5th International Symposium on Three-Dimensional Electromagnetics
Sapporo, Japan, May 7–9, 2013

Rigorous interpolation near interfaces in 3D finite-difference EM modeling

Daniil V. Shantsev[1], and Frank A. Maaø[2]
[1]EMGS, Trondheim, Norway
[2]Statoil, Trondheim, Norway

SUMMARY

We present a rigorous method for interpolation of electric and magnetic fields close to an interface with a conductivity contrast. The method takes into account not only a well-known discontinuity in the normal electric field, but also discontinuity in all the normal derivatives of electric and magnetic tangential fields. The proposed interpolation method is applied to marine 3D controlled-source electromagnetic modeling where sources and receivers are located close to the interface between conductive seawater and resistive formation. It is shown that for the finite-difference time-domain method based on the Yee grid, the interpolation error at the interface can dominate the numerical dispersion error in a broad range of cell sizes. The error is dramatically reduced if the proposed interpolation scheme is used. The proposed interpolation operators can have arbitrary length and can handle either uniform or non-uniform grids as well as arbitrary orientation of the interface with respect to the grid.

Keywords: forward modeling, interpolation, marine CSEM, finite difference, discontinuity

INTRODUCTION

In finite-difference modeling, nodes of the simulation grid do not necessarily coincide with exact positions of the modeled sources and receivers, and therefore one needs to interpolate. For example, in a conventional marine controlled-source electromagnetic (CSEM) survey the receivers are placed on the seafloor which is never ideally flat, and hence does not conform to the modeling grid. Moreover, the staggered Yee grid often used for EM modeling places different field components at different nodes, while a CSEM receiver measures all of them at the same location. Hence, in order to find all components at a desired recording position, one always needs to interpolate the field values computed at the nearby nodes.

Interpolation should be done with a special care when there exists a sharp conductivity contrast close to the interpolation site. This is exactly the case for a typical marine CSEM survey since both the source and receivers are located very close to the seabed where the conductivity abruptly changes by a factor of 3 – 10. It is well-known that the normal electric field experiences a jump at such an interface. In this paper we will however focus on the tangential field components, E_x, E_y, H_x and H_y (the z-axis is assumed normal to the interface), which are most extensively used in CSEM data analysis. Even though the tangential fields are continuous across the interface, their derivatives over z experience an abrupt jump. In other words, the field profiles as a function of z exhibit a sharp bend when crossing the interface and disregarding this bend in interpolation can lead to significant errors.

One common approach to tackle this problem is to interpolate using only nodes above the seafloor or, strictly speaking, to extrapolate, see e.g. (Abubakar *et al.*, 2008). A more powerful method is the essentially non-oscillatory interpolation where the interpolation stencil is chosen on the fly to minimize oscillations in the interpolation polynomial (Wirianto, Mulder, & Slob, 2011). When the modeling is performed by computing separately the primary and secondary fields, the interpolation error can be reduced if only the secondary field is interpolated (Streich, 2009).

All the mentioned approaches aim at *minimizing* the interpolation error arising due to discontinuities at the interface. In this work, we propose a more fundamental solution: to *directly compute* the derivative jumps at the interface and take them into account when performing the interpolation. We compute jumps in dE_x/dz, dE_y/dz, dH_x/dz and dH_y/dz from the Maxwell equations using conductivity values below and above the interface. The proposed rigorous interpolation allows improvement in the accuracy compared to the existing schemes because (i) it uses nodes on both sides of the interface and (ii) it utilizes the available information about the conductivity contrast at the interface. Note that the CPU cost of interpolation is orders of magnitudes smaller than the cost of computing fields at the nodes by solving the Maxwell equations, therefore it definitely pays off to use the most accurate interpolation approach.

DERIVATIVE JUMPS AT INTERFACE

The Maxwell equations without sources, in a non-magnetic medium and with negligible displacement current read

$$\mu_0 \dot{\mathbf{H}} = -\text{curl}\,\mathbf{E} \quad (1)$$
$$\sigma \mathbf{E} = \text{curl}\mathbf{H}. \quad (2)$$

Let us consider a horizontal interface – parallel to the (x, y) plane – between two media with conductivities σ_1 and σ_2. The boundary conditions require that the normal electric field has a jump at the interface, $\sigma_{1z}E_{1z} = \sigma_{2z}E_{2z} \equiv J_z$. Here J_z is the vertical current density, and a triaxial anisotropy in σ was assumed. All the other electric and magnetic field components are continuous across the interface.

