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The Inside Story on Rates of Return

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Does a day go by when you don't hear someone quoting a rate of return on real estate? Do you remember once knowing how to calculate it, but after the exam something more interesting displaced those memory cells? This article reviews the concept and calculation of investment return and shows how to approximate the return when you lack the time, information, or computing power to produce an exact answer.

Exact Rates of Return

Consider a simple case. An investor purchased an asset yesterday at 3:30pm for \$100; the investor sells the asset at 3:30pm today for \$105. The one day holding-period return is (\$105 − \$100)/\$100, or 5%. Now suppose the investor also receives a cash dividend of \$1, paid at 3:30pm today. The holding-period return is now (\$105 + \$1 − \$100)/\$100, or 6%. Adding the dividend does not complicate the return computation because there is an observable market price (\$105) for the asset at the date the dividend is received, and the relevant period over which the return is measured coincides with the receipt of the dividend.

Contrast this with the following case. An asset was purchased four days ago for \$100, a \$1 dividend was received yesterday, and the stock is sold today for \$105; no market price for the stock was reported yesterday. What is the rate of return?

The standard technique for solving for a return with values at a beginning and ending period and a receipt or disbursement of cash at some interim point is to compute the *internal rate of return* or *investment yield*. This requires finding a number r that makes the following equation true:

$$\$100 = \frac{\$1}{(1 + r)^3} + \frac{\$105}{(1 + r)^4} \qquad (1)$$

The solution is r = 0.01478, or 1.478% per day. Over the four-day period, the rate of return is $(1 + r)^4 - 1 = (1.01478)^4 - 1 = 0.06044$, or 6.044%.

Another way is to multiply both sides of Equation (1) by $(1 + r)^4$ and rearrange as:

$$\$105 = \$100\,(1 + r)^4 - \$1\,(1 + r) \qquad (2)$$

This can be interpreted as follows: \$100 is "deposited" (invested) for four periods (days) at the unknown yield r, a withdrawal of \$1 is made after three days, and the balance "on deposit" (available to be withdrawn) at the end of the fourth day is \$105. The rate r is, of course, still 1.478% per day. Finance texts typically present the problem of finding the internal rate of return in the context of present value problems; that is, using Equation (1) as the preferred format. However, for this study, the investment yield intuition underlying the format of Equation (2) may be more appealing.

To develop the measurement of true returns, the following notation is used:

V_0 = market value at beginning of period (date 0)
V_1 = market value at end of period (date 1)
N = number of cash receipts and payments at dates between 0 and 1
C_j = amount of the jth cash flow (j ranges from 1 to N)
t_j = the date when the jth cash flow is received or paid, $0 < t_j < 1$
C_{N+1} = cash flow received or paid at date 1
r = rate of return over the period (date 0 to date 1)

C_j may be either positive or negative: positive when it represents a "deposit" or additional investment in the asset (as would be the case with a capital improvement), and negative when it represents a "withdrawal," such as a partial sale or receipt of income. To clarify this, we restate Equation (2) using

V_0 = \$100
V_1 = \$105
N = 1
C_1 = −\$1
t_1 = 0.75 (\$1 distributed on the third day, i.e., three-quarters of the way through the four-day period)
C_{N+1} = 0

giving

$$\$105 = \$100\,(1 + r) - \$1\,(1 + r)^{0.25} \tag{3}$$

The fractional exponent 0.25 has an intuitive explanation. The dividend receipt occurs on day 3, three-fourths of the way through the four-day period. In Equation (3), the \$1 removed from the account one day before the end of the period is reflected in the exponent of 0.25. Hence, we can interpret Equation (3) as saying the final value of \$105 is generated by investing \$100 over the period (four days) at the rate r, reduced by the "withdrawal" of the \$1 dividend on day 3 and the subsequent loss of one day's interest on that \$1. The value of r is 6.044%, exactly what was earned for the *overall* four-day return above.

When we use Equation (3) and similar expressions involving fractional exponents, r is the holding-period return over the designated time interval. This is in contrast to the formulation with integral exponents [Equation (2)], which breaks the holding period into discrete segments, e.g., days. In that case, we compute a daily return (1.478%), then compound the daily return to get the holding-period return.

The general expression for obtaining the holding-period return r is:

$$V_1 - C_{N+1} = V_0(1 + r) + \sum_{j=1}^{N} C_j(1 + r)^{1-t_j} \tag{4}$$

In most cases, r in Equation (4) cannot be directly solved for because of the presence of the exponential term $1 - t_j$. To solve it an algorithm for finding the roots of an equation must be used. Standard routines are available in most numerical analysis texts. A well-designed computer program can implement these quite efficiently.

When a computer program is not available, an approximate solution for the rate of return r may be used. The approximation is:

$$r = \frac{V_1 - C_{N+1} - V_0 - \sum_{j=1}^{N} C_j}{V_0 + \sum_{j=1}^{N} C_j(1 - t_j)} \tag{5}$$

The formal derivation of the approximation is provided in the appendix.

Approximate Rates of Return

The information requirements in Equation (5) (timing and magnitude of all interim cash flows, as well as beginning and ending market values) are identical to those for the exact method given in Equation (4). If it is not possible to compute the exact return, an assumption about the timing must be made. Each assumption will result in a specialized version of Equation (5). We discuss the various assumptions and derivations next.

