri has published a series of book in this subject as well on his research on SWs; the results of his research are presented in this book.

Chapter 1
Foundation of Electromagnetic Theory

To study the subject of a scalar wave and its physics as well as its behavior as a source driving various applications of energy, we need to have some understanding of the fundamental knowledge of electromagnetic theory; such background is essential. This chapter introduces Maxwell's equations—particularly Ampère's Law as part of his other equations. We mainly are concerned with the law's missing term as part of the complete version of Maxwell's equations. We also examine this law to show that it sometimes fails, and to find a generalization that always is valid in classical electromagnetics, whereas it fails in electrodynamics because of the missing term, which is an important factor to develop the basic scalar wave equation [1].

1.1 Introduction

Although Maxwell formulated his equations (now known as *Maxwell's equations*) more than 100 years ago, the subject of electromagnetism never has been stagnate. Production of so-called clean energy, driven by magnetic confinement of hot plasma via a controlled thermonuclear reaction between two isotopes of hydrogen—namely, deuterium (D) and tritium (T)—results in some behavior in plasma that is known as *magneto hydrodynamics* (MHD). Study of such phenomena requires knowledge of and understanding of fundamental electromagnetism and fluid dynamics combined, where the *fluid dynamics equation* and *Maxwell's equations* are joined [1].

In the study of electricity and magnetism, as part of understanding the physics of plasma, however, we need to have some knowledge of notation that may be accomplished by using vector analysis. By providing a valuable shorthand for electromagnetics (EM) and electrodynamics, vector analysis also brings to the forefront the physical ideas involved in these equations; therefore, we briefly formulate some of these vector analysis concepts and present some of their uniqueness in this chapter.

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1.2 Vector Analysis

Several kinds of quantities are encountered in the study of the fundamental science of physics; in particular, we need to distinguish *vectors* and *scalars*. For our purposes, it is sufficient to define them as follows:

1. *Scalar:* A *scalar* is a quantity that is characterized completely by its magnitude. Examples of scalars are mass and volume. A simple extension of the idea of a scalar is a *scalar field*—a function of position that is entirely specified by its magnitude at all points in space.
2. *Vector:* A *vector* is a quantity that is characterized completely by its magnitude and direction. Examples of vectors are: the position from a fixed origin, velocity, acceleration, and force. The generalization to a *vector field* gives a function of position that is entirely specified by its magnitude and direction at all points in space.

Detailed review of *vector analysis* is beyond the scope of this book; thus, we briefly formulate the fundamental layout of vector analysis here for purposes of its operation for the operator developing essential electromagnetics and electrodynamics that are the foundation for understanding plasma physics.

1.2.1 Vector Algebra

Most everyone is familiar with scalar algebra from basic algebra courses; the same algebra can be applied to develop vector algebra. For the time being we use a Cartesian coordinate system to develop a three-dimensional analysis of vector algebra. The Cartesian system allows representation of a vector by its three components, denoting them with x, y, and $_z$; or, when it is more convenient, we use notation x_1, x_2, and x_3. With respect to the Cartesian coordinate system, a vector is specified by its $x-$, $y-$, and $z-$ components. Thus, a vector $\vec{V}$ (note that the vector quantities are denoted by symbol of vector $\rightarrow$ on top) is specified by its components, V_x, V_y, and V_z, where $V_x = \left|\vec{V}\right| \cos\alpha_1, V_y = \left|\vec{V}\right| \cos\alpha_2$, and $V_z = \left|\vec{V}\right| \cos\alpha_3$. The α's are the angles between vector $\vec{V}$ and the appropriate coordinate axes of the Cartesian system.

The scalar $\left|\vec{V}\right| = \sqrt{V_x^2 + V_y^2 + V_z^2}$ is the *magnitude* of the vector or its length. On the basis of Fig. 1.1, in the case of vector fields, each of the components is to be regarded as a function of x, y, and $_z$. It should be emphasized for the simplicity of analysis that we are using the Cartesian coordinate system, yet the similarity of these analyses applies to the other coordinates, such as cylindrical and spherical as well.