Since H_y is continuous, its time derivative, $\dot{H}_y = \partial E_x/\partial z - \partial E_z/\partial x$ should also be continuous. On the other hand, E_z has a jump across the interface and correspondingly $\partial E_z/\partial x$ also has a jump. It must be compensated by a similar jump in $\partial E_x/\partial z$, namely,

$$\frac{\partial E_{1x}}{\partial z} - \frac{\partial E_{2x}}{\partial z} = \left(\frac{1}{\sigma_{1z}} - \frac{1}{\sigma_{2z}}\right)\frac{\partial J_z}{dx} \quad (3)$$

Similarly, from the continuity of $\partial E_z/\partial y - \partial E_y/\partial z$ it follows that

$$\frac{\partial E_{1y}}{\partial z} - \frac{\partial E_{2y}}{\partial z} = \left(\frac{1}{\sigma_{1z}} - \frac{1}{\sigma_{2z}}\right)\frac{\partial J_z}{dy} \quad (4)$$

The x projection of the second Maxwell equation reads: $\sigma_x E_x = \partial H_z/\partial y - \partial H_y/\partial z$. Here H_z and hence $\partial H_z/\partial y$ are continuous, therefore a jump in $\sigma_x E_x$ across the interface must be equal and opposite in sign to the jump in $\partial H_y/\partial z$. A similar consideration can be applied to the y projection, which gives us the two following conditions:

$$\frac{\partial H_{1y}}{\partial z} - \frac{\partial H_{2y}}{\partial z} = (\sigma_{2x} - \sigma_{1x})\,E_x$$
$$\frac{\partial H_{1x}}{\partial z} - \frac{\partial H_{2x}}{\partial z} = (\sigma_{1y} - \sigma_{2y})\,E_y \quad (5)$$

APPLICATION TO CSEM MODELING

Fig. 1 shows an example where the proposed interpolation technique is applied to 3D CSEM modeling. We consider a simple model consisting of air, a 2 km water layer and a half-space formation with resistivity of 1 Ωm. A harmonic horizontal electric dipole with dipole moment of 1 Am at the frequency of 1 Hz is placed 50 m above the seafloor. The inline electric field E_x is recorded at the 4 km horizontal offset from the source. The field profile along the vertical z axis is computed using a finite-difference time-domain modeling code described in (Maaø, 2007) and (Mittet, 2010).

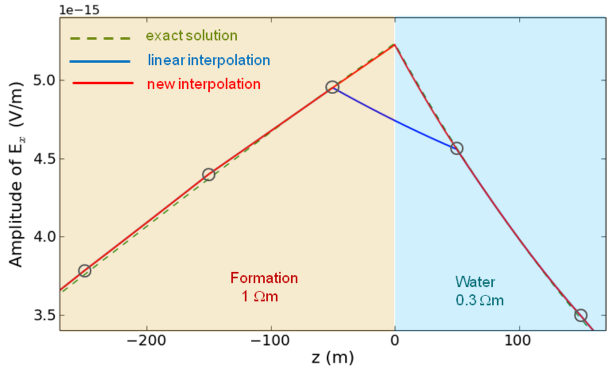

Figure 1. Interpolated profiles of inline electric field along the z axis crossing water–formation interface. The new interpolation method reproduces very well the behavior of the exact solution showing a derivative discontinuity at the interface.

The absolute field values computed at the grid nodes are indicated as circles in Fig. 1. They match almost perfectly the exact analytical solution (Løseth & Ursin, 2007) E_x^{exact} for this problem shown by the dashed line. To evaluate field values between the nodes, we first use a linear interpolation. It gives satisfactory results in most cases, however fails at the water–formation interface where the field derivative dE_x/dz is discontinuous. For a receiver located exactly at the interface the error in the interpolated E_x is as high as 10%.

Much better accuracy can be achieved by using the proposed interpolation scheme (red curve). Here the jump in the derivative dE_x/dz was evaluated using Eq. (3). As a result, the interpolated field profile consists of two line segments instead of one and fits very well the true behavior of $E_x(z)$.

In a similar way the proposed interpolation improves the accuracy for the phase of the electric field. The total error in the complex inline electric field $|E_x - E_x^{exact}|/|E_x|$ averaged over source–receiver offsets from 2 to 10 km is plotted as a function of the cell size dz in Fig. 2. One can see that for the linear interpolation scheme the error is first order (proportional to dz). This is expected since disregarding the derivative discontinuity at the interface leads to an error proportional to the cell size. Use of the new interpolation essentially removed the first-order error. The remaining error is due to numerical dispersion of our finite-difference scheme based on the Yee grid. The numerical dispersion error is second-order despite presence of a sharp conductivity contrast because a proper conductivity averaging is performed over the volume of cells crossed by the interface, see (Davydycheva, Druskin, & Habashy, 2003) for details.

Fig. 2 considers the case when the interface falls exactly between the E_x nodes, which corresponds to the maximal interpolation error for the linear interpolation scheme. When using the new interpolation, the error in E_x turned out to be almost independent of the relative position of source and receivers within the Yee cell. It implies that the interpolation error is essentially removed by the present scheme.

