# Commentary
Mars, I

1. **(8)** Most students will first add the two groups of marbles they have, 3 and 2, to get 5. The students can then subtract 13 - 5 to find the missing marbles, or use the *counting up* method from 5 to 13.

2. **(95)** The student can use coins to count out the change: 25, 50, 75, 85, 90, 95. The values of each coin can be added for the total: 3 quarters = 75 cents; 1 dime = 10 cents; and 2 nickels = 10 cents, so 75 + 10 + 10 = 95 cents.

3. **($30)** The student can add $7.50 four times or group by two sums of $15. Counting the money like change could be used: $7.50, $15.00, $22.50, $30.00. This leads to the concept of multiplication -- some students might even perform $7.50 X 4 on their calculator.

4. **(12, 9, 14)** The repeating pattern is to **add 5**, then **subtract 3**. Once discovered, the student should check to see if the pattern continues on the next few numbers. It does, so they would conjecture that the next three numbers are obtained by: *7 + 5 = 12; 12 -3 = 9; 9 + 5 = 14.*

   Notice that there is no way for the student to be sure they have discovered a pattern that always holds true; also note that students might discover another pattern that would give the numbers 1, 6, 3, 8, 5, 10, and 7, thus arriving at different numbers than 12, 9, and 14.

5. **(15)** The student can *count up* from 8 to 12, or solve 12 - 8 to find that ❋ = 4. Then the student substitutes 4 for the ❋ in ❋ + 11. So, 4 + 11 = 15.

6. **(13)** There are 12 people (6 + 6) in the movie ticket line, excluding Sue. When Sue is counted in the line there would 12 + 1 or 13 people.

7. **(5)** The student can physically mark the turtle's progress and slides to get to the top.
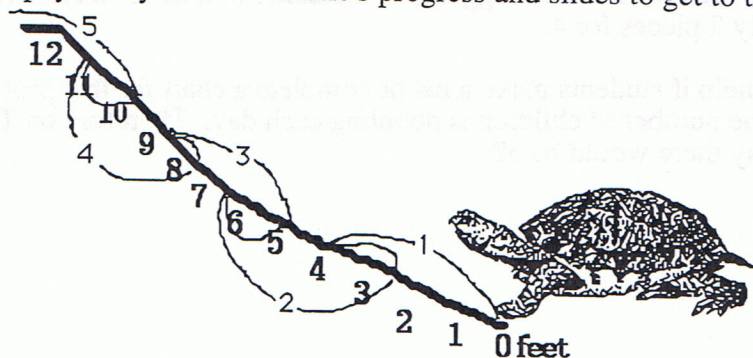
8. **(Tom, Sally, Maria, Bob)** Drawing a picture as each clue is used is a way for the student to find the students places' from tallest to shortest:

Tom is taller then Sally:	Tom	Sally	
Sally is taller than Bob:	Tom	Sally	Bob
Maria is taller than Bob but shorter than Sue:	Tom	Sally	Maria	Bob

Commentary
Mars, II

1. **(7, 0, 17, 8)** Students can subtract 7 from the number in column A to get the number in the column B. Students must reverse the thought process to do the last part. The number in B is given, so they must ask themselves "What number, if I subtracted 7, would give me 1."

2. **(59)** Give the students this problem posted where several can read it at one time:

   $34 + 25 = ?$

   and have them write only the answer on their paper.

3. **($0.22)** The class would have to buy 3 small packages of napkins which would cost $2.97. Most students will find this number by adding 99¢ three times, but some might multiply on a calculator. In either case, they must then subtract $2.75.

4. **(15)** Students might first label the two sides of the patio for which they know the length. That would be 20 feet of the 50-foot perimeter. Then students would subtract 20 feet from 50 feet and realize they have 30 feet left for the other two sides. They will use various methods to divide 30 feet into two equal pieces.

5. **(300)** 8 feet is not a reasonable length for a home run. 2,500 feet is also not reasonable, as a mile is about 5,000 feet, so 2,500 feet is about 1/2 mile. 300 feet is reasonable. That's the length of a football field.

6. **(6-3-5-6-2-3-1-2-5 is one solution)** All successful solutions have these in common: they either start at 6 and end at 5, or start at 5 and end at 6. That's because 5 and 6 are the only places in this network that have an odd number of paths going in and coming out.

7. **(a. 3; b. 1)** The area for 3 is twice as much as that for 2, so 3 is twice as likely as a landing for the spinner. The area for 1 is also bigger than the area for 4, as there are three equal sized pieces that make up 1 and only 2 pieces for 4.

8. **(32)** It will help if students make a list or complete a chart for this problem. If so, they will likely notice that the number of children is doubling each day. Therefore on Thursday there would be 16, and on Friday there would be 32.

Commentary
Mars, III

1. **(26)** The student can count up from 19 to 45, or subtract 19 from 45 to get 26.

2. **(7:00)** A clock for hands-on exploration would assist the student in adding 30 minutes to find 6:45, then adding 10 minutes to find 6:55, and adding 5 minutes to reach 7:00 AM.

3. **(21)** The student can add 3 groups of 7 or use the multiplication fact, 3 x 7 = 21.

4. **(No)** The student could start at $1.25 and count the change left if buying only the crayons. If 75¢ is left, then the paste for 79¢ would make the cost over $2.00. Most students will simply add $1.25 and $0.79 and realize that $2.04 is more than Drew has.