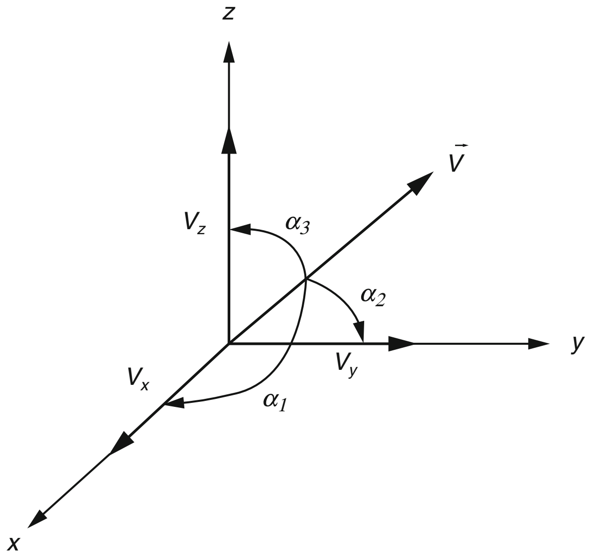

Fig. 1.1 Presentation of a vector along with its components in the Cartesian coordinate system

1. *Sum of Two Vectors*

The sum of two vectors, $\vec{A}$ and $\vec{B}$, is defined as vector $\vec{C}$ with components that are the sum of corresponding components in the original vectors. Thus, we can write:

$$\vec{C} = \vec{A} + \vec{B} \tag{1.1}$$

and

$$\begin{aligned} C_x &= A_x + B_x \\ C_y &= A_y + B_y \\ C_z &= A_z + B_z \end{aligned} \tag{1.2}$$

This definition of the vector sum is completely equivalent to the familiar parallelogram rule for vector addition.

2. *Subtraction of Two Vectors*

Vector subtraction is defined in terms of the negative of a vector, which is the vector with components that are the negative of the corresponding components of the original vector. Thus, if $\vec{A}$ is a vector, $-\vec{A}$ is defined by

$$\begin{aligned} (-A_x) &= -A_x \\ (-A_y) &= -A_y \\ (-A_z) &= -A_z \end{aligned} \tag{1.3}$$

The operation of subtraction is then defined as the addition of the negative and is written:

$$\vec{A} - \vec{B} = \vec{A} + \left(-\vec{B}\right) \tag{1.4}$$

Because the addition of real numbers is associative and commutative, it follows that vector addition and subtraction are also associative and commutative. In vector form notation, this appears as

$$\begin{aligned} \vec{A} + \left(\vec{B} + \vec{C}\right) &= \left(\vec{A} + \vec{B}\right) + \vec{C} \\ &= \left(\vec{A} + \vec{C}\right) + \vec{B} \\ &= \vec{A} + \vec{B} + \vec{C} \end{aligned} \tag{1.5}$$

In other words, the parentheses are not needed, as indicated by the last form.

3. *Multiplication of Two Vectors*

Now, we proceed to multiplication of two vectors and its process. We note that the simplest product is a scalar multiplied by a vector. This operation results in a vector, each component of which is the scalar times the corresponding component of the original vector. If c is a scalar and $\vec{A}$ is a vector, the product $c\vec{A}$ is a vector, $\vec{B} = c\vec{A}$, defined by

$$\begin{aligned} B_x &= cA_x \\ B_y &= cA_x \\ B_z &= cA_z \end{aligned} \tag{1.6}$$

It is clear that if $\vec{A}$ is a *vector field* and c is a *scalar field*, then $\vec{B}$ is a new vector field that is *not* necessarily a constant multiple of the origin field.

If we want to multiply two vectors, there are two possibilities; they are known as the *vector* and the *scalar products*—sometimes called *cross* or *dot products*, respectively.

