CONTINUITY

#Geometrical meaning of Continuity

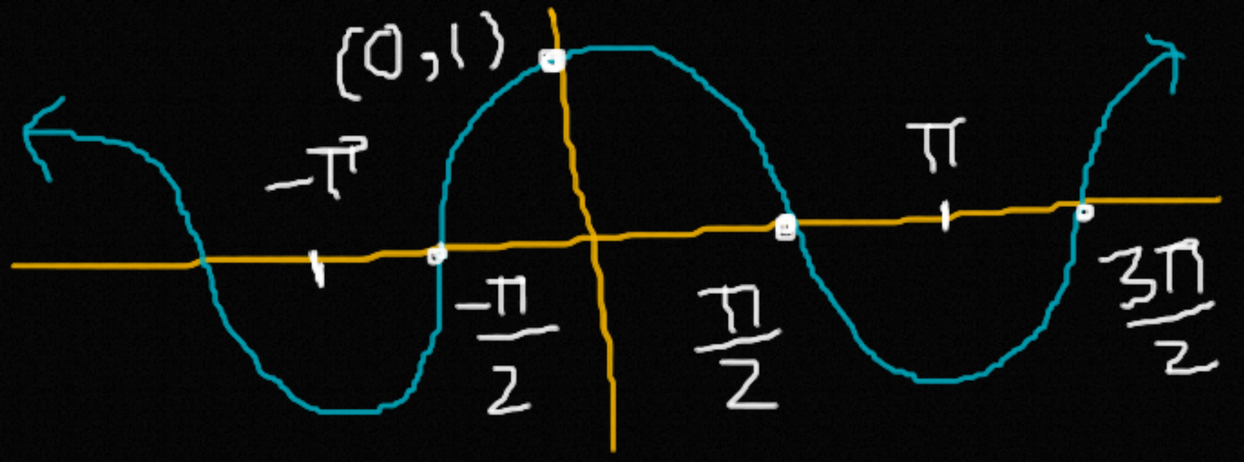
① If we need not to pick up our pen while drawing the graph of a function, then that function is continous everywhere, that is, throughout its domain.

example:

a) $f(x) = \cos x$

domain $= \mathbb{R}$

↖ set of all real numbers

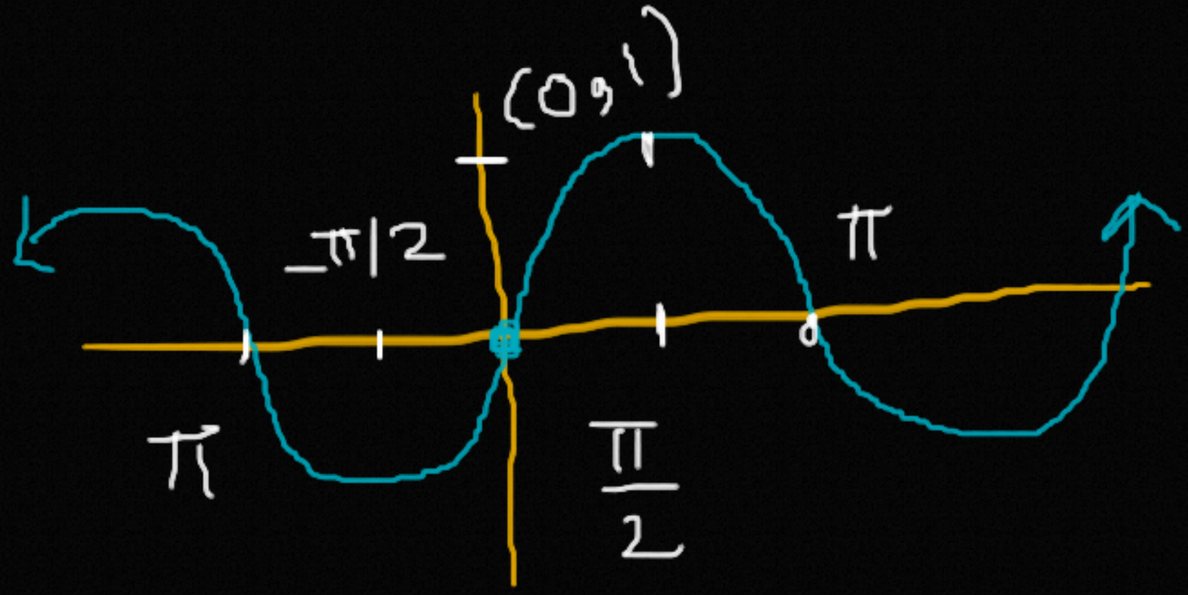


b) $f(x) = \sin x$

domain $= \mathbb{R}$

or

$(-\infty, \infty)$



$\pi^c/4 = 45^\circ$

$\pi^c/2 = 90^\circ$

radian → $\pi^c = 180^\circ$ → degree

$2\pi^c = 360^\circ$

$\pi^c/6 = 30^\circ$

$\pi^c/3 = 60^\circ$

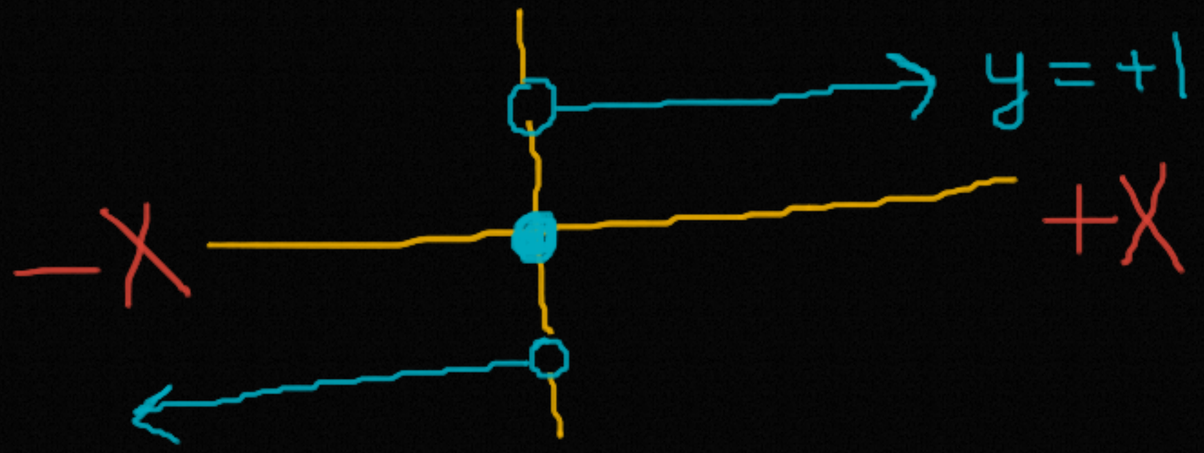
(2) But if I pick my pen at a point, while drawing the graph of a function, then that function is discontinous at THAT POINT now

examples:

a) $f(x) = \operatorname{sgn}(x)$

$y = \text{signum } x$

$$y = \begin{cases} +1 & ; \text{ if } x > 0 \\ 0 & ; \text{ if } x = 0 \\ -1 & ; \text{ if } x < 0 \end{cases}$$