5. **(21)** The pattern involves adding one more at each step than the step before. Start with 1 on Monday, then add 2 to get Tuesday's total, then 3 for Wednesday's total, then add 4 for Thursday and 5 for Friday, and finally 6 for Saturday. The total is 21.

6. **(10)** This problem resembles the handshake problem. It can be solved by assigning the 5 teams a letter or number and drawing a picture that shows team A plays B, C, D & E; Team B plays C, D, and E (they've already played A). Team C plays D & E as they have already played A and B. Team D plays E. Then the games are added: 4 + 3 + 2 + 1 = 10. Repeated work with this type of problem shows a pattern in the solutions.


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AB  BC  CD  DE
AC  BD  CE
AD  BE
AE
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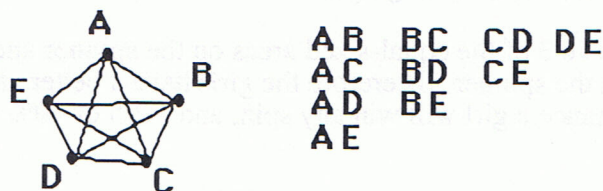
7. **(5 coins; 1 quarter, 1 dime, 1 nickel, and 2 pennies)** Some students may choose 4 dimes and 2 pennies (6 coins) to make 42¢. Extra work with using quarters in change will increase their skill with the least amount of coins in making change.

8. **(The answers are shown below.)** Using the concepts of counting up, counting back, or addition and subtraction sense, the missing numbers can be found. Problems B & C involve regrouping ones and tens.

A	B	C
2 3	5 4	6 5
+ 4 6	+ 2 7	+ 7 3
6 9	8 1	1 3 8

Commentary
Mars, IV

1. **(40)** Students will need good spatial skills to be able to count the cubes that aren't visible, or the students might actually build such a set of steps and count the cubes they use.

2. **(8; +; + or -)**

3. **(25)** The pattern is that the numbers increase by five each time: 5, 10, 15, ... The next two numbers would be 20 and 25.

4. **($15)** There are a number of ways students will solve this problem. One is with a calculator, adding $2.50 six times or possible multiplying $2.50 by 6. Another is that they might add $2.50 plus $2.50 to get $5, and then add $5 three times.

5. **(37)** Students might add the two sides then subtract from 96. Or they might subtract one side from 96, then the other side from the difference. If students have trouble with the problem, encourage them to label the sides of the triangle shown with the two numbers given.

6. **(13)** Students might count by twos for the dark candles, then count by ones for the light candles.

7. **(a. John, Mary, Sue, and Tom; b. 15; c. Mary and Sue; d. 7)** The problem involves reading and interpreting a bar graph.

8. **(girl)** Since the girls have 3 of the equal-sized areas on the spinner and the boys have 2, the girls have more area on the spinner. Therefore the girls have a better chance of winning. There's a 3/5 or 60% chance a girl will win any spin, and a 2/5 or 40% chance that a boy will win.

Commentary
Mars, V

1. **(5,738)** The purpose of this problem is for students to unscramble the place values before writing the answer. Students can use a place value chart to check their number.

2. **($\frac{5}{12}$)** There are 12 marbles in the bag. Since there are 5 red marbles, then there is a 5 in 12 chance of pulling out a red marble. "Five in twelve" can be written as the fraction $\frac{5}{12}$.

3. **(7)** The 2 absent students can be removed from 30, which leaves 28. Then the situation becomes a division problem: $28 \div 4 = 7$. The student could use counters or marks to "act out" the last part of the problem -- taking 28 counters and removing them in groups of four, asking *how many groups are removed* -- as many students will not have met division yet..

4. **(9)** Numbering the small rectangles provides an organized way to count them.

1	2
3	4

   1 big rectangle - 1&2&3&4
   4 small rectangles - 1, 2, 3, 4
   4 medium rectangles - 1&2, 3&4, 1&3, 2& 4

5. **(25)** Students might write the numbers less than 40 as they count by 5: *5, 10, 15, 20, 25, 30, 35* . The sum of the digits adding to 7 means that 25 is the number.

6. **(6)** From the top left scale, taking half of each side means that 2 marbles balance 1 tape dispenser. So 2 marbles can be substituted for the tape dispenser in the top right scale, giving that 2 marbles balance 4 pencils. This means each marble balances 2 pencils. Therefore 3 marbles balance 6 pencils. This type of thinking is a precursor to algebraic thinking in that students gain an intuitive notion of substituting equal quantities for other quantities, multiplying or dividing both sides of a balanced situation by the same amount, and so on.

7. **(3)** Dan has $3.00 left to spend ($20.00 - $17.00). Each disk costs 90¢ which is almost a dollar each. So the student reasons he can get 3 disks with the remaining $3.00. The more advanced student might multiply $0.90 times 3 which is $2.70.

8. **(4 measures long; 3 measures wide)** (Paper size being 8 1/2 inches by 11 inches.) Students might mark the length on a piece of paper and use it to measure. Making a small mark at the end of each measure will help them count the number of times they measure.

Commentary
Mars, VI

1. **(8)** Students might find this answer by drawing pictures of hot dogs and labeling each one "2 ounces", and counting by twos until they reach sixteen. The problem also relies on students knowing that 16 ounces is one pound -- many third graders might have to be told this.

2. **(7 + 5 - 9 + 3 = 6 is one solution)** Students can try writing the numbers and signs on small pieces of paper or index cards, and moving them around until they reach a solution. They might try lining up the numbers in a certain order, and just manipulating the signs to see if they can get a number sentence that works. If not, change the order of the numbers and try again.