3.1 *Scalar Product of Two Vectors*

First, considering the scalar or dot product of two vectors, $\vec{A}$ and $\vec{B}$, we note that sometimes the scalar product is called the *inner product*, which is derived from the scalar nature of the product. The definition of the scalar product is written as

$$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z \tag{1.7}$$

This definition is equivalent to another, and perhaps more familiar, definition—that is, as the product of the magnitudes of the original vectors times the cosine of the angle between these vectors if they are perpendicular to each other.

$$\vec{A} \cdot \vec{B} = 0 \tag{1.8}$$

Note that the scalar product is commutative. The length of $\vec{A}$, then, is:

$$|\vec{A}| = \sqrt{\vec{A} \cdot \vec{A}} \tag{1.9}$$

3.2 *Vector Product of Two Vectors*

The vector product of two vectors is a vector, which accounts for the name and alternative names: *outer product* and *cross product*. The vector product is written as $\vec{A} \times \vec{B}$. If $\vec{C}$ is the vector product of $\vec{A}$ and $\vec{B}$, then

$$\vec{C} = \vec{A} \times \vec{B} \tag{1.10}$$

or in terms of their components it can be written as:

$$\begin{aligned} C_x &= A_y B_z - A_z B_y \\ C_y &= A_z B_x - A_x B_z \\ C_z &= A_x B_y - A_y B_x \end{aligned} \tag{1.11}$$

It is important to note that the cross product depends on the order of the factors; interchanging the order of the cross product introduces a minus sign:

$$\vec{B} \times \vec{A} = -\vec{A} \times \vec{B} \tag{1.12}$$

Consequently,

$$\vec{A} \times \vec{A} = 0 \tag{1.13}$$

This definition is equivalent to the following: The vector product is the product of the magnitudes times the sine of the angle between the original vectors with the direction given by the right-hand screw rule (Fig. 1.2). Note that if we let $\vec{A}$ be rotated into $\vec{B}$ through the smallest possible angle, a right-hand screw rotated in this manner will advance in a direction perpendicular to both $\vec{A}$ and $\vec{B}$; this direction is the direction of $\vec{A} \times \vec{B}$.

The vector product may be easily expressed in terms of a determinant via the definition of unit vectors as $\widehat{i}$, $\widehat{j}$, and $\widehat{k}$, which are vectors of unit magnitude in the x-, y-, and z-directions, respectively; then we can write:

$$\vec{A} \times \vec{B} = \begin{vmatrix} \widehat{i} & \widehat{j} & \widehat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} \tag{1.14}$$

If this determinant is evaluated by the usual rules, the result is precisely our definition of the cross product of two vectors.

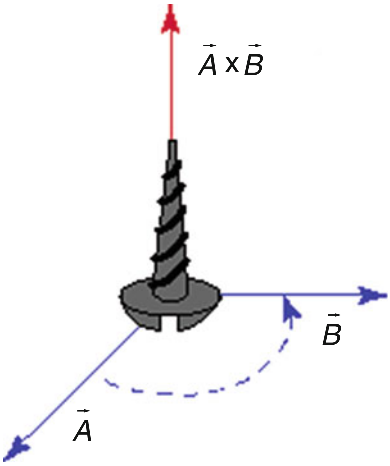

Fig. 1.2 Right-hand screw rule

The determinant in Eq. 1.14 may be combined in many ways, and most of the results obtained are obvious; however, two triple products of sufficient importance need to be mentioned. The triple scalar product, $D = \vec{A} \cdot \vec{B} \times \vec{C}$, is easily found and given by the determinant as

$$D = \vec{A} \cdot \vec{B} \times \vec{C} = \begin{vmatrix} A_x & A_y & A_z \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{vmatrix} = -\vec{B} \cdot \vec{A} \times \vec{C} \tag{1.15}$$

This product in Eq. 1.15 is unchanged by an exchange of dot and cross or by a cyclic permutation of the three vectors. Note that parentheses are not needed because the cross product of a scalar and a vector is undefined.

