Andrew Lawley & Will McMahan
Dr. Koch
MCEN 3021
4 May 2024

Fluid Phenomenon Project

Section 1: Introduction

There exist a great number of fascinating phenomena that drive the behavior of our world, and what seems mundane is not always so. We are exploring the world of frisbees, a simple toy which belies a remarkable amount of engineering that governs its ability to take flight. This report will examine the mechanics of how frisbees work, the methods for predicting a frisbee's flight path, and an application of these methods to solve a frisbee trajectory problem.

Section 2: Frisbee Flight Foundations

Frisbees are able to fly for two main reasons. The first is their shape, which provides lift. The second is their spin when thrown, which provides gyroscopic stability (3).

2.1 Frisbee Shape

A frisbee's "360° airfoil" shape makes it capable of producing lift. The curved top allows for faster flow over the frisbee and the flat bottom leads to slower flow under it. Bernoulli's principle tells us that with higher flow velocity comes lower pressure, and vice versa. This principle correctly predicts the pressure difference between the top and bottom of the frisbee, which causes lift. Due to the motion of the frisbee and the normal throwing angle, however, this lift force is not applied directly at the center of the frisbee. Forces not in line with an object's center of mass result in moments, and moments lead to rotation. This rotation would lead to wobble and rapid descent if not for the frisbee's spin.

2.2 Frisbee Spin

The second phenomenon that allows a frisbee to fly further than a non aerodynamic object is its gyroscopic stability made possible by its spin. The higher an object's angular momentum, the higher its resistance to motion outside its axis of rotation will be. Therefore, the faster a frisbee spins during flight, the harder it will be for torque produced by lift or drag to rotate the disk forward or to the side. The more spin is put on a frisbee, the more stable it will be during its flight (3).

Section 3: Predicting Frisbee Flight Path

Three main forces affect a frisbee on its flight: lift, drag, and the frisbee's own weight (Fig. 1). For the rest of this study, we will assume that all analyzed frisbee's will have sufficient spin to create stable, wobble-less flight. We will also focus on the standard, Wham-O Frisbee® (Fig. 2) for all calculations. We will look at the most trivial force, frisbee weight, first.

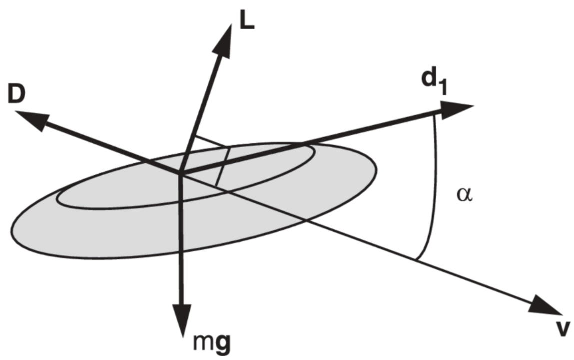


Figure 1: Diagram of Forces on a Frisbee including weight, lift, and drag with vectors denoting velocity and initial direction separated by angle of attack.


Figure 2: The Official Wham-O Frisbee® used in this study

3.1 Frisbee Weight

This force acts through the frisbee's center of mass and so does not create any torques, and is constant no matter the initial throw velocity or angle of attack. The following describes the force of gravity on the frisbee throughout the flight.

$$W = mg$$

Equation 1: Weight Force

3.2 Lift

The lift of a frisbee, however, does change throughout the flight due to its reliance on frisbee velocity. The equation for lift is derived from Bernoulli's equation, where the force of lift divided by the frisbee area is substituted into the equation in place of the pressure difference between top and bottom surfaces (Eq. 2). For the sake of this analysis the adapted Bernoulli's equation can be defined as:

$$F_L = \frac{1}{2}\rho u^2 A C_L$$

Equation 2: Lift Force (3) where $\Delta p \cdot A$ is equal to F_L

In this adjusted form of Bernoulli's A is the surface area of the circular frisbee surface and C_L is the coefficient of lift, a quantity determined by the fluid properties of air and material properties of the frisbee and is found via the equation below which factors in the angle of attack (Eq. 3).