piece-wise function

$f(x) = \operatorname{sgn}(x)$ is not continuous at $x = 0$

$\circ \rightarrow$ Not including

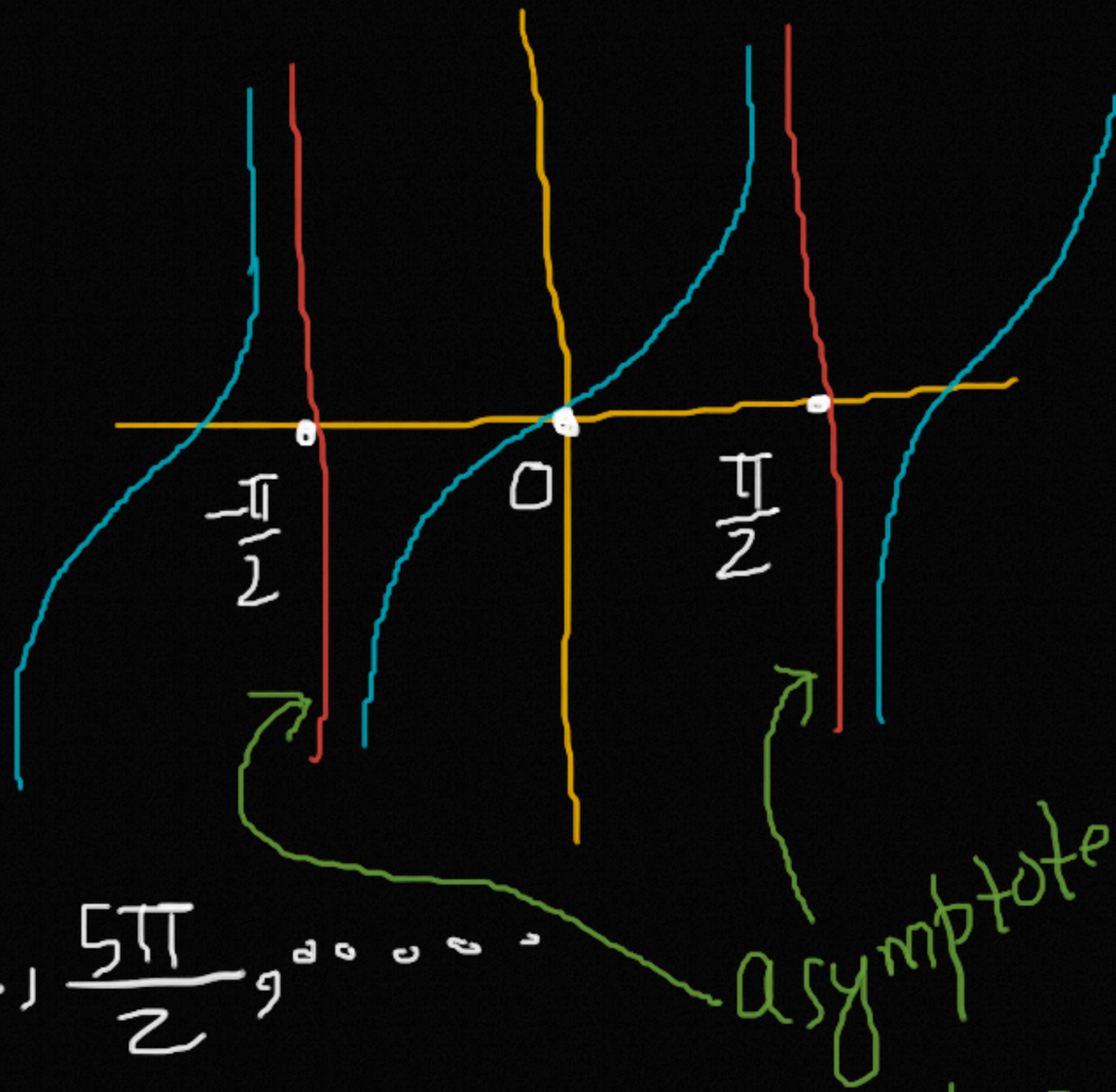
$\bullet \rightarrow$ including

b) $f(x) = \tan x$

is discontinuous at $x = \begin{pmatrix} \text{odd} \\ \text{multiple} \end{pmatrix} \frac{\pi}{2}$

$$x = (2n+1)\frac{\pi}{2}$$

$$x = \ldots, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots$$



(line which touches a curve at ∞)

Two functions:

If f and g are two cts. functions at $x=a$, then

1. $f+g$
2. $f-g$
3. $f \cdot g$
4. $\frac{f}{g}$; $g \neq 0$

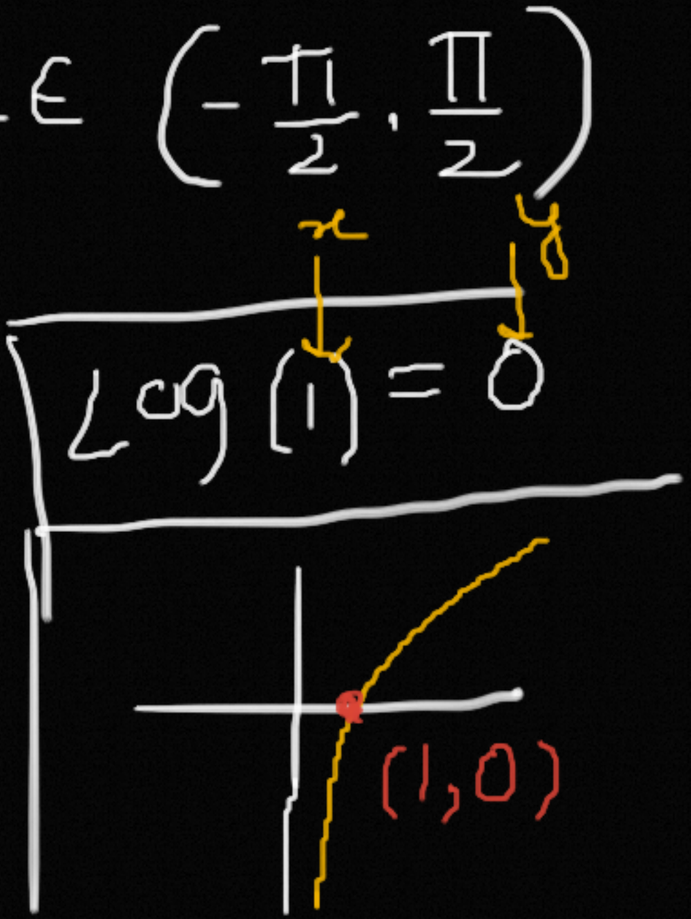
are also cts. at $x=a$

Q If $f(x) = \tan x$ and $g(x) = \log_{10} x$ for $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, then will $f(x) + g(x)$ also continuous?

Sol: $f(x) = \tan x$ is cts. $\Big\}$ in $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$g(x) = \log_{10} x$ is cts $\Big\}$

Hence $f(x) + g(x)$ will also be cts. function in $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$



Composition of functions :