3. **(83,472)** The problem has students unscramble the order of the numbers given, according to place value.

4. **(28)** The pattern involves increasing the number of cookies by four, for each new grade level.

5. **(40)** The problem tests students' number sense, in that 400 is far too many students for a school bus, and 4 is obviously too few. Therefore 40 is the only reasonable number.

6. **(26)** The four sides can be added together and that sum subtracted from the perimeter. Some students might prefer to subtract each number in turn from the perimeter.

7. **(The figure is shown below.)** The repeating pattern involves adding another vertical line to the circle, and then another horizontal line to the circle, each time you move to the right.

8. **(llama)** There are 4 llama cards and 2 giraffe cards out of the 13 in the box. This problem does not ask directly what is the probability of pulling each card out of the box, but gives a hint that there is some mathematical basis for such a question. The chances of pulling out a llama card is 4/13, while the chances of pulling out a giraffe card is 2/13.

9. **(6)** The problem involves several steps, and is a precursor to algebraic thinking. Students know a hat weighs 3 pounds from the scale on the right. On the scale to the left, the two hats would then weigh 6 pounds out of the 18 total, leaving 12 pounds for the two rabbits. Each rabbit then weighs 6 pounds. In later grades, equations such as "$2r + 2h = 18$ and $h = 3$" might be used to show the existing situations, and students would solve the equations for r.

Commentary
Mars, VII

1. **(3)** The first number in each pair is 4 times the second number. Students who have mastered their multiplication facts might have discovered this pattern. Other students might be having trouble if they are looking for an addition or subtraction relationship.

2. **(9)** Some students might choose to draw marks or use counters. If so, they will find that 8 boxes are needed for 48 golf balls, with 4 balls left over. This means a ninth box is needed.

3. The student should first add to find the sum of the diagonal which has all three numbers showing. Then each box can be solved by adding the two numbers and subtracting to find the missing number. See the magic squares below:

6	1	8
7	5	3
2	9	4

12	7	14
13	11	9
8	15	10

4. **(9, 5)** The *guess and check* method is one that can be used. A quicker method is to think of the fact families of 14.
   $7 + 7 = 14$ but $7 - 7 = 0$
   Then you look for a difference of 4 between the numbers. $8 + 6 = 14$ but $8 - 6 = 2$
   The numbers 9 and 5 meet both conditions. $9 + 5 = 14$ and $9 - 5 = 4$ √
   $10 + 4 = 14$ but $10 - 4 = 6$

5. **(28)** Students should be encouraged to approach this problem in an organized way. For example, they might count all of the small rectangles first, those made by the individual lines, and get 7. Then they count all the next larger size, those formed by putting two small rectangles together -- this gives 6. They proceed in this fashion, finding 5 of the next size, 4 of the next, then 3, 2, and 1, which is the whole card itself.

6. **(7)** Either *guess-check-revise* or *work backwards* strategies can be used to find the starting number. With *working backward*, you would ask yourself "What number multiplied by 3 gives 30?" The answer is 10. You would then ask "What number, less 4, gives 10?" The answer is 14. Finally, "What number, when 7 has been added, gives 14?" The answer is 7.

7. **(6)** Once students organize their plan, finding these 6 numbers will be easy.
   Starting with the 2 as the hundreds digit : 234, 243
   Starting with the 3 as the hundreds digit: 324, 342
   Starting with the 4 as the hundreds digit: 423, 432
   The condition of using each number only once limits the number to 6.

8. **(20, 10, 20)** Students with good number sense can intuitively find half of numbers such as 40 and 20 at this time.   Other students might need to actually make 40 or 20 marks on a sheet of paper, or work with cubes or other concrete materials to represent the beads, and divide them into two piles with the same amount in each.

Commentary
Mars, VIII

1. **(b)** The given picture shows a rectangle that is one-half of the square. In (b) the half-circle is one-half of the circle. In (a) and (c), the two shapes are not similar and their areas are not in the same relationship as in the given figure. However, if students choose (a) or (c), listen to their reasons -- they might have used some other logical reason for selecting them.

2. **(The chances are the same that she'll pick either color.)** The question is designed to measure both the child's sense of probability, and their confidence. The confidence factor comes in because the question is asked in such a way that they think they should answer with one particular color.

3. **(356)** The challenge is for the student to put the place values in the correct relationship, before finding the total. Most textbooks show pictures like this, but the tens and hundreds blocks have already been placed in their correct, left-to-right order.

4. **(10)** The students can count by 20's, and get to 80 books on 4 shelves. Therefore 10 books, the difference in 80 and 90, will not have a shelf.

5. **(12 rose and 8 holly bushes)** Students might draw a picture of the nature trail, and sketch and label the five bushes at each stop. They would continue until they have 20 bushes in all, then go back and count the rose and holly bushes separately. Making a chart is another way for students to organize this information.

6. **(25)** Students might make such stacks using index cards or some other manipulative. They can then see physically why the answer is 25. This problem is a physical introduction to the concept of the *mean*.

7. **(first row: 5 4 9; second row: 10 6 2; third row: 3 8 7)** Students can begin this magic square by finding the sum along the diagonal which is complete -- 18. Then they look for rows and columns for which there is one missing number, and knowing the sum must be 18, they can find that number.

