

3D finite-difference time-domain forward modeling with convolutional perfectly matched layers (CPML) absorbing boundary condition for marine CSEM

Zhou Xiaochen¹, Hu Xiangyun¹, Han Bo¹

¹Institute of Geophysics and Geomatics, China University of Geosciences, Wuhan, China

SUMMARY

Marine CSEM surveying has been viewed as a new exploration technology for mapping offshore hydrocarbon in recent years. To interpret the received signals correctly, we need new forward modeling and inversion tools. This paper has developed 3D finite-difference time-domain (fdtd) forward modeling codes with the convolutional perfectly matched layers (CPML) absorbing boundary condition to reduce the error of numerical solutions. We compare numerical solution to semi-analytic solution for a 1D model to provide a self-check on the fdtd solution in 2000 m deep waters. The agreement between numerical solution and semi-analytic solution proves the viability and validity of the fdtd codes. Then we simulate electric field responses of a 3D model in the same water depth as 1D model. 3D numerical experiment shows that we can identify the 3D reservoir boundaries with amplitudes and phases information. However, the characters of boundaries become hard to confirm when frequencies are gradually increasing. We also find phase responses are more sensitive than amplitudes responses when anomalous body is presenting, which is in according with plane-layered modeling. Hence phase responses are very useful in marine CSEM explorations. In conclusion, this paper focuses on high accuracy of numerical solutions and may have great impact on 3D survey planning, imaging, data processing, and inversion.

Keywords: marine CSEM, finite-difference time-domain (fdtd), forward modeling, convolutional PML

INTRODUCTION

Marine controlled-source electromagnetic (CSEM) methods, which has been applied to explore hydrocarbon bearing reservoirs in deep water areas for decades (Constable, 2010). It uses a high-powered mobile source to transmit a low-frequency signal through the Earth to an array of receivers, which are placed on the seafloor in and around a target area (Eidesmo et al., 2002; Ellingsrud et al., 2002; Amundsen et al., 2006). Marine CSEM data provide rock and fluid properties information which can reduce exploration risk particularly when the seismic interpretation is uncertain (Constable and Srnka, 2007).

To reduce erroneous interpreting of data, we need accurate forward modeling and inversion tools. The finite-difference method is a widespread numerical analysis tool by solving partial differential equations on computers. Yee (1966) introduced the finite-difference time-domain (fdtd) method for solving Maxwell's equations. Berenger (1994) introduced the perfectly matched layers (PML) method, which added reflectionless boundaries to calculate region. 3D numerical experiments indicated that the efficiency of the PML method of free-space simulation (Berenger, 1996). Liu (1997) extended Berenger's PML theory to conductive media. Roden and Gedney (2000) introduced the convolutional PML (CPML) method which provided a number of advantages over the traditional

implementations of the PML, especially in low frequencies condition.

Marine CSEM is a low frequency method. It is suggested that accurate numerical results could be acquired if we implement the CPML absorbing boundary conditions in modeling. The Marine CSEM signals is extremely weak because of the attenuation of the electromagnetic waves, which provides a challenge to our modeling work. This paper pays attention to improving the accuracy of numerical solutions of fdtd modeling.

THEORY

The CPML is based on the stretched-coordinate form of the PML. The CPML is more accurate than the classic PML, more efficient, and better suited for the application of domain with generalized materials (Taflove and Hagness, 2005).

The explicit update for E_x is given by:

$$E_x \Big|_{i+1/2,j,k}^{n+1/2} = CA \Big|_{i+1/2,j,k} E_x \Big|_{i+1/2,j,k}^{n-1/2} + CB \Big|_{i+1/2,j,k} \left(\frac{H_z \Big|_{i+1/2,j+1/2,k}^n - H_z \Big|_{i+1/2,j-1/2,k}^n}{\kappa_{y\Delta y}} - \frac{H_y \Big|_{i+1/2,j,k+1/2}^n - H_y \Big|_{i+1/2,j,k-1/2}^n}{\kappa_{z\Delta z}} + \Psi_{E_x,y} \Big|_{i+1/2,j,k}^n - \Psi_{E_x,z} \Big|_{i+1/2,j,k}^n \right) \quad (1)$$

