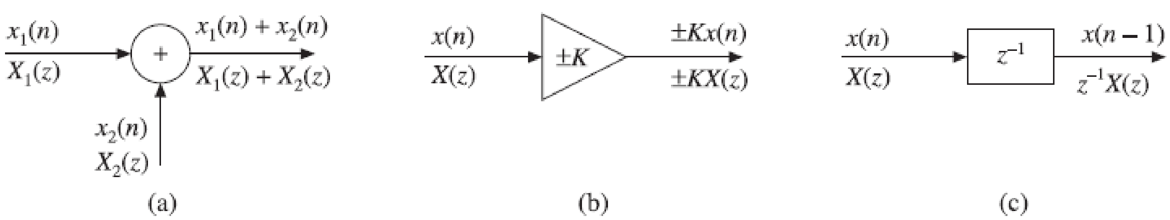
UNIT 3 Digital Systems Structure

Introduction

Systems may be continuous-time systems or discrete-time systems. Discrete-time systems may be FIR (Finite Impulse Response) systems or IIR (Infinite Impulse Response) systems. **FIR** systems are the systems whose impulse response **has finite number of samples** and **IIR** systems are systems whose impulse response **has infinite number of samples**. Realization of a discrete-time system means obtaining a network corresponding to the **difference equation or transfer function of the system**. In this chapter, various methods of realization of discretetime systems are discussed.

Realization of Discrete time Systems

To realize a discrete-time system, the given difference equation in time domain is to be converted into an algebraic equation in z-domain, and each term of that equation is to be represented by a suitable element (a constant multiplier or a delay element). Then using adders, all the elements representing various terms of the equation are to be connected to obtain the output. The symbols of the basic elements used for constructing the block diagram of a discrete-time system (adder, constant multiplier and unit delay element) are shown in Figure 3.1



(a) Adder (b) Constant multiplier and (c) Unit delay element.

Transfer Functions of Discrete-Time Systems

An LTI discrete-time system is characterized by a linear constant coefficients difference equation given by

$$\sum_{k=0}^{N} a_k y(n-k) = \sum_{k=0}^{M} b_k x(n-k),$$

where x(n) is the input and y(n) is the output of the system, ak and bk are constant coefficients. The order of the system is the max(N,M).

Such a system has an impulse response of infinite length (IIR) but is realizable since it is implemented using a finite sum of products terms from the linear constant coefficient difference equation.

$$y(n) = \sum_{k=0}^{M} b_k x(n-k) - \sum_{k=1}^{N} a_k y(n-k).$$

We can obtain the z-transform of the system by using the linearity and the time-shifting properties as follows:

$$Y(z)\sum_{k=0}^{N} a_k z^{-k} = X(z)\sum_{k=0}^{M} b_k z^{-k}.$$

The transfer function of the system is obtained as

$$H(z) = \frac{Y(z)}{X(z)} = \frac{\sum_{k=0}^{M} b_k z^{-k}}{\sum_{k=0}^{N} a_k z^{-k}}$$

$$= \frac{b_0 + b_1 z^{-1} + b_2 z^{-2} + \cdots + b_M z^{-M}}{a_0 + a_1 z^{-1} + a_2 z^{-2} + \cdots + a_N z^{-N}}.$$

If all the **denominator coefficients were zero except a0 = 1**, H(z) will have a transfer function given by $H(z) = b_0 + b_1 z^{-1} + b_2 z^{-2} + \cdots + b_M z^{-M}$. This represents the transfer function of a Finite Impulse Response (**FIR**) filter.

→ Direct form Realisation of FIR filter:
(Transverse / Tapped delay line)