8. **(4)** Some students will not know a key fact here, which is that 1 kilogram is 1,000 grams. Once they have been reminded of this, they might think of 251 grams as 250 grams, since the problem involves an estimation. Then 250 and 250 is 500, and another 500 would be 1000. Therefore four cans of soup would be about 1000 grams, or 1 kilogram.
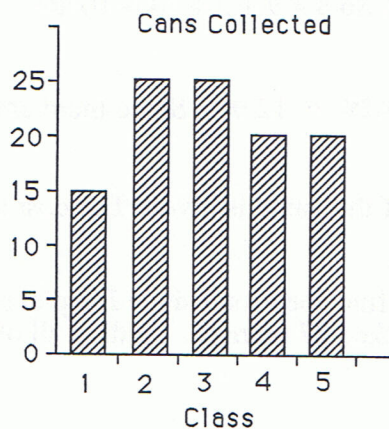
Commentary
Mars, IX

1. **(5)** *Working backwards* is one strategy to use. The student asks, "What number, when I subtract 3, leaves 17?" The answer is 20. Continuing, the student asks, "What number multiplied by 4 gives 20?" The answer is 5. By *working backward* the student arrives at 5.

2. **(a. >; b. >; c. <; d. =)** If students compute on both sides of the box, they'll find in (a) that they get 65 on the left and 61 on the right. For (b), they get 20 and 18, for (c) they get 14 and 23, and for (d) they get 8 and 8.

3. **(4)** The student needs to subtract the 68 students that ride the bus from the total of 84. That leaves 16 students to ride in cars. Since 4 students can ride in each car, counting by 4's will show that four cars are needed.

4. **(at least 9)** This problem can be solved by multiplying 4 x 2, or adding 2 four times, since the doorbell rang 4 times and 2 friends arrived at each ring. But the student must remember to add Gina herself to the 8 friends, so there are at least 9 people at the party -- there may be more than 9 since Gina might have someone else at her home that attends the party.

5. **(43)** The perimeter is found by adding all the sides together. So 8 + 9 + 2 + 14 + 10 are added together to find 43 feet.

6. **(6)** The student needs to substitute 3 ✈'s for each ☎. So ✄✄ = 12 ✈'s. Since there are 2 ✄'s, then each ✄ is worth 6 ✈'s.

7. **($9.00)** The student should use subtraction since the cost of the game is given. The cost is taken from the total spent ($28 - $19 = $9).

8. **(65)** The student can use the number Bill picked -- 23 -- to find Joe's total since Joe picked 8 less (23 - 8 = 15). Tom picked 12 more than Joe's 15, so Tom picked 27 oranges. Adding all of these together gives 65.

Commentary

Mars, X

1. **(2/13; 5/13)** It might help students to draw the correct number of each shape mentioned, then look at them as parts of a total set. 2 figures out of 13 figures are squares; 5 figures out of 13 are circles.

2. **(c)** The figures can be traced and then cut out of paper, for students to set how (c) folds into a box. Students who can do this problem without such an aid have very good spatial sense.

3. **(4; 7)** Line segments do not include curved lines. Therefore 2, 3, and 5 are eliminated.

4. **($2.25)** The problem tests a student's number sense and knowledge of the real world. $10.25 would be too much for twelve pencils -- that would be almost $1 per pencil. Likewise, 10¢ is too little -- that would be less than a penny per pencil. $2.25 is the only reasonable answer -- this would be almost 20¢ per pencil.

5. **(35 minutes)** Students are likely to start at 7:00 and add on a half-hour to get 7:30, and then add on the other intervals individually to arrive at 7:55 when she's through. This leaves her 5 minutes till 8:00 arrives to read, and 30 minutes after that, totally 35 minutes.

6.

Cans Collected

A bar graph titled "Cans Collected" with the vertical axis labeled in increments of 5 from 0 to 25, and the horizontal axis labeled "Class" with classes 1 through 5. Class 1: 15, Class 2: 25, Class 3: 25, Class 4: 20, Class 5: 20.

7. **(21)** The problem is an intuitive introduction to finding the mean of a collection. At this point, students will simply add the number of cans together to get 105, then use their intuition and number sense to divide 105 cans into 5 groups. One concrete way would be to make 105 marks on their paper and divide these marks fairly. A more sophisticated strategy would be to estimate that each group would have 20, which would be 100 marks altogether, then distribute the remaining five marks.

8. **(128)** Have the problem 4 x 32 written on chart paper or index cards so that several students can see it at the same time, when they turn their papers in. They have to do the problem mentally, and put their answers correctly on their papers.

9. **(8)** This problem is an introduction to the concept of *ratio*. Students might find the answer by drawing the tables and placing the right number of markers on each, until they have used up 24 markers. This would require four tables. Then they would draw 2 pieces of poster board on each table.

Commentary
Mars, XI

1. **(107; 184)** The student might subtract 288 from 395 to find the first missing number, 107. Other students might "add on" to 288, till they get to 395. Similar methods will work for the second problem.

2. **(28)** Students can solve this problem by drawing a row of 35 seeds, and grouping them into sets of 5, and crossing out one seed in each group. Counting the remaining seed gives 28.

3. **(9)** The student might draw a picture to help visualize the problem, or to actually count the places for more stamps. If 3 1/2 rows are full, then 1 1/2 rows are empty. This is one empty row of 6 and another half row of 3, gives 9 more places for stamps.

4. **(5/9; 4/9)** The problem involves writing a part-whole relationship for a collection. The entire collection has 9 disks, so each part is written as a correct numerator over the denominator of 9.