The other interesting triple product is the triple vector product, $\vec{D} = \vec{A} \times \left(\vec{B} \times \vec{C}\right)$. Through repeated application of the definition of the cross product, Eqs. 1.10 and 1.11, we find:

$$\vec{D} = \vec{A} \times \left(\vec{B} \times \vec{C}\right) = \vec{B}\left(\vec{A} \cdot \vec{C}\right) - \vec{C}\left(\vec{A} \cdot \vec{B}\right) \tag{1.16}$$

which frequently is known as the *back-cab rule*. We should bear in mind that in the cross product the parentheses are vital as part of the operation; without them the product is not well defined.

4. *Devision of Two Vectors*

At this point one might be interested in the possibly of vector division. Division of a vector by a scalar can, of course, be defined as multiplication by the reciprocal of the scalar. Division of a vector by another vector, however, is possible only if the two vectors are parallel. On the other hand, it is possible to write a general solution to vector equations and so accomplish something akin to division.

Consider this equation:

$$c = \vec{A} \cdot \vec{X} \tag{1.17}$$

where c is a known scalar, $\vec{A}$ is a known vector, and $\vec{X}$ is an unknown vector. A general solution to Eq. 1.17 is given as follows:

$$\vec{X} = \frac{c\vec{A}}{\vec{A} \cdot \vec{A}} + \vec{B} \tag{1.18}$$

where $\vec{B}$ is an arbitrary vector that is perpendicular to $\vec{A}$—that is, $\vec{A} \cdot \vec{B} = 0$. What we have done is very nearly to divide c by vector $\vec{A}$; more correctly, we have found the general form of vector $\vec{X}$ that satisfies Eq. 1.17. There is no unique solution, and this fact accounts for vector $\vec{B}$. In the same fashion, we can consider the vector equation as

$$\vec{C} = \vec{A} \times \vec{X} \tag{1.19}$$

In Eq. 1.19 both vectors $\vec{A}$ and $\vec{C}$ are known; $\vec{X}$ is an unknown vector. The general solution of this equation is then given by

$$\vec{X} = \frac{\vec{C} \times \vec{A}}{\vec{A} \cdot \vec{A}} + k\vec{A} \tag{1.20}$$

where k is an arbitrary scalar. Thus, $\vec{X}$, as defined by Eq. 1.20, is very nearly the quotient of $\vec{C}$ by $\vec{A}$; scalar k takes into account the non-uniqueness of the process. If $\vec{X}$ is required to satisfy both Eqs. 1.17 and 1.19, then the result is unique if it exists and is given by

$$\vec{X} = \frac{\vec{C} \times \vec{A}}{\vec{A} \cdot \vec{A}} + \frac{c\vec{A}}{\vec{A} \cdot \vec{A}} \tag{1.21}$$

1.2.2 Vector Gradient

Now that we have covered basic vector algebra, we pay attention to vector calculus, which extends to vector gradient, integration, vector curl, and differentiation of vectors. The simplest of these is the relation of a particular vector field to the derivative of a scalar field.

For that matter, it is convenient to introduce the idea of a *directional derivative* of a function of several variables; we leave it to the reader to find these analyses in any vector calculus book—that is, details of such a derivative are beyond the intended scope of this book. Thus, we jump to the definition of the vector gradient.

The *vector gradient* of a scalar function, φ, is a vector with a magnitude that is the maximum directional derivative at the point being considered and with a direction that is the direction of the maximum directional derivative at the point. We put this definition into some perspective using the geometry of Fig. 1.3, and it is evident that the gradient has the direction to the level surface of φ through the point, as we said that is being coinsured.