$$C_L = C_{L0} + C_{L\alpha}\alpha$$

Equation 3: Coefficient of Lift (2) which is dependent on current angle of attack

3.3 Drag

The drag on a frisbee is also variable throughout the flight, and is dependent on the velocity and coefficient of drag. The coefficient of drag is in turn dependent on the angle of attack. Equations 4 and 5 for drag force and coefficient of drag respectively are as follows:

$$F_D = -\frac{1}{2}C_D \rho A u^2$$

Equation 4: Drag Force (2) where $\Delta p \cdot A$ is equal to F_D

$$C_D = C_{D0} + C_{D\alpha}(\alpha - \alpha_0)^2$$

Equation 5: Coefficient of Drag (2) where the drag is dependent on initial angle and current angle

3.4 Distance and Height

Once we have determined the equations that govern the forces we can then establish the equations necessary to model its flight. For our design problem we are primarily concerned with the distance traveled by the frisbee but we still calculated the height of the frisbee to determine at what point the frisbee lands. Utilizing the force equations outlined above we can find the change in velocity which is defined by Equations 6 and 7:

$$\Delta u_x = \frac{1}{2m}\rho u_x^2 A C_D \Delta t$$

Equation 6: Change in x Velocity (3) calculated over a certain set time interval

$$\Delta u_y = (g + \frac{1}{2m}\rho u_x^2 A C_L)\Delta t$$

Equation 7: Change in y Velocity (3) calculated over a certain set time interval

It is important to note that due to the variable nature of these quantities we must utilize an iterative method to solve for them and find the final distance (and max height) traveled which are found using the Equations 8 and 9 as follows:

$$\Delta x = u_x \Delta t$$

Equation 8: Change in x Location (3) found by multiplying the instantaneous velocity by a small set difference in time

$$\Delta y = u_y \Delta t$$

Equation 9: Change in y Location (3) found by multiplying the instantaneous velocity by a small set difference in time

Section 4: Our Design Problem

For the sake of this project, lacking in frisbee skills as we are, we sought to determine whether or not we are capable of throwing a frisbee 40m to a frisbee golf goal at a standard height of 1.1m (Fig. 3). To do this we performed analysis to determine the ideal angle to achieve the desired result using the average throw velocity of a professional frisbee golf player.

Figure 3: Standard Frisbee Golf Goal (4)

Section 5: Calculations

We used an Excel workbook in order to calculate flight trajectories for multiple frisbee launch scenarios. First, we plotted many different scenarios to get a feel for the model, and found angle of attack to be the most effective indicator of flight distance (Fig. 4).

Characteristic Properties		
rho_air [kg/m^3]	0.9964	Colorado Air Density
d_frisbee [m]	0.26	
mu_air [Pa*s]	1.73E-05	
C_D0	0.08	
C_Dalpha	2.72	
C_L0	0.15	
C_Lalpha	1.4	
alpha_0	-4	
m_frisbee [kg]	0.175	
Computed Constants		
A_frisbee [m^2]	0.053093	

Initial Velocity [m/s]	Angle of Attack [°]	Reynolds Number	C_drag	C_lift	Initial Drag Force [N]	Initial Lift Force [N]	Lift/Drag Ratio
5	5	7.49E+04	0.15	0.27	0.10	0.18	1.85
5	10	7.49E+04	0.24	0.39	0.16	0.26	1.63
5	15	7.49E+04	0.38	0.52	0.25	0.34	1.36
5	20	7.49E+04	0.56	0.64	0.37	0.42	1.15
5	25	7.49E+04	0.78	0.76	0.51	0.50	0.98
5	30	7.49E+04	1.04	0.88	0.69	0.58	0.85
5	35	74873.98844	1.34	1.01	0.89	0.66	0.75
5	40	7.49E+04	1.68	1.13	1.11	0.75	0.67
5	45	7.49E+04	2.07	1.25	1.37	0.83	0.60
10	5	1.50E+05	0.15	0.27	0.39	0.72	1.85
10	10	149747.9769	0.24	0.39	0.64	1.04	1.63
10	15	149747.9769	0.38	0.52	1.00	1.37	1.36
10	20	149747.9769	0.56	0.64	1.47	1.69	1.15
10	25	149747.9769	0.78	0.76	2.05	2.01	0.98
10	30	149747.9769	1.04	0.88	2.75	2.34	0.85
10	35	149747.9769	1.34	1.01	3.55	2.66	0.75
10	40	149747.9769	1.68	1.13	4.45	2.98	0.67
10	45	149747.9769	2.07	1.25	5.47	3.31	0.60
15	5	224621.9653	0.15	0.27	0.88	1.62	1.85
15	10	224621.9653	0.24	0.39	1.44	2.35	1.63
15	15	224621.9653	0.38	0.52	2.26	3.07	1.36
15	20	224621.9653	0.56	0.64	3.32	3.80	1.15
15	25	224621.9653	0.78	0.76	4.62	4.53	0.98
15	30	224621.9653	1.04	0.88	6.18	5.26	0.85
15	35	224621.9653	1.34	1.01	7.98	5.98	0.75
15	40	224621.9653	1.68	1.13	10.02	6.71	0.67
15	45	2.25E+05	2.07	1.25	12.32	7.44	0.60