5. **(22¢)** The student must first find out how much Sally spent. 32¢ can be added 4 times or can be multiplied by 4 to give $1.28. To find the amount of change, the student can count out the change with real or play money, or subtract ($1.50 − $1.28 = $0.22).

6. **(There are 11 lines of symmetry as shown below .)**
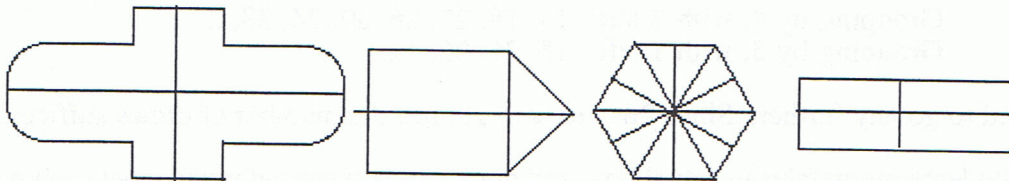
7. **(5; 2)** The student can draw pictures of tables and count the chairs. Since all tables hold at least 4 people, there can be no more than 8 tables since 8 groups of 4 is 32. But 8 is an even number, so there can be no more than 7 tables. Check and see if it's possible to have a combination of 4- and 6-person tables that total 32 chairs. Seven 4-person tables would be 28 chairs, so take the extra 4 chairs and turn 2 of the tables into 6-person tables, and the problem is solved.

8. **(77)** The pattern of dots is shown below:

dots:	5	9	14	20	?	?	?	?	?	?
figure:	1	2	3	4	5	6	7	8	9	10 ...

The pattern involves adding successive numbers -- 4, 5, 6, 7, etc. -- each time to get the number of dots for the next figure.

Commentary
Mars, XII

1. **(56)** Some students will add 22 and 22 and then 12 more, and others will add 12 to 22 first, and then add 22 and 34.

2. **(21; 14)** Students might draw a diagram of trees and label the birds, and count up until they have 35 birds. This would be in the seventh tree. Then they could count the types of each type of birds in the seven trees. Another method is to make a chart that shows the ratio, such as the one started to the right.

M	C	total birds
3	2	5
6	4	10
9	6	15
.	.	.

3. **(100)** Give the problem 4 x 25. If students think of this as money, as they were encouraged to do, this would represent 4 quarters, which they should know is 100 cents or 1 dollar.

4. **(no; yes)** Students will need to add 35¢ and 25¢ to get the amount that Jamie needs -- 60¢. He doesn't have that much. But that total plus another 5¢ would be 65¢ to buy all three items, and Katie has more than enough.

5. **(26)** Students might work with real or play coins to decide this. More advanced students might write down a list of how many coins she might have under both methods of grouping, and look for a common number.

   Grouping by 4, with 2 left: 14, 18, 22, 26, 30, 34, 38, ...
   Grouping by 5, with 1 left: 16, 21, 26, ...

   No need to go any further. Since 26 is in both groups, that number of dimes suffices.

6. **(68)** Students might take an actual box, and draw a ribbon around it and label each part with the correct length. They should find that there are two 10-inch parts, two 6-inch parts and four more 6-inch parts, for a total of 56 inches. Then adding the 12 inches for the bow produces 68.

7. **(201)** Students can use logical reasoning to find this number. Since the number is less than 300, the hundreds digit is a 1 or 2. It must be a 2 so that the ones and tens digits can both be less than the hundreds digit.

8. **(pentagon)** Other students might name the shape as "arrowhead" or "sideways house," which should be accepted. The most important part of the problem is to see the correct drawing, which is shown to the right.
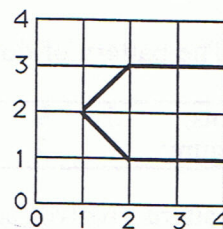
9. **(5 cats won)** Students' reasoning might proceed along the following lines.

   From the second picture, 2 donkeys match 6 dogs so I know that 1 donkey matches with 3 dogs, by dividing both sides in half. Then I can substitute 1 donkey for the 3 dogs in the top picture, and know that 1 donkey matches 4 cats. So in the bottom picture, 5 cats would win over 1 donkey.

   This type of reasoning is important when students begin algebraic experiences with equations.

Commentary
Mars, XIII

1. **(18, 16, 21)** The pattern involves repeatedly adding 5, then subtracting 2. Then the sequence continues as: $13 + 5 = 18$; $18 - 2 = 16$; $16 + 5 = 21$.

2. **(6)** The student can solve this problem by an organized guess and check strategy such as below.

#	quarters	dimes	nickels	total
3	75¢	30¢	15¢	$1.20
4	$1.00	40¢	20¢	$1.60
5	$1.25	50¢	25¢	$2.00
6	$1.50	60¢	30¢	$2.40 ✔

    Some students might see that 3 of each is $1.20, so 6 must be $2.40. Others might start with 1 of each coin being 40¢, and then add 40¢ six times to get $2.40.

3. **(a. Bill; b. 10; c. Tom)** The student can visually see from the pictograph that Bill has the largest collection. A student may answer 2 for how many more Alan has than Tom; but each insect is worth 5 so the answer would be 5 x 2 or 10 more. Students can visually see that Tom has half of Bill's, or they may count Bill's as 6 and look for half of that, which is 3.

4. **(3:45)** A clock can be used to *work backwards* to the time he got home. He walked the dog for 30 minutes, and 30 minutes before 5:00 is 4:30. Counting back 45 minutes from 4:30 might be done in stages, first counting back by 30 minutes to get to 4:00, then 15 more minutes before 4:00 would be 3:45.