The most common mathematical symbol for gradient is $\vec{\nabla}$; in text form it is *grad.* In terms of the gradient, the directional derivative is given by

$$\frac{d\varphi}{ds} = |\text{grad}\,\vec{\varphi}|\cos\theta \tag{1.22}$$

where θ is the angle between the direction of $d\vec{s}$ and the direction of the gradient. This result is evident immediately from Fig. 1.3. If we write $d\vec{s}$ for the vector displacement of magnitude ds, then Eq. 1.22 can be written as

$$\frac{d\varphi}{ds} = \text{grad}\,\vec{\varphi}\cdot\frac{d\vec{s}}{ds} \tag{1.23}$$

Equation 1.23 enables us to seek the explicit form of the gradient and find it in any coordinate system in which we know the form of $d\vec{s}$. In a Cartesian or a rectangular coordinate system, we know that $d\vec{s} = \widehat{i}dx + \widehat{j}dy + \widehat{k}dz$. We also know from differential calculus that

$$d\varphi = \frac{\partial\varphi}{\partial x}dx + \frac{\partial\varphi}{\partial y}dy + \frac{\partial\varphi}{\partial z}dz \tag{1.24}$$

From Eq. 1.22, the results are:

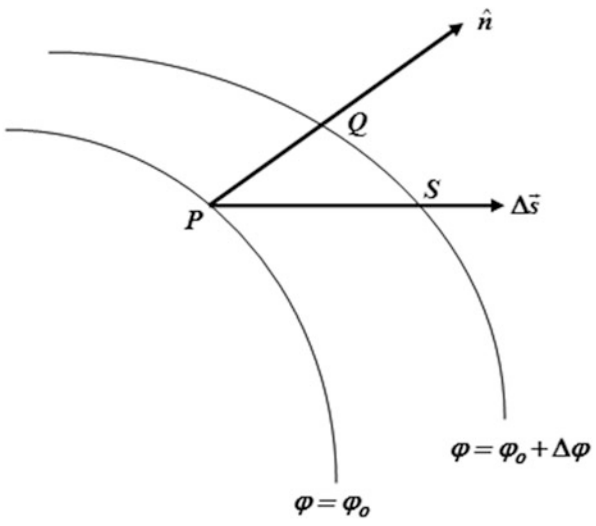

Fig. 1.3 Parts of two level surfaces of the function $\varphi(x, y, z)$

$$d\varphi = \frac{\partial \varphi}{\partial x}dx + \frac{\partial \varphi}{\partial y}dy + \frac{\partial \varphi}{\partial z}dz$$
$$= (\text{grad}\varphi)_x dx + (\text{grad}\varphi)_y dy + (\text{grad}\varphi)_z dz \tag{1.25}$$

Equating the coefficient of independent variables on both sides of the equation in a rectangular coordinate gives:

$$\text{grad}\,\vec{\varphi} = \frac{\partial \varphi}{\partial x}\hat{i} + \frac{\partial \varphi}{\partial y}\hat{j} + \frac{\partial \varphi}{\partial z}\hat{k} \tag{1.26}$$

In a more complicated case, the procedure is very similar as well. In spherical polar coordinates, by using Fig. 1.4 with denotation of r, θ, and ϕ, we can write Eq. 1.24 in the following form:

$$d\varphi = \frac{\partial \varphi}{\partial r}dr + \frac{\partial \varphi}{\partial \theta}d\theta + \frac{\partial \varphi}{\partial \varphi}d\phi \tag{1.27}$$

and

$$d\vec{s} = \hat{a}_r dr + \hat{a}_\theta r d\theta + \hat{a}_\phi r \sin\theta d\phi \tag{1.28}$$

where $\hat{a}_r$, $\hat{a}_\theta$, and $\hat{a}_\phi$ are unit vectors in the r, θ, and ϕ directions, respectively. Applying Eq. 1.23 and equating coefficients of independent variables yields:

$$\text{grad}\,\vec{\varphi} = \hat{a}_r \frac{\partial \varphi}{\partial r} + \hat{a}_\theta \frac{1}{r}\frac{\partial \varphi}{\partial \theta} + \hat{a}_\phi \frac{1}{r\sin\theta}\frac{\partial \varphi}{\partial z} \tag{1.29}$$

Equation 1.29 is established in a spherical coordinate system.