Notation and General Assumptions

Some additional notation is needed. In the preceding section, C_1, C_2, ..., C_{N+1} designate general cash flows, which may be positive or negative. Each cash flow may be the aggregate of several components, and each asset is assumed to be a real property financed completely with equity. Each cash flow may have up to three components: distributions from partial sales (PS), contributions for capital improvements (CI), and distributions of net operating income (NOI). Each cash flow C_j is then:

$$C_j = CI_j - NOI_j - PS_j \tag{6}$$

where CI_j, NOI_j, and PS_j are all positive values or zero. Note that if, at date t_j, the only cash flow is an additional contribution CI_j, then C_j will be a positive quantity at that date, i.e., funds were "deposited" into the asset. Similarly, if at date t_j the only cash flow is a distribution from a partial sale PS_j or a distribution of income NOI_j, C_j will be negative, i.e., funds were "withdrawn" from the asset. Also, note that many other cash flow decompositions are possible. Thus Equation (5) may be used with many different assets, including leveraged properties, for example, by appropriate decomposition of the cash flows C_j.

For convenience, assume the time period from dates 0 to 1 is one calendar quarter. Asset values V_0 and V_1 are established at the beginning and end of the quarter, respectively. The values come from either appraisals or sales.

The Russell/NCREIF Property Index

The Russell/NCREIF Property Index (RNPI), probably the most widely used performance measure for real estate, computes quarterly returns using one version of Equation (5). Specifically, RNPI makes the following assumptions:

1. Net operating income for the quarter is received in equal installments at the end of each month; and
2. Any capital improvements or partial sales take place midway through the quarter.

These assumptions mean there is at most one each of NOI_j, CI_j, and PS_j during the period, so the subscript j may be conveniently dropped.* In terms of the formal notation:

$$\begin{aligned}
N &= 3 \\
C_1 &= -0.33\ NOI \\
t_1 &= 0.33 \\
C_2 &= CI - PS \\
t_2 &= 0.5 \\
C_3 &= -0.33\ NOI \\
t_2 &= 0.67 \\
C_{N+1} &= -0.33\ NOI
\end{aligned}$$

Using these values, along with the estimates of beginning and ending market value V_0 and V_1, the FRC total return measure is as follows:

$$r = \frac{V_1 - V_0 - CI + PS + NOI}{V_0 + 0.5(CI - PS) - 0.33\ NOI} \tag{7}$$

Note that this estimate is a variation of Equation (5), in which NOI is assumed to be distributed in three equal portions, CI is assumed to be paid in at the midpoint of the period, and PS is assumed to be distributed at the midpoint. FRC divides the total return into "appreciation" and "income" components:

$$r_a = \frac{V_1 - V_0 - CI + PS}{V_0 + 0.5(CI - PS) - 0.33\ NOI} \tag{8}$$

$$r_i = \frac{NOI}{V_0 + 0.5(CI - PS) - 0.33\ NOI} \tag{9}$$

Alternative Measures

The number of alternative return computations for unlevered properties derived from Equation (5) is large, because many different cash flow timing assumptions are possible. It is worth noting, however, that *all* computations have the same numerator — cash flow timing only appears in the denominator. Although cash flows and asset values are known (with some imprecision when appraisals are used), uncertainty about the timing of cash flows leads to different estimated values of r, according to the timing assumption employed.

There are, therefore, *two* sources of potential errors to consider when using an approximation to estimate the true return. The first stems from the fact that all simplified versions of Equation (5) are approximations of the return r in Equation (4). The second is that there are differences between the actual timing of cash flows and the timing assumed in the measures.

Knowing how to calculate investment returns may not enhance your cocktail party conversation but it should enable you to penetrate some of the haze around the actual or prospective performance of a real estate asset. Besides, now you know why those pesky 0.5 and 0.33 factors crop up in the RNPI return.

Appendix -- Rate of Return Approximation

In this section Equation (5), the approximation for Equation (4) in the text, is developed. The approximation eliminates the exponential term in Equation (4), so that a direct solution for r is possible.

Begin by defining $x = 1 + r$. Rewrite Equation (4):

$$V_1 - C_{N+1} = V_0 x + \sum_{j=1}^{N} C_j x^{1-t_j} = f(x) \qquad \text{(A1)}$$

The Taylor series expansion of a differentiable function $g(x)$ about the point $x = a$ is:

$$g(x) = g(a) + g'(a)\frac{(x-a)}{1!} + g''(a)\frac{(x-a)^2}{2!} + \ldots \qquad \text{(A2)}$$

The function $g(x)$ may be approximated by truncating the above infinite series. To approximate (A1) and eliminate the exponential terms, only the first two terms in (A2) are used and the expansion is taken around $a = 1$:

$$f(x) \sim f(1) + f'(1)(x-1) \qquad \text{(A3)}$$

First, the derivative $f'(x)$ is needed. This is obtained by differentiating Equation (A1) with respect to x:

$$f'(x) = V_0 + \sum_{j=1}^{N} C_j \left(1-t_j\right) x^{-t_j} \qquad \text{(A4)}$$

Then,

$$f(1) = V_0 + \sum_{j=1}^{N} C_j \qquad \text{(A5)}$$

$$f'(1) = V_0 + \sum_{j=1}^{N} C_j \left(1 - t_j\right) \qquad \text{(A6)}$$

Substituting (A5) and (A6) into (A3), and noting that $x - 1 = r$, gives:

$$V_1 - C_{N-1} = f(x) \sim V_0 + \sum_{j=1}^{N} C_j + r\left[V_0 + \sum_{j=1}^{N} C_j\left(1 - t_j\right)\right] \qquad \text{(A7)}$$

Finally, to the order of approximation (A7), the rate of return r can be computed as:

$$r = \frac{V_1 - C_{N-1} - V_0 - \sum_{j=1}^{N} C_j}{V_0 + \sum_{j=1}^{N} C_j\left(1 - t_j\right)} \qquad \text{(A8)}$$

This completes the derivation.