The difference equation of FIR filter is given by,

$$y(n) = \sum_{k=0}^{M-1} b_k x(n-1).$$

w.r.t, $h(n) = \begin{cases} b_n & , 0 < n \leq M-1 \\ 0 & \text{otherwise} \end{cases}$

substituting for b_k,

$$y[n] = \sum_{k=0}^{M-1} h(k) x(n-k).$$

$y[n] = h(0)x(n) + h(1)x(n-1) + h(2)x(n-2) + \ldots + h(M-1)x[n-(M-$

Finite Impulse Response (FIR) filter

The **FIR filter is represented by** Equation

$$H(z) = h(0) + h(1)z^{-1} + h(2)z^{-2} + h(3)z^{-3} \ldots h(M-1)z^{M-1}$$

If we select a filter **order of 4** then the transfer function becomes

$$H(z) = h(0) + h(1)z^{-1} + h(2)z^{-2} + h(3)z^{-3} + h(4)z^{-4}.$$

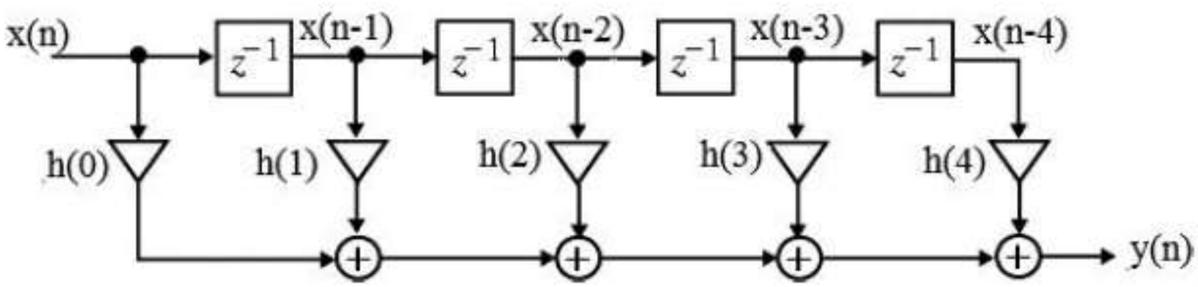
If we take the inverse z-transform of Equation we get the impulse response of the filter which is given by

$$\begin{aligned} h(n) = & \; h(0)\delta(n) + h(1)\delta(n-1) + h(2)\delta(n-2) \\ & + h(3)\delta(n-3) + h(4)\delta(n-4). \end{aligned}$$

To obtain the output we must convolve the input and the impulse response to get

$$
\begin{aligned}
y(n) &= h(n) \otimes x(n) \\
&= (h(0)\delta(n) + h(1)\delta(n-1) + h(2)\delta(n-2) \\
&\quad + h(3)\delta(n-3) + h(4)\delta(n-4)) \otimes x(n) \\
&= h(0)x(n) + h(1)x(n-1) + h(2)x(n-2) \\
&\quad + h(3)x(n-3) + h(4)x(n-4).
\end{aligned}
$$

The **output y(n)** can be obtained by the structure of Figure below



4.4 Direct form FIR realization.

Numericals

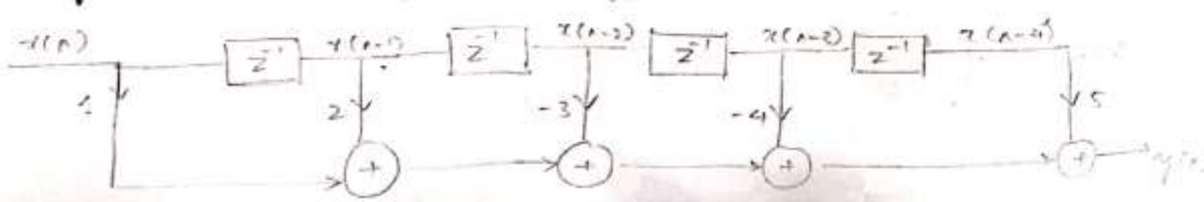
1. Given $H(z) = 1 + 2z^{-1} + 3z^{-2} - 4z^{-3} + 5z^{-4}$, draw direct form realization.

Soln: $H(z) = \dfrac{Y(z)}{X(z)} = 1 + 2z^{-1} - 3z^{-2} - 4z^{-3} + 5z^{-4}$

$Y(z) = X(z) + 2z^{-1}X(z) - 3z^{-2}X(z) - 4z^{-3}X(z) + 5z^{-4}X(z)$

$\downarrow$ IZT

$y(n) = x(n) + 2x(n-1) - 3x(n-2) - 4x(n-3) + 5x(n-4).$



Linear Phase FIR filter:

Problems :)

1. Determine $H(z)$ of an FIR filter to implement, $h(n) = \delta(n) + 2\delta(n-1) + \delta(n-2)$. Draw the linear phase FIR filter for $N = 3$.