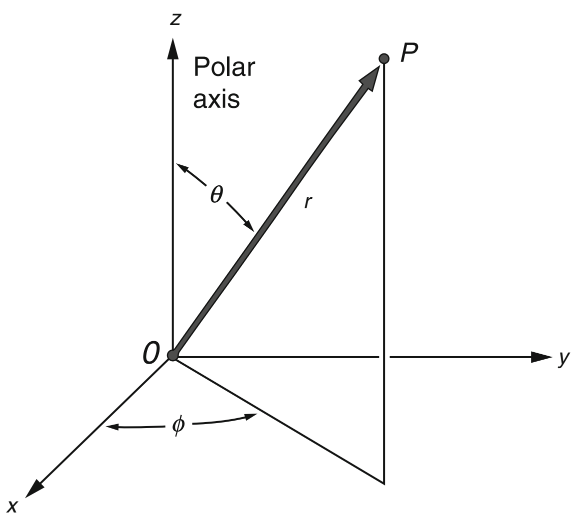

Fig. 1.4 Definition of the polar coordinates

1.2.3 Vector Integration

Although there are other aspects of vector differentiation, first we need to consider vector integration. The details of such analyses are left for the reader to look up in any vector calculus book; we discuss them just briefly here. For our purposes of vector integration, we consider three kinds of integrals, according to the nature of the differential appearing in them:

1. Line integral
2. Surface integral
3. Volume integral

In either case, the integrand may be either a vector or a scalar field; however, certain combinations of integrands and differentials give rise to uninteresting integrals. Those of most interest here are the scalar line integral of a vector, the scalar surface integral of a vector, and the volume integral of both vectors and scalars.

If $\vec{F}$ is a vector field, a line integral of $\vec{F}$ is written as

$$\int_{a(C)}^{b} \vec{F}(\vec{r}) \cdot d\vec{l} \tag{1.30}$$

where C is the curve along which the integration is performed, a and b are the initial and final points on the curve, and $d\vec{l}$ is an infinitesimal vector displacement along curve C.

It is obvious that because the result of the dot product of $\vec{F}(\vec{r}) \cdot d\vec{l}$ is scalar, the result of the linear integral in Eq. 1.30 is scalar. The definition of line integral follows closely the Riemann definition of the definite integral; thus, the integral can be written as a segment of curve C between the lower and the upper bounds of a and b, respectively, and then it can be divided into a large number of small increments, $\Delta\vec{l}$. For an increment, an interior point is chosen and the value of $\vec{F}(\vec{r})$ at that point is found. In other words, Eq. 1.30 can form the following equation as

$$\int_{a(C)}^{b} \vec{F}(\vec{r}) \cdot d\vec{l} = \lim_{N\to\infty} \sum_{i=1}^{N} \vec{F}_i(\vec{r}) \cdot \Delta\vec{l} \tag{1.31}$$

It is important to emphasize that the line integral usually depends not only on the end points of a and b, but also on curve C along which the integration is to be done, because the magnitude and direction of $\vec{F}_i(\vec{r})$ and the direction of $d\vec{l}$ depend on curve C and its tangent, respectively. The line integral around a closed curve is of sufficient importance that a special notation is used for it—namely,

$$\oint_C \vec{F} \cdot d\vec{l} \tag{1.32}$$

Note that the integral around a closed curve usually is not zero. The class of vectors for which the line integral around any closed curve is zero is of considerable importance. Thus, we normally write line integrals around undesignated closed paths as

$$\oint \vec{F} \cdot d\vec{l} \tag{1.33}$$

The form of integral in Eq. 1.33 around a closed curve C is for those cases where the integral is independent of contour C within rather wide limits.