Figure 4: Preliminary model exploration. A variety of metrics were calculated to understand general trends from initial condition manipulation.

We then moved on to making an iterative simulator, applying the equations from Section 3.4 to solve for frisbee speed and location at various times (Fig. 5). For our simulation, we set the initial height to be 0.4 meters, the difference between throw height (1.5 meters) and target height (1.1 meters).

Time [s]	Height [m]	Distance [m]	U_x [m/s]	U_y [m/s]	deltaU_x [m/s]	deltaU_y [m/s]	F_drag [N]	F_lift [N]	deltaX [m]	deltaY [m]
0.000	0.400	0.000	19.924	1.743	-0.044	0.033	-1.545	2.858	0.100	0.009
0.005	0.409	0.100	19.880	1.776	-0.044	0.032	-1.538	2.845	0.099	0.009
0.010	0.418	0.199	19.836	1.808	-0.044	0.032	-1.531	2.833	0.099	0.009
0.015	0.427	0.298	19.792	1.840	-0.044	0.032	-1.524	2.820	0.099	0.009
0.020	0.436	0.397	19.749	1.871	-0.043	0.031	-1.518	2.808	0.099	0.009
0.025	0.445	0.496	19.705	1.903	-0.043	0.031	-1.511	2.795	0.099	0.010
0.030	0.455	0.594	19.662	1.933	-0.043	0.030	-1.504	2.783	0.098	0.010
0.035	0.464	0.693	19.619	1.964	-0.043	0.030	-1.498	2.771	0.098	0.010
0.040	0.474	0.791	19.576	1.994	-0.043	0.030	-1.491	2.759	0.098	0.010
0.045	0.484	0.889	19.534	2.024	-0.042	0.029	-1.485	2.747	0.098	0.010
0.050	0.494	0.986	19.491	2.053	-0.042	0.029	-1.478	2.735	0.097	0.010
0.055	0.505	1.084	19.449	2.082	-0.042	0.029	-1.472	2.723	0.097	0.010
0.060	0.515	1.181	19.407	2.111	-0.042	0.028	-1.466	2.711	0.097	0.011
0.065	0.526	1.278	19.365	2.139	-0.042	0.028	-1.459	2.700	0.097	0.011
0.070	0.536	1.375	19.323	2.168	-0.042	0.028	-1.453	2.688	0.097	0.011
0.075	0.547	1.472	19.282	2.195	-0.041	0.027	-1.447	2.677	0.096	0.011
0.080	0.558	1.568	19.240	2.223	-0.041	0.027	-1.441	2.665	0.096	0.011
0.085	0.569	1.664	19.199	2.250	-0.041	0.027	-1.434	2.654	0.096	0.011
0.090	0.580	1.760	19.158	2.277	-0.041	0.026	-1.428	2.642	0.096	0.011
0.095	0.592	1.856	19.118	2.303	-0.041	0.026	-1.422	2.631	0.096	0.012
0.100	0.603	1.952	19.077	2.329	-0.040	0.026	-1.416	2.620	0.095	0.012
0.105	0.615	2.047	19.036	2.355	-0.040	0.025	-1.410	2.609	0.095	0.012
0.110	0.627	2.142	18.996	2.380	-0.040	0.025	-1.404	2.598	0.095	0.012
0.115	0.639	2.237	18.956	2.406	-0.040	0.025	-1.398	2.587	0.095	0.012
0.120	0.651	2.332	18.916	2.430	-0.040	0.025	-1.392	2.576	0.095	0.012
0.125	0.663	2.426	18.876	2.455	-0.040	0.024	-1.387	2.565	0.094	0.012