Soln) $h(n) = \delta(n) + 2\delta(n-1) + \delta(n-2)$

$\downarrow$

$$H(z) = 1 + 2z^{-1} + 2z^{-2}$$

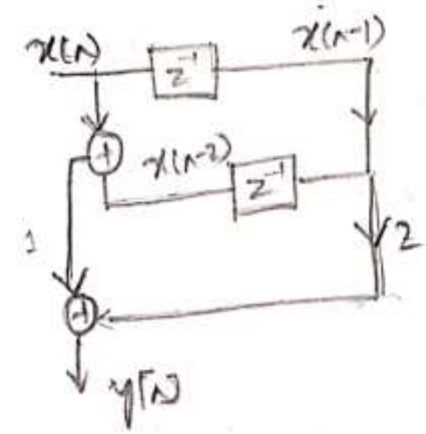
$$\frac{Y(z)}{X(z)} = 1 + 2z^{-1} + 2z^{-2}$$

$$Y(z) = X(z) + 2z^{-1}X(z) + 2z^{-2}X(z)$$

$\downarrow$ IZT

$$y(n) = x(n) + 2x(n-1) + x(n-2)$$

$$y(n) = 1[x(n) + x(n-2)] + 2x(n-1).$$



2. $h(n) = \delta(n) + \frac{1}{4}\delta(n-1) + \frac{1}{8}\delta(n-2) + \delta(n-4) - \frac{1}{4}\delta(n-3)$

Soln

$$H(z) = 1 + \frac{1}{4}z^{-1} + \frac{1}{8}z^{-2} + z^{-4} + \frac{1}{4}z^{-3}$$

$$\downarrow$$

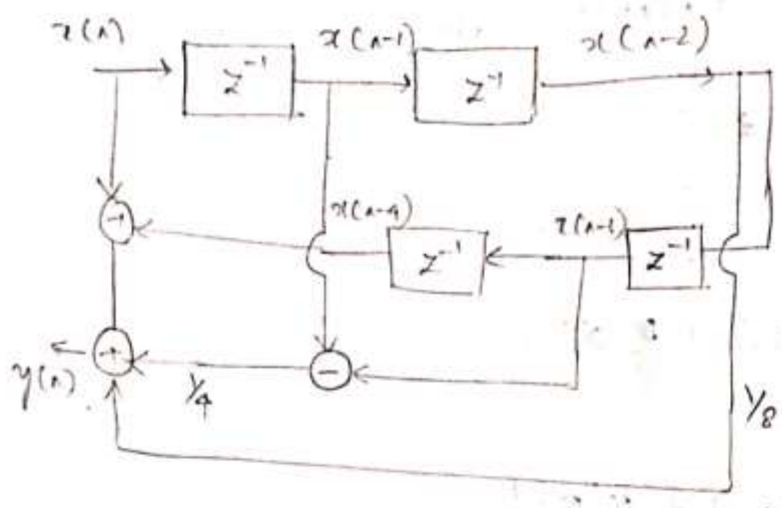
$$Y(z) = X(z) + \frac{1}{4}z^{-1}X(z) + \frac{1}{8}z^{-2}X(z) + z^{-4}X(z) + -\frac{1}{4}z^{-3}X(z)$$

$\downarrow$ IZT

$$y(n) = x(n) + \frac{1}{4}x(n-1) + \frac{1}{8}x(n-2) + x(n-4) - \frac{1}{4}x(n-3)$$

$$y(n) = x(n) + \frac{1}{4}x(n-1) + \frac{1}{8}x(n-2) - \frac{1}{4}x(n-3) + x(n-4)$$

$$y(n) = \{x(n) + x(n-4)\} + \frac{1}{4}\{x(n-1) - x(n-3)\} + \frac{1}{8}x(n-2)$$



Structures for IIR Systems -DF1(direct form1) and DF2(direct form2):