Where $\Psi_{E_x,y}$ and $\Psi_{E_x,z}$ are discrete convolution terms stored only in PML regions with y-normal and z-normal

interface boundaries, respectively. These convolution terms are updates as follows:

$$\Psi_{Ex,y} \Big|_{i+1/2,j,k}^n = b_{yj} \Psi_{Ex,y} \Big|_{i+1/2,j,k}^{n-1} + C_{yj} \left(\frac{H_z \Big|_{i+1/2,j+1/2,k}^n - H_z \Big|_{i+1/2,j-1/2,k}^n}{\Delta y} \right) \quad (2)$$

$$\Psi_{Ex,z} \Big|_{i+1/2,j,k}^n = b_{zk} \Psi_{Ex,z} \Big|_{i+1/2,j,k}^{n-1} + C_{zk} \left(\frac{H_y \Big|_{i+1/2,j,k+1/2}^n - H_y \Big|_{i+1/2,j,k-1/2}^n}{\Delta z} \right) \quad (3)$$

And the coefficients CA and CB are given by:

$$CA \Big|_{i+1/2,j,k} = \left(1 - \frac{\sigma_{i+1/2,j,k} \Delta t}{2 \varepsilon_{i+1/2,j,k}} \right) / \left(1 + \frac{\sigma_{i+1/2,j,k} \Delta t}{2 \varepsilon_{i+1/2,j,k}} \right) \quad (4)$$

$$CB \Big|_{i+1/2,j,k} = \left(\frac{\Delta t}{\varepsilon_{i+1/2,j,k}} \right) / \left(1 + \frac{\sigma_{i+1/2,j,k} \Delta t}{2 \varepsilon_{i+1/2,j,k}} \right) \quad (5)$$

The other five field components, Ey, Ez, Hx, Hy and Hz, have similar expressions.

NUMERICAL EXAMPLE

We have developed 3D ftd forward modeling codes with the CPML absorbing boundary condition to simulate marine CSEM responses of reservoir targets. Above all, we compare numerical solution to semi-analytic solution for a 1D model to provide a self-check on the FDTD solution. The 1D semi-analytic solution is given by plane-layered modeling (Løseth, Pedersen et al., 2006; Løseth and Ursin, 2007; Løseth, 2011). Here, we calculate electric field responses to detect reservoir in 2000 m deep waters. Then we present the results of ftd modeling to infer some useful characteristics of the 3D marine CSEM response.

1D model self-check

Modeling was performed with a plane-layered model and a 3D model. The two models are shown in Figure 1. Both models consist of a 2000m-deep-water layer. A regular grid with 100-m spacing in all three dimensions was used. The horizontal electric-dipole source is placed 100 m above seafloor level, pointing along the source-receiver line. The electro-magnetic field is recorded at the seafloor at stations with 400-m horizontal intervals along a line, paralleling x- coordinates.

In Figure 1(a), the total size of the 1D model is $N_x = 221$, $N_y = 60$, and $N_z = 71$ along the x-, y-, and z- coordinates, respectively. It includes the PML absorbing boundaries of 10 grid points at each side. And the calculate region is symmetric along the x- and y- coordinates. So we show only the responses for the transmitter on the right because the two sets of data are same.

Figure 2 shows the inline amplitudes and phases at 0.25 Hz. No significant differences between the two methods are found in MVO (magnitude variation with offset) plot,

but we can see slight deviation in PVO (phase variation with offset) plot, especially at the very near offsets and large offsets. This diversity is a result of the spatial and time discretization of Maxwell's equations. We also find that phase responses are more sensitive than amplitudes responses to the diversity.

Figure 3 shows the inline amplitudes and phases at 0.75 Hz and displays the similar characters as Figure 3. At this frequency, in PVO plot, the ftd modelling begins to deviate from the plane-layered modeling at very small and large offsets. The characters of the magnetic phase responses are similar to that of electric responses.

After implementing a self-check on the ftd solution, we obtain the field data of a 1D model. Though a small phase error can be seen in PVO plot we also conclude that agreement is excellent in both the amplitude and phase responses.

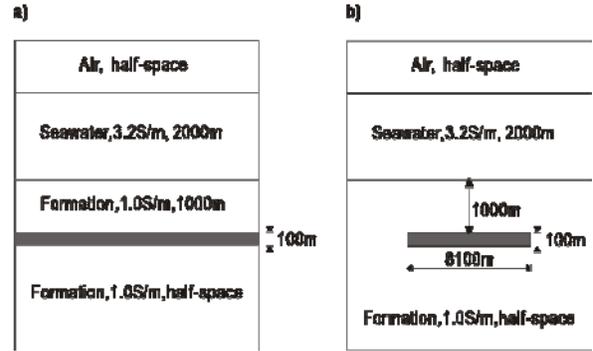


Figure 1. Two marine models. (a) a plane-layered model used for comparison of results. (b) a 3D reservoir model. Both models consist of a 2000m-deep-water layer.