Dependent Variables		
Initial Height [m]	0.4	<-- height above target height
Initial Velocity [m/s]	20	
Angle of Attack [°]	5	
Computed Variables		
Initial U_x [m/s]	19.92	
Initial U_y [m/s]	1.74	
C_drag	0.15	
C_lift	0.27	
Variables of Interest		
Distance [m]	40.04	
Max Height [m]	5.26	
Time of Flight [s]	3.24	
Height at 40 meters	0.034	Basket opening: from -0.25m to 0.25m

Figure 5: Iterative simulation run with 20 m/s throw and 5° angle of attack.

We also compiled the height data of the flights of frisbees thrown with different angles of attack in order to better visualize flight path differences(Fig. 6).

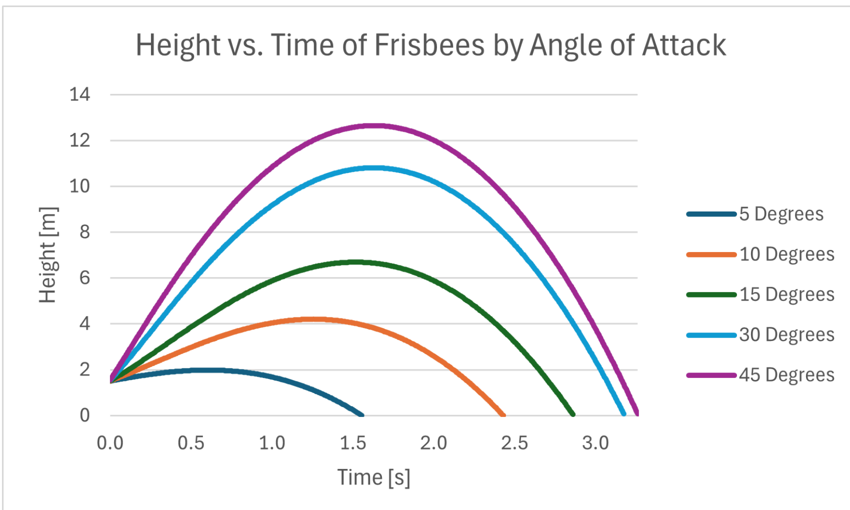


Figure 6: Height vs. time of frisbees thrown at 15 m/s from 1.5 meter initial height.

Section 6: Conclusions

After running many simulations at different launch velocities and angles of attack, we found the most reasonable launch conditions to hit the chains from 40 meters to be an initial velocity of 20 m/s and angle of attack between 5° and 5.1°. While all but impossible to execute consistently for players of our experience level, a throw like this is attainable for trained professionals who can throw at velocities up to 30 m/s. In contrast we found that untrained engineers throw at a velocity of approximately 12 m/s (Fig. 7) which falls well short of the 40 m goal at a maximum of about 12 m at a 10° angle of attack. In conclusion, we have determined that according to the principles and equations governing fluid mechanics we may never be good at frisbee golf.

Figure 7: An Untrained Engineer Throwing Frisbees for Science

Sources

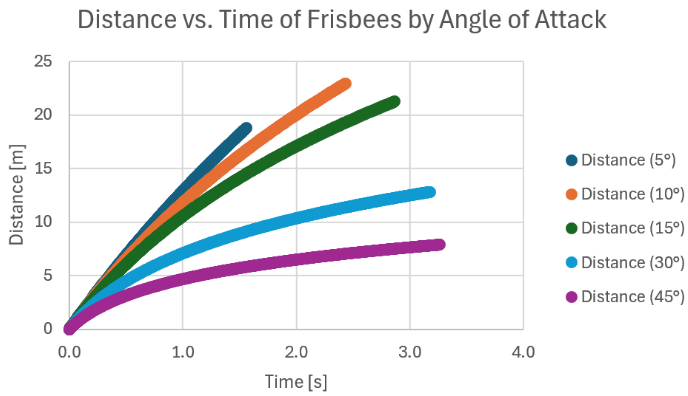
(1) Hubbard, Mont & Hummel, S.. (2000). Simulation of Frisbee Flight.