5. **(120)** At this grade level area is found by counting square units. The student can count all of the small squares shown, but many will take a short cut and add 12 ten times, or ten 12 times. Some might even multiply, if they have a calculator.

6. **(3, 4, 5 and 6)** Each block has a different number so the student can choose 4 of the numbers and add and then choose another 4 if the sum is not 18. The process can be repeated until the sum of $3 + 4 + 5 + 6 = 18$ is reached. (Another approach is to add all of the 4 numbers and get 20, and then see which number to remove to have 18 as the sum.)

7. **(22)** This problem is one which will later be called *finding the mean*. At this point, students will likely not add the number of books and divide by 4. Instead, they might add the numbers to get 88, and then distribute the 88 in chunks, equally, among the four shelves. For example, they would likely give 20 to each shelf first, then 1 to each shelf, and then 1 more, exhausting the total of 88 books.

8. **(36)** A clue that makes this problem accessible is that the sum of the digits is 9. By listing these numbers -- 18, 27, 36, 45, 54, 63, 72, and 81 -- you can then search the list for the number for which the ones digit is twice the tens digit.

9. **(1)** The problem involves finding a fraction of a set, and then a fraction of a subsequent set, and seeing what is left. One-third of 3 cookies is 1 cookie, so Henry ate 1, leaving 2 cookies on the plate. Marsha ate half of two cookies, so she ate 1, leaving 1 on the plate for Art.
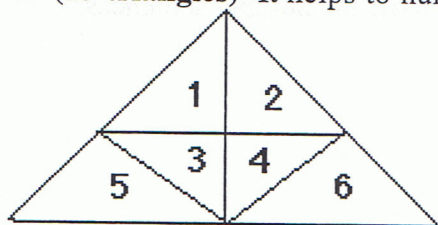
Commentary
Mars, XIV

1. **(b)** The number sentence is a one-step subtraction situation.

2. **(24; 12)** Students might *guess-check-revise* to find an answer. Another way to begin is to write down a list of numbers that sum to 36, and look for two addends where one is twice as much as the other. In the middle grades, this problem might be solved algebraically by letting x be Rex's weight and $2x$ be Fido's weight, and $x + 2x = 36$ so $3x = 36$. Then $x = 36 \div 3$ or 12, and $2x = 24$.

3. **(505)** The number being between 500 and 600 means the hundreds digit is 5. The ones digit is also 5, so the difference between the two is 0, giving 505 as the answer.

4. **(a, b, d)** Pan (a) is divided from the corner of one rectangle to the opposite corner, implying the two parts are equal in area. Students might count whole and half squares to find the area of the surface of each of the last three pans, since they aren't divided symmetrically. The area is 3 for each part of a, b, and d. In (c), the two parts have areas of 2 1/2 and 3 1/2.

5. **(a. grams; b. kilograms; c. kilograms; d. grams)** The problem gives a sense of whether the student has number sense related to the weight of common objects, and the metric units used to measure them.

6. **(2 bananas, 7 apples, 11 oranges, and 20 pieces of fruit)** Students can begin with the fact they know -- 2 bananas -- and find the number of apples by adding 5, and the number of oranges by adding 4 to the number of apples.

7. **(part one: $1.50; part two: $0.75)** The problem encourages students to use mental mathematics, as they must do in such problems in the world outside of school.

8. **(32)** Students have not been introduced to the formula for finding the area of a triangle, so they will find it by counting whole and half unit squares. There are 28 whole unit squares, and then they put together the 8 half squares to make another 4 whole squares, for a total of 32.

9. **[(14, 3); (3, 3); (3, 9); (14, 9)]** The problem measures the student's knowledge of the Cartesian coordinate system in which the first number of an ordered pair gives the horizontal distance from the axis, and the second number gives the vertical distance. The problem also involves "clockwise," a term that may be new to some students, and "90°." Some students will associate the problem with the computer program known as Logo, since a turtle's movement around a grid is common to both.

Commentary
Mars, XV

1. **(<)** The student should solve each side of the number sentence first. $81 \div 9 = 9$; $5 \times 3 = 15$. When 9 and 15 are compared, $9 < 15$.

2. **(Tina; 38 marbles)** The student can find the number of marbles each person has by building on Ben's total of 5. Kate has 7 more than Ben's 5, so she has 12. Tina has 9 more marbles than Kate's 12, so Tina has 21. To find the total, all 3 numbers must be added: $5 + 12 + 21 = 38$ marbles.

3. **(a)** The student might find this problem easier by *counting up* from $87.95 to $90.00.

4. **(🌴 = $ 5; 🍇 = $4, 🦂 = $9)** The student can find the statement that can be solved as it exists, and solve the rest of the sentences by using that answer. 🦂 + 🦂 = $18, so 🦂 = $9; 🍇 + $9 = $13, so 🍇 = $4; 🌴 + $4 = $9, so 🌴 = $5.

5. **(A)** The student must find the area of all the rectangles to find the greatest area. The area can be found by counting unit squares. (A) has 36 ft^2, B has 27 ft^2. C has 32 ft^2, and D has 35 ft^2. So the square that is 6 by 6 has the greatest area.

6. **(58, 166, 620)** The 'kewees' are even numbers and the odd numbers are not 'kewees'. Once this feature is noticed, the student can look at the numbers: 43, 58, 166, 369, 620, and 891. 58, 166, and 620 are even so they are 'kewees'. 43, 369, and 891 are odd so they are not 'kewees. Other answers may be possible, as students may notice other characteristics.

