$$a f(x) + b f\left(\frac{1}{x}\right) = \frac{1}{x} - 5 \quad \text{①} \quad , a \neq b \quad , \text{find } f(x)$$

Evaluate at $\frac{1}{x}$

$$a f\left(\frac{1}{x}\right) + b f(x) = x - 5 \quad \text{②}$$

$a \times$ ① $\quad a^2 f(x) + ab f\left(\frac{1}{x}\right) = \frac{a}{x} - 5a \quad$ — ③

$b \times$ ② $\quad ab f\left(\frac{1}{x}\right) + b^2 f(x) = bx - 5b \quad$ — ④

③ − ④

$$(a^2 - b^2) f(x) = \frac{a}{x} - bx - 5a + 5b$$

$$f(x) = \frac{1}{a^2 - b^2}\left[\frac{a}{x} - bx - 5a + 5b\right]$$

$$\frac{1}{\frac{1}{x}} = x$$

$$\frac{\frac{-a}{-b}}{\frac{-c}{-d}} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc}$$

$$= \frac{\text{Product of Extremes}}{\text{Product of Means}}$$

$$3x = 18$$

$$x = \frac{18}{3}$$

$f(x) = \frac{ax+b}{cx+d}$, $f(f(x)) = x \Rightarrow \boxed{d = -a}$

$\frac{ax+b}{d}$ $f(f(x)) = x$.

$f(f(x)) = f\left(\frac{ax+b}{cx+d}\right) = \dfrac{a\left(\frac{ax+b}{cx+d}\right) + b}{c\left(\frac{ax+b}{cx+d}\right) + d}$

$\Rightarrow \dfrac{x \cdot 1 + 0}{x \cdot 0 + 1}$

$= \dfrac{x}{1}$

$= x$

$= \dfrac{\left[a(ax+b) + b(cx+d)\right] / \cancel{(cx+d)}}{\left[c(ax+b) + d(cx+d)\right] / \cancel{(cx+d)}}$

$= \dfrac{x\boxed{(a^2+bc)} + \boxed{(ab+bd)}}{x\boxed{c(a+d)} + \boxed{(bc+d^2)}} = x$

($a^2+bc = 1$, $ab+bd = 0$, $c(a+d) = 0$, $bc+d^2 = 1$)

$c(a+d) = 0$

$\Rightarrow \boxed{c = 0}$ or $\underline{a+d = 0}$

$\underline{\boxed{d = -a}}$.

$A = \{1, 2, 3\}$, $B = \{1, 2, \cdots, 8\}$

$f : A \to B$

$f = \{(a, b) : b = a^2, \ a \in A, b \in B\}$

f is not a function

$\in$ belongs to

(1,1)

(2,4)

(3,9)

$f : A \to \mathbb{N}$

$A = \{9, 10, 11, 12, 13\}$.

B

$f(9) = 3$

$f(10) = 5$

$f(11) = 11$

$f(12) = 3$

$f(13) = 13$

Absolute Value Equations

Absolute Value Inequalities

$|2x-1| < 5$

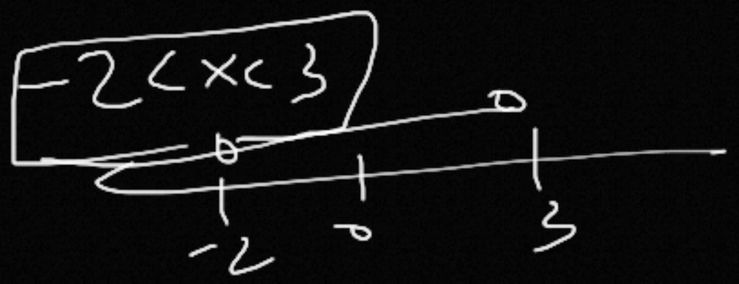
$-5 < 2x-1 < 5$

$-5 < 2x-1$ and $2x-1 < 5$

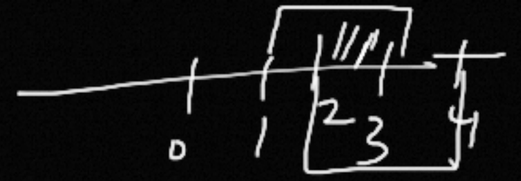
$-4 < 2x$ and $2x < 6$

$-2 < x$ and $x < 3$

$-2 < x < 3$

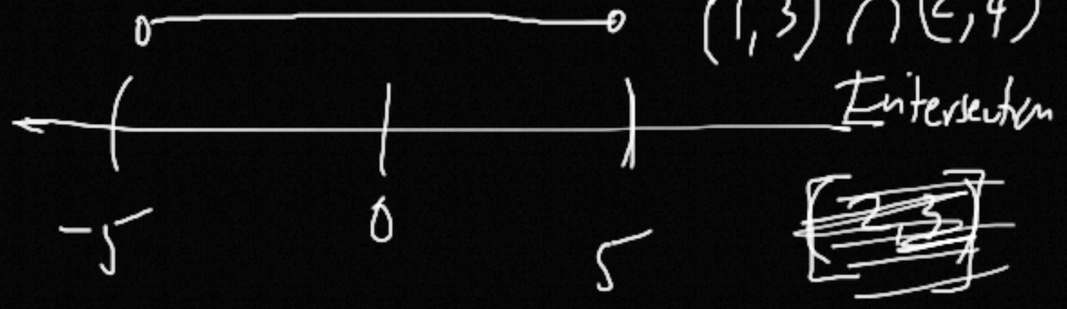


$|x| < 5$



$(1,3) \cap (2,4)$

Intersection



$(2,3)$

$|x| < 5$, $-5 < x < 5$

$|x| > 5$

$(-\infty, -5) \cup (5, \infty)$

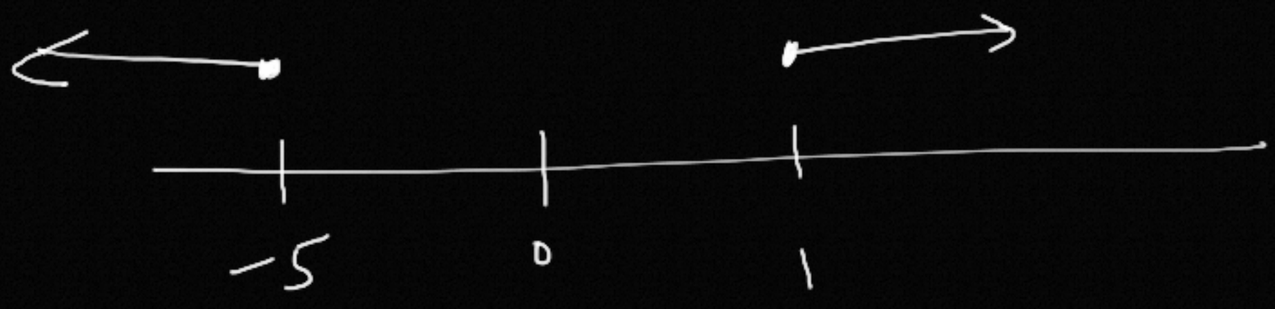
union

$|2x+4| \geq 6$

$2x+4 \leq -6$ or $2x+4 \geq 6$

$2x \leq -10$ or $2x \geq 2$

$x \leq -5$ or $x \geq 1$



$(-\infty, -5] \cup [1, \infty)$ ✓

Absolute Value functions

$f(x) = x^2$

$f(x) = |x|$