The transfer function of the system is obtained as

$$
\begin{aligned}
H(z) &= \frac{Y(z)}{X(z)} = \frac{\sum_{k=0}^{M} b_k z^{-k}}{\sum_{k=0}^{N} a_k z^{-k}} \\
&= \frac{b_0 + b_1 z^{-1} + b_2 z^{-2} + \cdots + b_M z^{-M}}{a_0 + a_1 z^{-1} + a_2 z^{-2} + \cdots + a_N z^{-N}}.
\end{aligned}
$$

Let us consider above Equation for the case when N = M = 2. it is straight forward to extend this analysis to higher orders. The transfer function will reduce to

$$
H(z) = \frac{Y(z)}{X(z)} = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2}},
$$

which we can write in the following format

$$
\begin{aligned}
H(z) &= \left(\frac{Y(z)}{W(z)}\right)\left(\frac{W(z)}{X(z)}\right) \\
&= \left(\frac{b_0 + b_1 z^{-1} + b_2 z^{-2}}{1}\right)\left(\frac{1}{1 + a_1 z^{-1} + a_2 z^{-2}}\right).
\end{aligned}
$$

Let

$$
H_1(z) = \frac{W(z)}{X(z)} = b_0 + b_1 z^{-1} + b_2 z^{-2}
$$

and

$$
H_2(z) = \frac{Y(z)}{W(z)} = \frac{1}{1 + a_1 z^{-1} + a_2 z^{-2}}.
$$

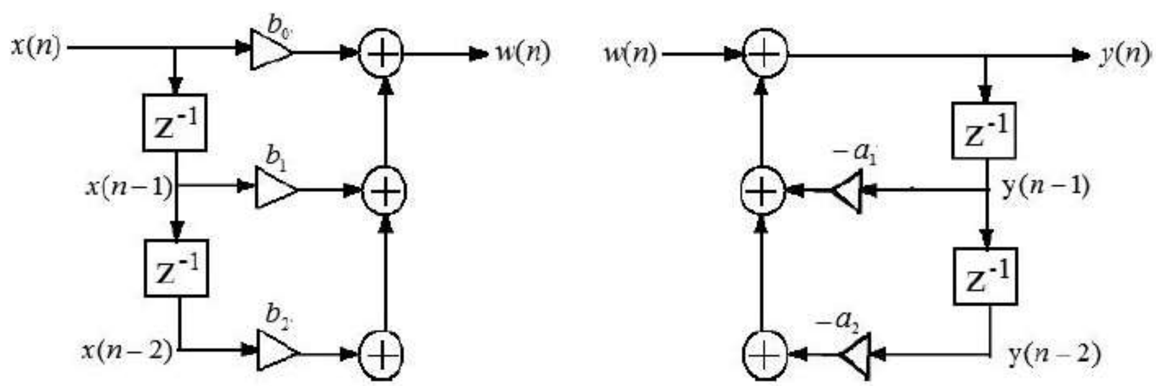


Fig. 4.6 Realization structures for $H_1(z)$ and $H_2(z)$.