7. **(14, 16, and 18)** Students can follow the examples and try other even numbers. They would find that $8 + 10 + 12 = 30$; $10 + 12 + 14 = 36$; $12 + 14 + 16 = 42$. Then $14 + 16 + 18 = 48$. A student might notice the increase by 6 in each 3 numbers and use that to reach 48.

8. **(13 triangles)** It helps to number the small triangles as shown below.

1 & 2 & 3 & 4 & 5 &6 = 1 large triangle
1 ; 2; 3; 4; 5; 6 = 6 small triangles
1&2; 1 & 3; 3 & 4; 2 & 4 = 4 double triangles
1 & 3 & 5; 2 & 4 & 6 = 2 tri-triangles
$1 + 6 + 4 + 2 = 13$ triangles.

Commentary

1. **(4)** The problem has students read and interpret a graph with a key. Blue has two more dots than red, which indicates 2 x 2 more people.

2. **(125-inch spool)** The students can add 14 six times, or multiply 6 x 14, to find that 84 inches of ribbon are needed. This is more than the 70-inch roll can supply.

3. **(27)** This problem encourages students to start a problem where it makes sense, not necessarily with the beginning words. Students can start with what they know -- there are 6 red crayons. Then they can determine the number of brown crayons from that (5), the number of blue crayons (10) from the number of brown, and finally the number of pink (6) from the number of blue.

4. One solution:
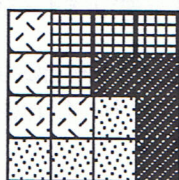
5. **(420)** Give the problem 42 x 10 to students as they hand in their papers. They should realize, after practice, that multiplying by ten simply appends a zero, and multiplying by 100 appends two zeros. This is extended, of course, to multiplying by any higher power of ten.

6. **(a. Yes, 28; b. no)** The problem points out to students that rectangles can have the same perimeter, or distance around the outside, but have different areas.

7. **(3 x 5 + 2 = 17; 17 + 5 - 4 = 18 or 15 + 7 - 4 = 18; 6 x 5 - 1 = 29)** Some students will come up with different, but equivalent, ways to write the number sentences.

Commentary
Mars, XVII

1. **(9)** The students may need to draw a picture with 8 rectangles and place a dot for the tacks to discover that 9 thumbtacks will be needed to hang all 8 pictures with overlapping corners.

2. **(24)** The problem states that Mike has 12 goldfish. This fact is used to find the number of Alan's goldfish. If Mike has 8 more than Alan, then Alan has 4 fish ($12 - 8 = 4$). Alan has 4 fewer than Mark, so Mark has $4 + 4$ more which is 8 goldfish. Then the student should add the fish totals together: $12 + 4 + 8 = 24$ goldfish.

3. **(8765, 8756, and 8675)** The student might first place the digits from greatest to least: 8765. Then if the 6 and 5 are exchanged, the second number is found: 8756. Since there is no other possibility with the 7 as the hundreds' digit, the student should use the 6: 8675. If the student exchanges the 7 and 5, they will have the next highest number: 8657 which is not needed for the answer. The student will become skilled with more problems like this one.

4. **(Garage = 38 ■' s; Stairs = 28 ■ 's; Triangle = 32 ■ 's)** The student will count the whole squares with little trouble. Then they must reason that 2 halves can be put together to make 1 whole square, and count the rest of the area of the garage and the triangle. Recounting is an excellent method for accuracy.

5. **(3 adult and 3 children's' tickets)** The *guess and check* strategy is excellent for this type of problem. The student might try 2 adult ($2 \times \$6 = \$12$) and 4 children ($4 \times \$4 = \$16$). But when \$12 and \$16 are added, they get \$28, not \$30. So they need to make another guess. If 3 adult ($3 \times \$6 = \$18$) and 3 children ($3 \times \$4 = \$12$) is tried, then the total of the \$18 and \$12 is \$30 -- the amount the family spent for the tickets!

6. **(12 won, 8 lost)** The student might make a list of the numbers that add to 20, since the wins and losses taken together must add to twenty. From the list, select the pair of numbers such that one number is four more than the other. A partial list is demonstrated below:

$10 + 10 = 20$	$10 - 10 = 0$	$13 + 7 = 20$	$13 - 7 = 3$
$14 + 6 = 20$	$14 - 6 = 8$	$12 + 8 = 20$	$12 - 8 = 4$ √
$11 + 9 = 20$	$11 - 9 = 2$		

   Some students will become skilled at doing such problems in their heads if they have a strong fact base knowledge.

7. **(11 cm ; $4\frac{5}{16}$ inches)** The student should receive credit if their answers are close to the above numbers. Accept from 10.9 to 11.1 cm, and $4\frac{1}{4}$ (or $4\frac{4}{16}$) as alternate answers

8. **(X)** The student might look at several of the equations to ensure that x is the correct sign. It is likely that, as the student places each of the other x signs in the circles, they will also check the mathematics quite naturally.

Commentary
Mars, XVIII

1. **(8 minutes)** This is a simple subtraction problem: 63 - 55. Some students may solve it by *counting up* from 55 to 63.

2. **(130)** A 3-digit number is called for, and the smallest such would have a 1 in the hundreds place, and a zero in the units place.