Now, paying attention to the second kind of integral—namely, surface integrals—we again can define $\vec{F}$ as a vector, and a surface integral of $\vec{F}$ is written as

$$\int_s \vec{F} \cdot \widehat{n}\, da \tag{1.34}$$

where S is the surface over which the integral is taken, da is an infinitesimal area on surface S, and $\widehat{n}$ is a unit vector normal to da.

There is an ambiguity of two degrees in the choice of unit vector $\widehat{n}$ as far as outward or downward direction to the normal surface S is concerned if this surface is a closed one. If S is not closed and is finite, then it has a boundary, and the sense of the normal is important only with respect to the arbitrary positive sense of traversing the boundary. The positive sense of the normal is the direction in which a right-hand screw would advance if rotated in the direction of the positive sense on the bounding curve, as illustrated in Fig. 1.5. The surface integral of $\vec{F}$ over a closed surface S is sometimes denoted by

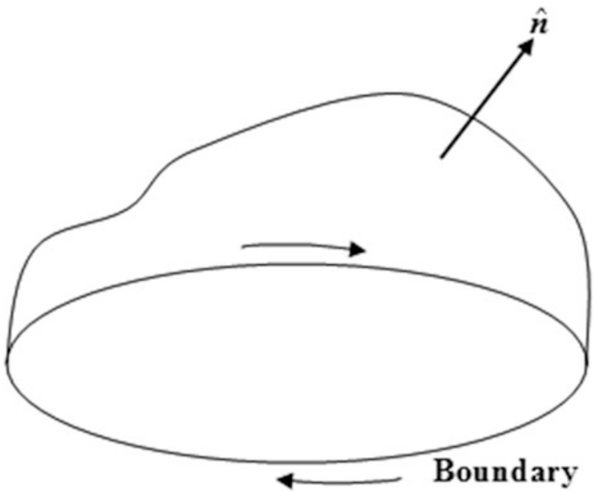

Fig. 1.5 Illustration of the relation of normal unit vector to surface and the direction of traversal of the boundary

$$\oint_S \vec{F} \cdot \hat{n}\, da \tag{1.35}$$

Comments exactly parallel to those made for the line integral can be made for the surface integral. This integral is clearly scalar, and it usually depends on surface S; cases where it does not are particularly important.

Now, we can pay attention to the third type of vector integral—namely, the volume integral—and we start with vector $\vec{F}$. Therefore, if $\vec{F}$ is a vector and φ is a scalar, then the two volume integrals in which we are interested are written:

$$J = \int_V \varphi dv \qquad \vec{K} = \int_V \vec{F} dv \tag{1.36}$$

Clearly, J is a scalar and $\vec{K}$ is a vector. The definitions of these integrals reduce quickly to just the Riemann integral in three dimensions, except that in $\vec{K}$ one must note that there is one integral for each component of $\vec{F}$. We are very familiar with these integrals, however, and require no further investigation nor any comments.

1.2.4 Vector Divergence

Another important vector operator, which plays an essential role in establishing electromagnetism equations, is a vector divergence operation; it is a derivative form. The divergence of vector $\vec{F}$, written as $\mathrm{div}\vec{F}$, is defined as follows.

The divergence of a vector is the limit of its surface integral per unit volume as the volume enclosed by the surface goes to zero. This statement can be presented mathematically as follows:

$$\mathrm{div}\vec{F} = \lim_{V \to 0} \oint_S \vec{F} \cdot \hat{n}\, da \tag{1.37}$$

The divergence is clearly a scalar point function; its resulting operation ends up with a scalar field, and it is defined at the limit point of the surface of integration.