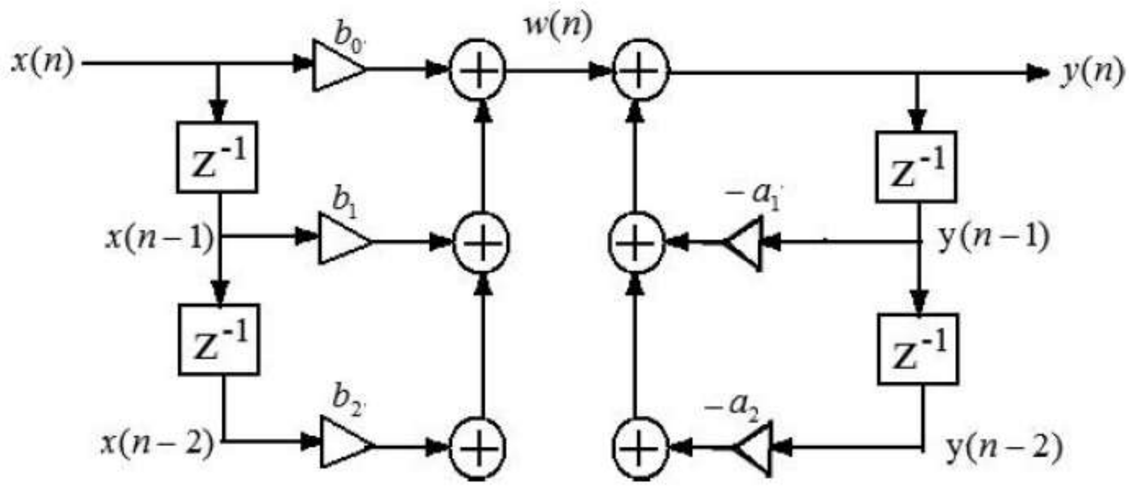
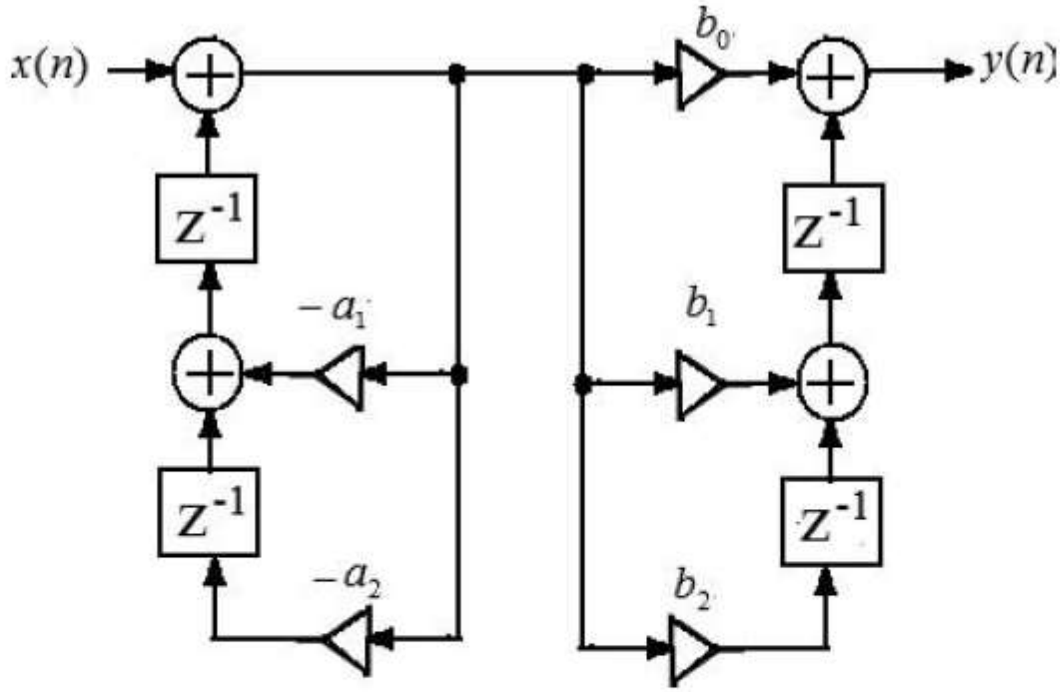


Fig. 4.7 Direct form 1 Realization structure for $H(z)$.