3. **(15)** Most students will trace the 12 ways on the map itself. A more advanced way to solve the problem would be to label each move to the left as L, and each move down as D. Then the student must make 2 L's and 4 D's to get to school, and they can come in any order. So the question is: How many ways can you arrange 2 L's and 4 D's. The ways are shown below:

   LLDDDD; LDLDDD; LDDLDD; LDDDLD; LDDDDL;
   DDDDLL; DDDLDL; DDLDDL; DLDDDL; DLDLDD
   DDDLLD; DDLLDD; DLLDDD; DLDDLD; DDLDLD

4. **(a. 15L; b. 1mL; c. 70mL)** The problem tests the number sense of students. They might need to be reminded that a mL of water is about a drop; a L of water is about half as big as a 2-liter bottle of soda.

5. **(30)** Work backwards by starting with what is known, the number of science books--4. Then find from that the number of music books--8. From that we know the number of history books--6, and then the number of art books--12. The total is 30.

6. **(15)** 1 large; 5 small; 4 rectangles made of 2 small; 3 rectangles made of 3 small; 2 rectangles made of 4 small.

7. **(93)** Give the problem 19 + 74 =

8. **(right angle - 90 degree angle: 3:00 or 9:00)** Check individually.
   (acute angle - less than 90 degrees) Check individually.
   (obtuse angle - more than 90 degrees) Check individually.

9. **(6)** In the top left picture, 3 apples balance 2 tomatoes. Therefore 3 apples can substitute twice for the 4 tomatoes in the right hand picture. In the bottom picture, 6 apples then balance with 1 cup of soup.

Commentary
Mars, XIX

1. **(a. 95; b. 98)** The student can use subtraction to find both missing numbers. Or the student might add-on to the smaller number, to get the larger number, and keep count of how much was added.

2. **(Sally is 6 yrs; Joan is 24 yrs.)** The student can solve this problem by building on the fact that Tara is 12. If Tara is double Sally's age, then Sally's age is $12 \div 2 = 6$. If Joan is double Tara's age, then Joan's age is $12 \times 2 = 24$.

3. **($3.96)** The student can add the tax of 7¢ to $1.25 to get $1.32 for each popcorn box, and then add this amount 3 times or multiply by 3. A student might decide to find 3 times $1.25 and then add the tax of 21¢.

4. **(The box is heavier.)** Solving the problem requires intuition about a balance scale, but this same intuition will help in algebraic thinking. The student can see that the ball is on both sides of the scale, and therefore the ball can be removed and the scale will stay balanced. This means that a box balances two pyramids. Therefore a box is twice as heavy as a pyramid, which will seem strange to some students because there is an *inverse* relationship between the number of items of each, and the relative weights.

5. **(11 rectangles)** Labeling the rectangles and listing them will help the student find them all as shown below:
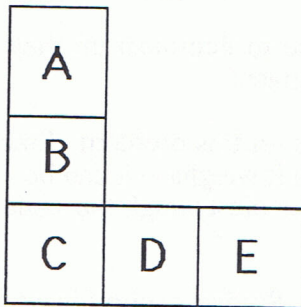
A, B, C, D, E; CD, DE, CDE; AB, BC, ABC

6. **(Greatest: 84 + 62 or 82 + 64 = 146 ; Least: 46 + 28 or 26 + 48 = 74)** The student should place the largest numbers in the ten's place for the largest sum. The student should place the smallest numbers in the ten's place for the smallest sum.

7. **(8)** Reading the problem carefully is a key to success. When 4 is subtracted from 12, the answer is 8. If 8 is subtracted from 16, the answer is also 8, so the secret number is 8.

8. **(26, 35, 40)** The student might reason that for a score of 101, some of the large numbers need to be chosen. If the student starts with the 2 largest numbers -- 35 and 40, which is 75 -- then 26 is needed to reach 101. Some students might solve the problem simply by *guess-check-revise*.

Commentary

Mars, XX

1. **(200-inch roll)** An estimation strategy would be to round 24 to 25 and think of it in terms of money. Four 25's is 100 and three more is 75; so she will need about 175 inches and will therefore buy the 200-inch roll. The exact answer for how much she needs (168 inches) will be obtained by some students by adding or multiplying.

2. **($\frac{5}{7}$ of the children went to the theater; $\frac{2}{7}$ of the children stayed home.)** The problem is a part:whole ratio problem. Students might want to draw a diagram of the seven children, and partition it accordingly, to find the answer.

3. **($7.95)** The problem involves reading a menu and making decisions from the context of the story. The answer is found by adding $3.50, $2.95, $0.75 and $0.75.

4. **(2, 2, and 4; or 1, 4, and 2)** *Guess-check-revise* or *make a list* are strategies that can be used with this problem. One creative approach is to notice that "one of each" means that the problem can be simplified by removing that much money (17¢) from the total, leaving 25¢ to be distributed among the three types.

5. **(a. school; b. store; c. bank)** The Cartesian coordinate system is used in this problem. The first number in each ordered pair tells the horizontal distance; the second number tells the vertical distance.

6. **(a. 5 LB; b. 1 oz. c. 70 LB)** This gives a student the chance to demonstrate they have real-world number sense. Unreasonable answers can be eliminated.

7. **(2)** Students should have intuitive knowledge about balance scales for this problem. Since the triangle is on both sides of the scale, it doesn't matter how much it weighs -- it can be removed and the scale still balances. Then the square and two circles must weigh the same amount. Therefore, one circle weighs half of a square.

8. **(c, a, b, d)** Visual estimation skills are required for this problem. Students might want to actually measure the lengths, to check their estimations.