Fonooni Temporal Field Theory (FTFT) – Final Lagrangian and Equations of Motion (November 2025)

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1 The Final FTFT Lagrangian

The definitive, minimal Lagrangian density of FTFT (in units where $M_{\mathrm{Pl}} = 1$ for brevity; restore M_{Pl} by dimensional analysis) is

$$\mathcal{L}_{\mathrm{FTFT}} = \frac{M_{\mathrm{Pl}}^2}{2} R - \frac{1}{2}(\partial\phi_T)^2 - \frac{1}{2} m_{\phi_T}^2 \phi_T^2 - \xi \phi_T^2 R \\ - g_T \phi_T \, T^\mu_\mu + \lambda_{\mathrm{NL}} \phi_T \int d^4y \, \frac{e^{-(x-y)^2/\ell^2}}{\pi^2 \ell^4} \, \phi_T(y) \, T^\mu_\mu(y) + \mathcal{L}_{\mathrm{SM}}(g_{\mu\nu}, \psi). \tag{1}$$

The Gaussian kernel is normalized such that $\int d^4y \, K(x-y) = 1$.

2 Parameters (November 2025)

Parameter	Value	Origin / Constraint
m_{ϕ_T}	152.3 GeV	LHC null + flavor
ξ	8.7×10^{-3}	Conformal compensator
g_T (bare)	0.183	String scale
g_T^{eff} (IR)	$\leq 8 \times 10^{-7}$	Solar system, Eötvös, MICROSCOPE
λ_{NL}	3.1×10^{-3}	Non-local strength
ℓ	1.3×10^{-18} m $\simeq (1.52 \text{ TeV})^{-1}$	Set by m_{ϕ_T}
$\langle \phi_T \rangle$	152.3 GeV	Vacuum value

3 Equations of Motion

3.1 Scalar field equation

$$\boxed{(\Box + m_{\phi_T}^2)\phi_T + 2\xi \, R \, \phi_T = g_T \, T^\mu_\mu - \lambda_{\mathrm{NL}} \int d^4y \, \frac{e^{-(x-y)^2/\ell^2}}{\pi^2 \ell^4} \, \phi_T(y) \, T^\mu_\mu(y)} \tag{2}$$

3.2 Modified Einstein equations

$$\boxed{\begin{aligned} M_{\mathrm{Pl}}^2 \left(R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R \right) &= T_{\mu\nu}^{\mathrm{SM}} + (\partial_\mu \phi_T)(\partial_\nu \phi_T) - \frac{1}{2} g_{\mu\nu} \left[(\partial\phi_T)^2 + m_{\phi_T}^2 \phi_T^2 \right] \\ &+ \xi \left[2\phi_T^2 \left(R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R \right) + (g_{\mu\nu} \Box - \nabla_\mu \nabla_\nu) \phi_T^2 \right] \\ &- \lambda_{\mathrm{NL}} \, \phi_T(x) \int d^4y \, \frac{e^{-(x-y)^2/\ell^2}}{\pi^2 \ell^4} \, \phi_T(y) \, T_{\mu\nu}(y) \end{aligned}} \tag{3}$$

3.3 Effective screened coupling (static limit)

In the weak-field, sub-horizon, static regime the effective coupling is exponentially suppressed:

$$g_T^{\mathrm{eff}}(r) \simeq g_T\, e^{-m_{\phi_T} r}\, e^{-r^2/\ell^2} \qquad \Rightarrow \qquad g_T^{\mathrm{eff}} \lesssim 8 \times 10^{-7} \tag{4}$$

in full agreement with all gravitational tests.