Let $f(x) = \cos x$

$g(x) = x^3$

$g \circ f$

$g \circ f(x)$

$g(f(x))$

$g(\cos x)$

$(\cos x)^3$

$f \circ g$

$f \circ g(x)$

$f(g(x))$

$f(x^3)$

$\cos(x^3)$

$f \circ g \neq g \circ f$

NOTE whenever we are finding $f(g(x))$

always consider

range of $g(x) \subseteq$ domain of $f(x)$

output of $g(x) \subseteq$ input of $f(x)$

$\subseteq$ ↑ Subset

always check

If $f(x)$ and $g(x)$ are cts. at $x=a$ then fog and gof are also cts. at $x=a$

Q Discuss the Continuity of function $h(x) = \sin|x|$

Sol: There are 2 functions in $h(x)$

Let $f(x) = \sin x$ and $g(x) = |x|$

$f \circ g$

$f(g(x))$

$f(|x|)$

$\sin|x|$

$g \circ f$

$g(f(x))$

$g(\sin x)$

$|\sin x|$

$f(x) = \sin x$ is cts. at $x \in \mathbb{R}$

$g(x) = |x|$ is cts. at $x \in \mathbb{R}$

Hence $f \circ g$ is also cts. at $x \in \mathbb{R}$

but do check that condition is

ouput of $g(x) \subseteq$ input of $f(x)$

HW

① assignment → today → X

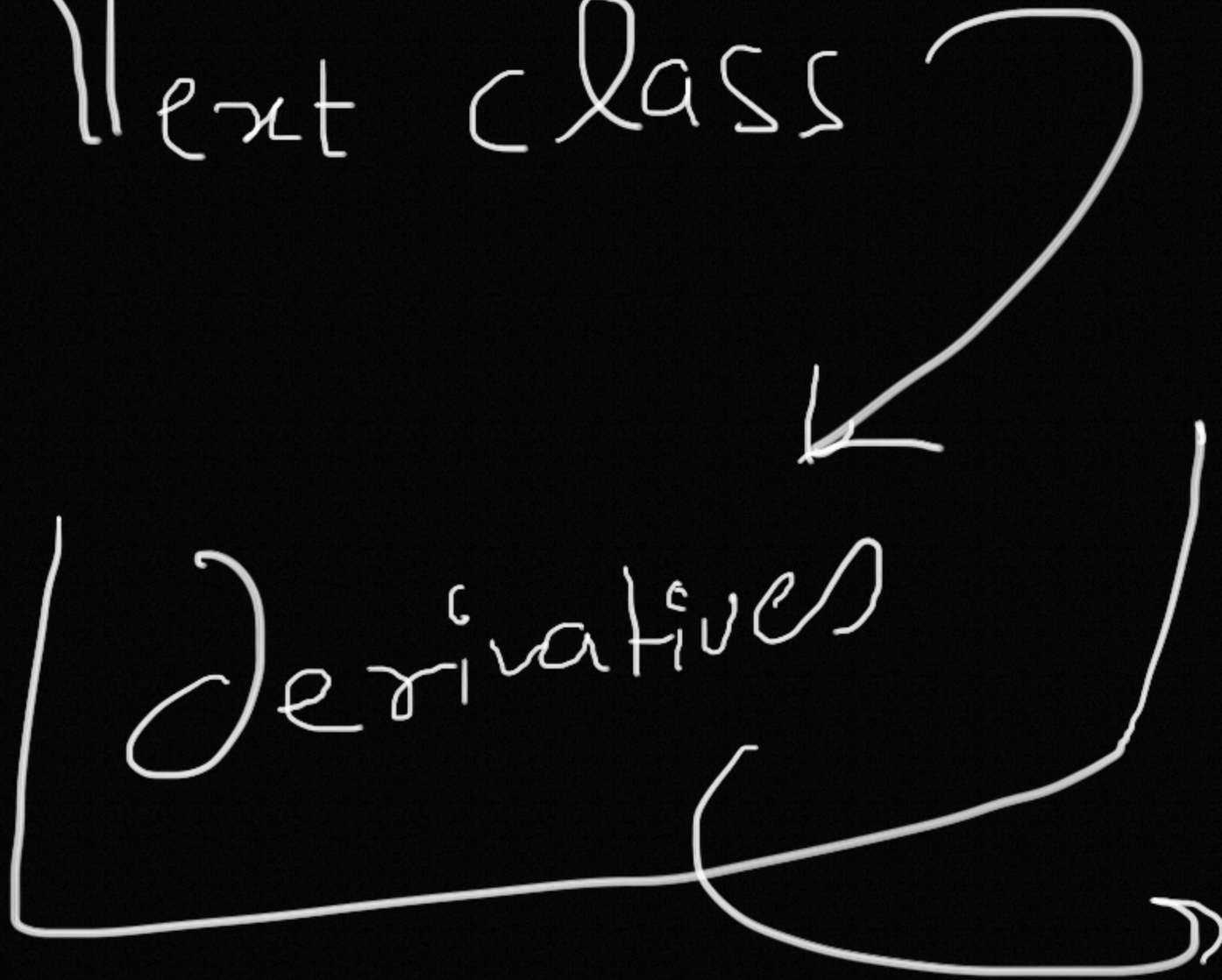
but
do complete Yesterday's assignment (set 2)

set 1 → 5 as limits
→ 5 as city
} done

(2) Find points of discontinuity for function $y = \frac{1}{t^2 - t - 6}$ if $t = \frac{1}{x - 2}$

If any doubt, ask tomorrow

Next class



Great chapter