In the time domain we can write two equations

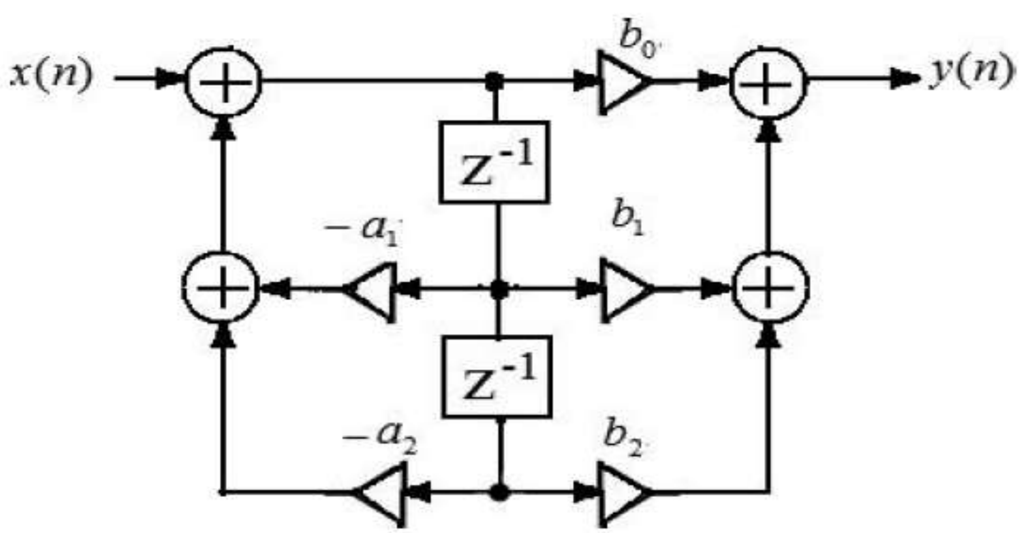
$$w(n) = b_0x(n) + b_1x(n-1) + b_2x(n-2), \quad \text{and}$$
$$y(n) = w(n) - a_1y(n-1) - a_2y(n-2).$$

The transpose of the Direct form 1 structure is shown below



this structute is a Direct Form 1 structure. We also note there are 4 delays and the order of the transfer function is 2.

The Direct form 1t is also noncanonic. It is possible with further manipulations of these structures to obtain a canonic structure. For instance, in Figure above , moving all delays to be done before the multipli-ers it will not change the transfer function. It follows that the delays on the parallel arms have the same inputs and will have the same outputs. These delays can be merged to form the structure which is canonic and is shown below. This structure is referred to as Direct form II structure.



Direct form II structure.

Cascade form Realization:

The cascade form structure is nothing, but a cascaded or series interconnection of the sub transfer functions or sub system functions which are realized by using the direct form structures (either direct form-I or direct form-II or a combination of both). Hence, in cascade form realization, the given transfer function $H(z)$ is expressed as a product of a number of second order or first order sections as indicated below:

$$H(z) = \frac{Y(z)}{X(z)} = \prod_{i=1}^{k} H_i(z)$$

where $$H_i(z) = \frac{C_{0i} + C_{1i}z^{-1} + C_{2i}z^{-2}}{d_{0i} + d_{1i}z^{-1} + d_{2i}z^{-2}} \quad \text{[second order section]}$$

or $$H_i(z) = \frac{C_{0i} + C_{1i}z^{-1}}{d_{0i} + d_{1i}z^{-1}} \quad \text{[first order section]}$$

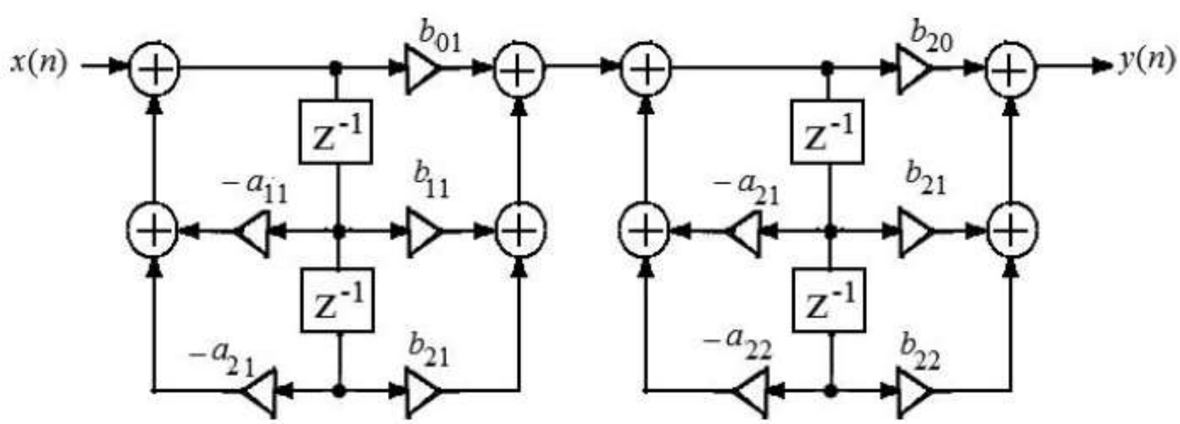


Fig. 4.10 A cascade of two second-order sections.