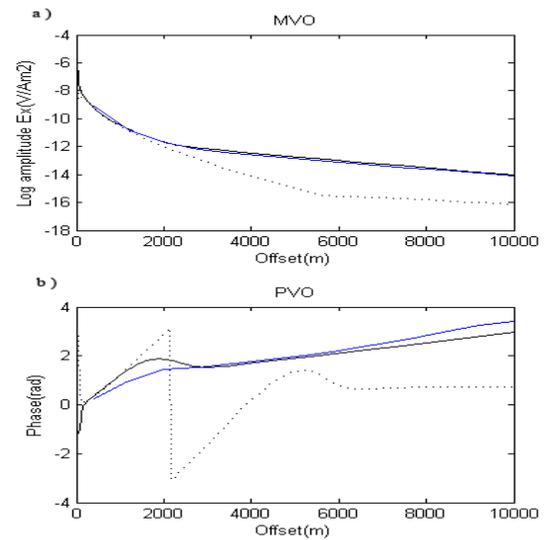


Figure 2. A comparison of the ftd (blue line) and semi-analytic (black line) solutions for 1D model (Figure 1a) at 0.25 Hz. The (a) amplitude and (b) phase of the inline horizontal electric field are shown. The black dashed line shows the background responses.

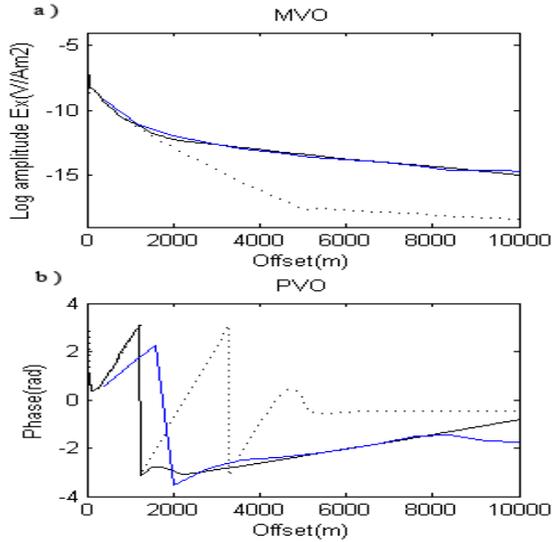


Figure 3. A comparison of the ftd (blue line) and semi-analytic (black line) solutions for the 1D model (Figure 1a) at 0.75 Hz. The (a) amplitude and (b) phase of the inline horizontal electric field are shown. The black dashed line shows the background responses.

3D model study

We have checked the viability and validity of the ftd codes in last section. Now, we simulate electric responses of the 3D reservoir model.

In Figure 1(b), the size of 3D reservoir is $N_x=81$, $N_y=30$, and $N_z=1$. And the 3D reservoir is situated in centre of the formations and is symmetric along the x- and y-coordinates. So we show only the responses for the transmitter on the right like 1D model.

Figure 4 shows the inline amplitudes and phases at 0.25 Hz. We can't see significant amplitudes differences between the 1D model and the 3D reservoir model at small offsets, but 3D reservoir model responses begin to deviate from the 1D model responses at large offsets (> 4km). Because phase responses are more sensitive than amplitudes responses, this deviation also can be seen in PVO plot. So we can identify the 3D reservoir boundaries in packaging formations by carrying out the 3D ftd procedure.

Figure 5 and Figure 6 shows the inline amplitudes and phases at 0.5 Hz and 0.75 Hz, respectively. They displays the similar characters as Figure 4. But the identifiable characters of 3D reservoir boundaries are

becoming hard to confirm gradually along with the increasing frequencies.