A detailed proof of this concept is beyond the scope of this book, and it is left to readers to refer to any vector calculus book. Yet, the limit can be taken easily, and the divergence in rectangular coordinates is found to be:

$$\mathrm{div}\vec{F} = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z} \tag{1.38}$$

Equation 1.38 for the vector divergence operation designated for the Cartesian coordinate and in the spherical coordinate is written in the following form:

$$\operatorname{div}\vec{F} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 F_r\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta F_\theta) + \frac{1}{r\sin\theta}\frac{\partial F_\phi}{\partial\phi} \tag{1.39}$$

In the cylindrical coordinate it is represented by

$$\operatorname{div}\vec{F} = \frac{1}{r}\frac{\partial}{\partial r}(rF_r) + \frac{1}{r}\frac{\partial}{\partial\theta}(F_\theta) + \frac{\partial}{\partial z}(F_z) \tag{1.40}$$

The method to find the explicit form of the divergence is applicable to any coordinate system, provided that the forms of the volume and the surface elements, or, alternatively, the elements of the length, are known.

Now that we have the idea behind the vector divergence operator and its operation, we can establish the *divergence theorem*. The integral of the divergence of a vector over volume V is equal to the surface integral of the normal component of the vector over the surface bounding V—that is,

$$\int_V \operatorname{div}\vec{F}\,dv = \oint_S \vec{F}\cdot\hat{n}\,da \tag{1.41}$$

We leave it at that; for proof readers can refer to any vector calculus book.

1.2.5 Vector Curl

Another interesting vector differential operator is the vector *curl*. The curl of a vector, written as $\operatorname{curl}\vec{F}$, is defined as the limit of the ratio of the integral of its cross product with the outward drawn normal, over a closed surface, to the volume enclosed by the surface as the volume goes to zero—that is,

$$\operatorname{curl}\vec{F} = \lim_{V\to 0}\frac{1}{V}\oint_S \hat{n}\times\vec{F}da \tag{1.42}$$

Again, the details of proof are left to readers to find in a vector calculus book; we just write the final result of the curl operator in at least the rectangular coordinate, as follows:

$$\operatorname{curl}\vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_x & F_y & F_z \end{vmatrix} \tag{1.43}$$

Finding the form of the curl in other coordinate systems is only slightly more complicated and is left to reader to practice.

Now that we have an understanding of the vector curl operator, we can state *Stock's theorem* as follows. The line integral of a vector around a closed curve is equal to the integral of the normal component of its curl over any surface bounded by the curve—that is,

$$\oint_C \vec{F} \cdot d\hat{l} = \int_S \text{curl}\, \vec{F} \cdot \hat{n}\, da \tag{1.44}$$

where C is a closed curve that bounds surface S.

1.2.6 Vector Differential Operator

We now introduce an alternative notation for the types of vector differentiation that have been discussed—namely, gradient, divergence, and curl. This notation uses the vector differential operator, *del*, and it is identified by the symbol $\vec{\nabla}$ and written mathematically as:

$$\vec{\nabla} = \hat{i}\frac{\partial \varphi}{\partial x} + \hat{j}\frac{\partial \varphi}{\partial y} + \hat{k}\frac{\partial \varphi}{\partial z} \tag{1.45}$$

Del is a differential operator in that it is used only in front of a function of (x, y, z), which it differentiates; it is a vector in that it obeys the laws of vector algebra. It is also a vector in terms of its transformation properties, and in terms of del, Eqs. 1.25, 1.38, and 1.43 are expressed as follows:

Grad $= \vec{\nabla}$:

$$\vec{\nabla} F = \hat{i}\frac{\partial F_x}{\partial x} + \hat{j}\frac{\partial F_y}{\partial y} + \hat{k}\frac{\partial F_z}{\partial z} \tag{1.46}$$

Div $= \vec{\nabla}$:

$$\vec{\nabla} \cdot \vec{F} = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z} \tag{1.47}$$

Curl $= \vec{\nabla}\times$:

$$\vec{\nabla} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ F_x & F_y & F_z \end{vmatrix} \tag{1.48}$$