Each of these sections is realized separately and all of them are connected in cascade (series). Therefore, the cascade form realization is also called a series structure in which one sub transfer function is the input to the other transfer function and so on. The cascade form realization is shown below

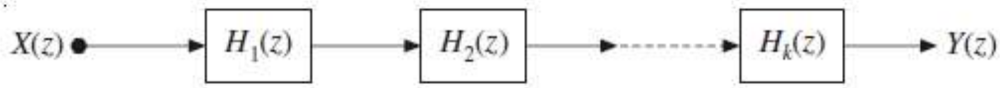


Figure 4.12 Cascade realization of IIR system.

Parallel Form Realization:

In this case a high order direct form transfer structure is
expressed as a parallel realization of first- and second-order sections.

$$H(z) = \frac{Y(z)}{X(z)} = \frac{\sum_{k=0}^{M} b_k z^{-k}}{\sum_{k=0}^{N} a_k z^{-k}}$$
$$= D + \sum_{k} \left(\frac{d_{0k} + d_{1k} z^{-1} + d_{2k} z^{-2}}{1 + c_{1k} z^{-1} + c_{2k} z^{-2}} \right).$$

One can obtain first-order sections if $d_{2k} = c_{2k} = 0$ for any section k. In the fig. a first-order section is connected in parallel to a second-order section. The transfer function is given as

$$H(z) = \frac{Y(z)}{X(z)}$$
$$= D + \left(\frac{d_{00} + d_{10} z^{-1}}{1 + c_{10} z^{-1}} \right) + \left(\frac{d_{01} + d_{11} z^{-1}}{1 + c_{11} z^{-1} + c_{21} z^{-2}} \right). \quad (4$$

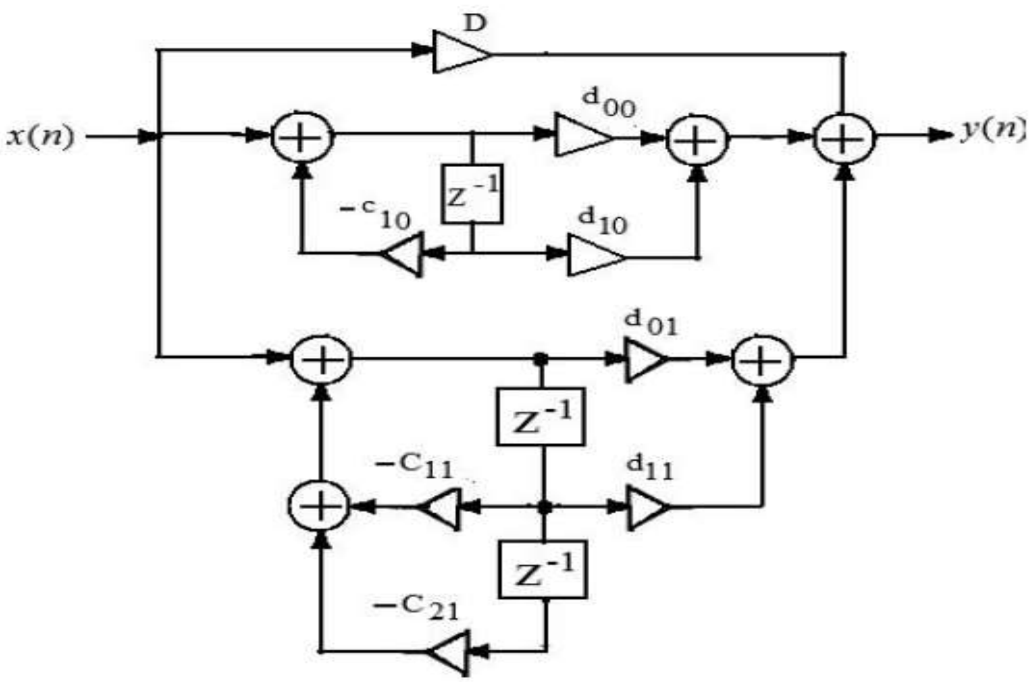


Fig. 4.11 Parallel realization of $H(z)$.