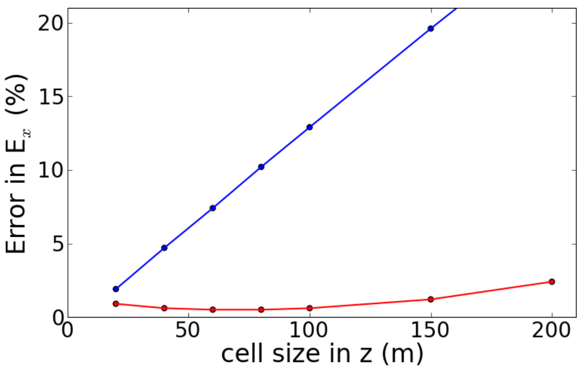

Figure 2. Error in the inline electric field recorded at the seafloor as a function of the cell size in z direction. The blue line is the conventional linear interpolation, the red line is the proposed rigorous interpolation.

IMPLEMENTATION

The computed derivative jumps can be readily included in either linear or non-linear interpolation schemes. In the latter case one uses longer interpolation stencil extending to both sides of the interface and a polynomial interpolation function. The polynomial coefficients can be found in the usual way (e.g. using the Taylor expansion), but the coefficient at the linear term will now experience a jump across the interface. The method will work fine both for uniform and for non-uniform grids.

A peculiar feature of the new interpolation is that it mixes different field component. It follows from Eq. (3) that evaluation of E_x at an arbitrary receiver location, requires not only values of E_x computed at the surrounding nodes, but also surrounding values of E_z. Similarly, to evaluate H_x one needs not only values at H_x nodes, but also values at E_y nodes.

The proposed interpolation can be applied not only at the recording positions, but also for the EM source interpolation. The interpolation coefficients will be identical for both cases due to the reciprocity property of the Maxwell equations. As a result, to model an electric dipole source in the x-direction located at the seafloor, we should place some source terms at the surrounding E_x nodes and some small "fictitious" source terms at the E_z nodes.

Equations above are derived under assumption that the interface lies in the (xy) plane. For an arbitrary interface orientation, the equations become much more complicated, however, the interpolation concept does not change. There is still a jump in the normal derivatives of the tangetial electric and magnetic field components at the interface. Practical implementation of the interpolation scheme then requires transformations from the Yee grid coordinate system to the coordinate system aligned with the interface, and then backwards. Two complications arise here. First, it is no longer possible to find interpolation coefficients separately for each dimension, and compose the 3D interpolation coefficents by simple multiplication. Instead, one has to set up the whole interpolation in 3D. Second, all the field component become mixed, e.g. computation of E_x at the interface will in general require knowledge of all electric and magnetic field components at the nearby nodes. The details of this interpolation framework will be published elsewhere, but we can refer to a similar framework recently developed to compute discrete differential operators in the presence of curved interfaces (Bauer, Werner, & Cary, 2011). One important difference from our approach is that Bauer *et al.* uses four nearest nodes for the interpolation stencil, while we use a much larger number of nodes than the number of coefficients in the interpolation polynomial and solve a minimization problem.

Use of the new interpolation scheme becomes especially important in the challenging case of very shallow water. For example, interpolation methods using only nodes in the water can hardly provide accurate results if the water layer contains only one or two nodes. By contrast, the proposed scheme turns out to be very accurate even if there is only one grid node in the water since it also uses nodes below the seabed, and the conductivity values at both sides of the interface.

REFERENCES

Abubakar, A., Habashy, T. M., Druskin, V. L., Knizhnerman, L., & Alumbaugh, D. (2008). 2.5d forward and inverse modeling for interpreting low-frequency electromagnetic measurements. *Geophysics*, *73*, F165-F177.

Bauer, C. A., Werner, G. R., & Cary, J. R. (2011). A second-order 3d electromagnetics algorithm for curved interfaces between anisotropic dielectrics on a yee mesh. *J. Comput. Physics*, *230*, 2060-2075.

Davydycheva, S., Druskin, V., & Habashy, T. (2003). An efficient finite-difference scheme for electromagnetic logging in 3d anisotropic inhomogeneous media. *Geophysics*, *68*, 1525-1536.

Løseth, L., & Ursin, B. (2007). Electromagnetic fields in planarly layered anisotropic media. *Geophys. J. Intl.*, *170*, 44-80.

Maaø, F. A. (2007). Fast finite-difference time-domain modelling for marine-subsurface electromagnetic problems. *Geophysics*, *72*, 19-23.

Mittet, R. (2010). High-order finite-difference simulations of marine csem surveys using a correspondence principle for wave and diffusion fields. *Geophysics*, *75*, 33-50.

Streich, R. (2009). 3d finite-difference frequency-domain modeling of controlled-source electromagnetic data: Direct solution and optimization for high accuracy. *Geophysics*, *74*, F95-F105.

Wirianto, M., Mulder, W., & Slob, E. (2011). Applying essentially non-oscillatory interpolation to controlled-source electromagnetic modelling. *Geophysical Prospecting*, *59*, 161-175.