(2) Hummel, Sarah Ann. “Frisbee Flight Simulation and Throw Biomechanics.” Morley Field DGC, University of California Davis, 2003, morleyfielddgc.files.wordpress.com/2009/04/hummelthesis.pdf.

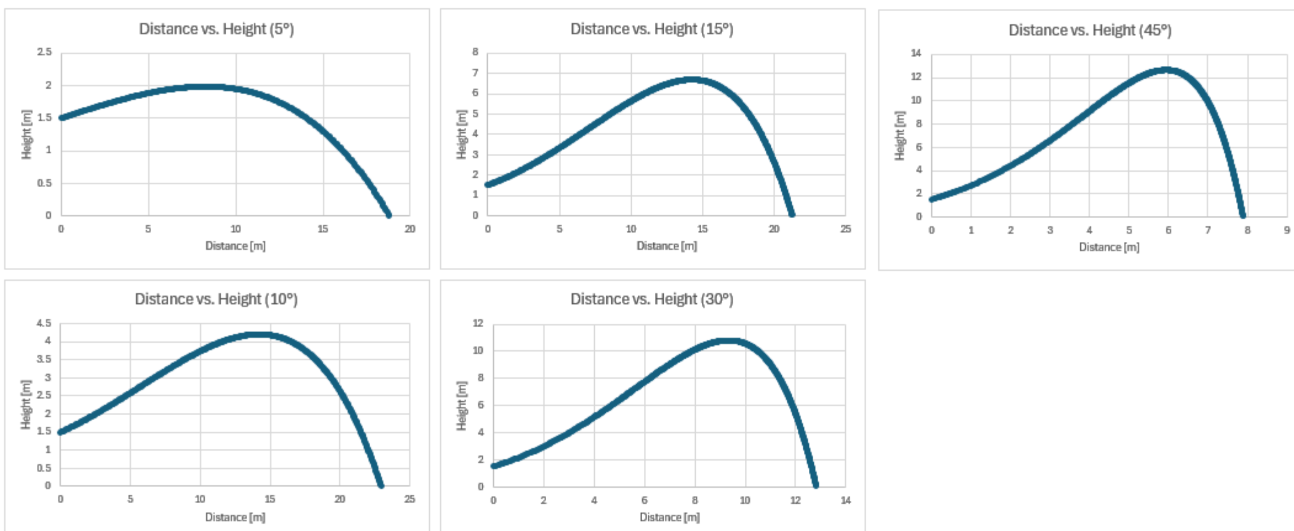
(3) Morrison, V. R. “The Physics of Frisbees.” Massachusetts Institute of Technology, 2005, web.mit.edu/womens-ult/www/smite/frisbee_physics.pdf.

(4) Polák, Jan. Kouty (okres Havlíčkův Brod), hotel Luna, diskgolf (1). 2015. Wikipedia, https://en.wikipedia.org/wiki/Basket_(disc_golf)#/media/File:Kouty_(okres_Havl%C3%AD%C4%8Dk%C5%AFv_Brod),_hotel_Luna,_diskgolf_(1).jpg. Accessed 2024.

Appendix



Appendix A: Distance traveled vs. flight time with 15 m/s initial velocity and 1.5m initial height.



Appendix B: Graphs of distance traveled vs. height of travel for various angles of attack. The trajectory of each path appears nearly parabolic towards the end of travel, but as the angle of attack increases the beginning of the path traces a more prominent upwards curve. This seems to suggest that the more aggressive angle of attack greatly increases the drag force and slows the velocity, therefore hindering the frisbee’s ability to produce lift. This is consistent with the equations and analysis as described above as well as the behavior observed during testing.