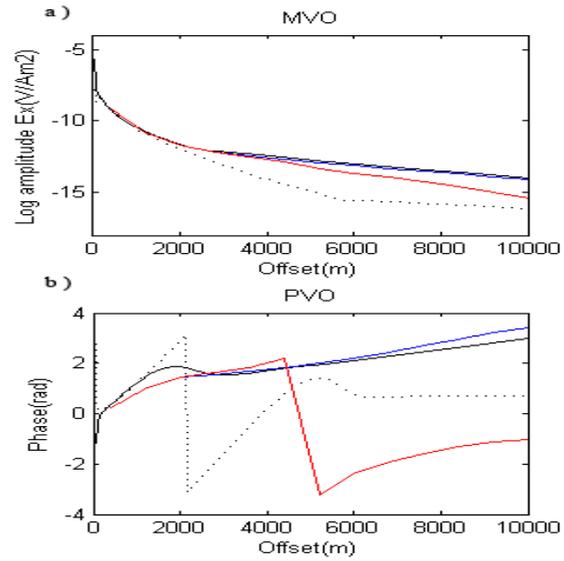


Figure 4. A comparison of the ftd (red line) and semi-analytic (black line) solutions for the 3D model (Figure 1b) at 0.25 Hz. The (a) amplitude and (b) phase of the inline horizontal electric field are shown. The black dashed line shows the background responses. The blue line shows the ftd solution of the 1D model.

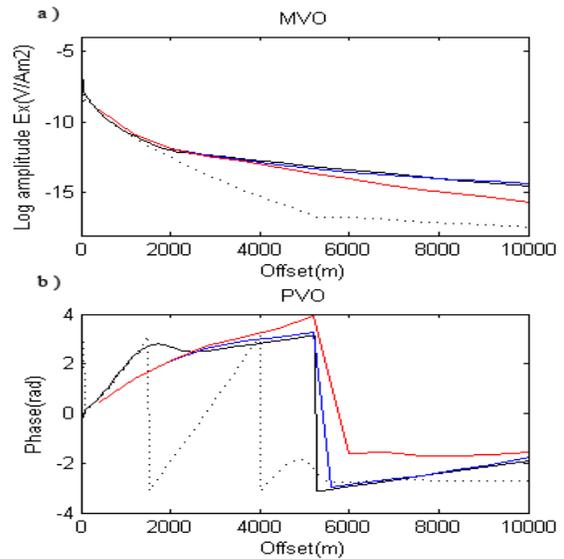


Figure 5. A comparison of the ftd (red line) and semi-analytic (black line) solutions for the 3D model (Figure 1b) at 0.5 Hz. The (a) amplitude and (b) phase of the inline horizontal electric field are shown. The black dashed line shows the background responses. The blue line shows the ftd solution of the 1D model.

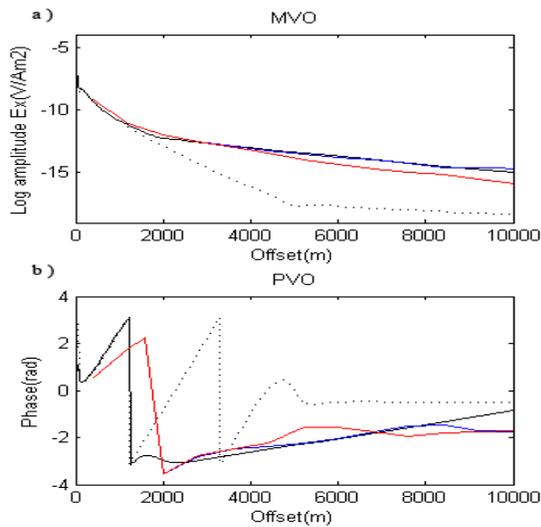


Figure 6. A comparison of the fdtd (red line) and semi-analytic (black line) solutions for the 3D model (Figure 1b) at 0.5 Hz. The (a) amplitude and (b) phase of the inline horizontal electric field are shown. The black dashed line shows the background responses. The blue line shows the fdtd solution of the 1D model.

CONCLUSIONS

Implementing 3D fdtd modeling with CPML absorbing boundary condition, we have learned some useful characters about 3D reservoir in low resistive formations. The numerical examples show that (1) we can identify the 3D reservoir boundaries in packaging formations, however, which become hard to confirm gradually along with the increasing frequencies. (2) we find that phase responses are more sensitive than amplitudes responses when anomalous body is presenting. Furthermore, this is in accordance with plane-layered modeling. Our work may have great impact on 3D survey planning, imaging, data processing and inversion. But our job needs revising to reduce CPU time. Because air layer is taken account of in the models, timestep is very small (10^{-6} s). These modelings consume too much time (about 20 days). The fast modeling method, which was introduced by Maaø (2007), will be taken into the next work.

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