Aim: How do we define rational expressions and write them in equivalent form?

DO NOW: Use the calculator to graph

a) $y = \frac{x^2}{x^2+1}$ b) $y = \frac{(x+3)(x+2)}{(x+2)}$ c) y = $\frac{x+4}{x-2}$

Question: What function has a domain of all real values? Why?

I – Rational Function

1) A rational function is a function that you can write in the form $f(x) = \frac{P(x)}{Q(x)}$

2) where $P(x)$ and $Q(x)$ are polynomial functions. The domain of $f(x)$ is all real numbers except those values for which $Q(x) = 0$.

3) From the DO NOW – Graph (a) is a continuous graph while the others are discontinuous graphs. What is a discontinuous/continuous graph?

II – Finding Points of Discontinuity

1) In some cases, you can redefine the function to remove the discontinuity.

2) The discontinuity caused by $(x – a)^n$ in the denominator is removable if the numerator also has $(x – a)^n$ as a factor.

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3) **What are the domain and points of discontinuity of each rational function? Are the points of discontinuity removable or non-removable? What are the x- and y-intercepts?**

a. $y = \frac{x+5}{x-2}$ **b.** $y = \frac{1}{x^2+2x+1}$ **c.** $y = \frac{x+4}{x^2+2x-8}$

III- Vertical Asymptotes

1) The graph of the rational function $f(x) = \frac{P(x)}{Q(x)}$ has a vertical asymptote at each real zero of $Q(x)$ if $P(x)$ and $Q(x)$ have no common zeros. If $P(x)$ and $Q(x)$ have $(x-a)^m$ and $(x-a)^n$ as factors, respectively and $m < n$, then $f(x)$ also has a vertical asymptote at $x = a$.

2) What is an example of a rational function that fits the first sentence? **[Sample: $\frac{x+3}{x-4}$.]** What is an example of a rational function that fits the second sentence? **[Sample: $\frac{(x-4)^2}{(x-4)^3}$.]**

3) What are the vertical asymptotes for the graph of the rational function?

a. $y = \frac{x-2}{(x-1)(x+3)}$ **b.** $y = \frac{x-2}{(x-2)(x+3)}$ **c.** $y = \frac{x^2-1}{x+1}$

IV- Horizontal Asymptote of a Rational Function

1) What is the maximum number of vertical asymptotes in a function with a denominator of degree n? Why? **[n; a polynomial of degree n has at most n real roots.]**

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2) While the graph of a rational function can have any number of vertical asymptotes, it can have no more than one horizontal asymptote.

3) To find the horizontal asymptote of the graph of a rational function, compare the degree of the numerator m to the degree of the denominator n.

If $m < n$, the graph has horizontal asymptote $y = 0$ (the x-axis).

If $m > n$, the graph has no horizontal asymptote.

If $m = n$, the graph has horizontal asymptote $y = \frac{a}{b}$ where a is the coefficient of the term of greatest degree in the numerator and b is the coefficient of the term of greatest degree in the denominator.

4) Find the horizontal asymptote of the graph of each rational function. To start, identify the degree of the numerator and denominator.

a. $y = \dfrac{x+1}{x+5}$ **b.** $y = \dfrac{x+2}{2x^2-4}$ **c.** $y = \dfrac{3x^3-4}{4x+1}$

$x+1$ □ degree 1

$x+5$ □ degree 1

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5) What is the graph of the rational function $y = \frac{x^2 + x - 12}{x^2 - 4}$?

$y = \frac{x^2 + x - 12}{x^2 - 4}$

horizontal asymptote: $y = \frac{1}{1} = 1$

$y = \frac{(x + 4)(x - 3)}{(x + 2)(x - 2)}$

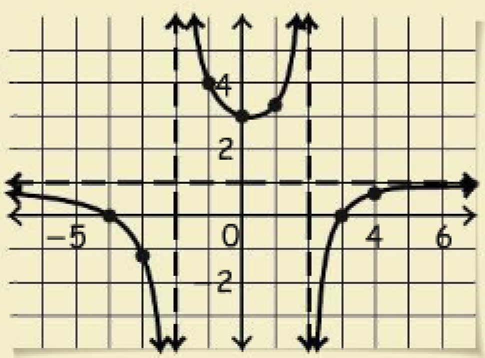
vertical asymptotes: $x = -2, x = 2$

When the numerator equals zero, $y = 0$.
x-intercepts: $(-4, 0)$ and $(3, 0)$

$y = \frac{(0 + 4)(0 - 3)}{(0 + 2)(0 - 2)} = 3$

y-intercept: $(0, 3)$

More points on the graph:
$(-3, -\frac{6}{5})$, $(-1, 4)$, $(1, \frac{10}{3})$, and $(4, \frac{2}{3})$



6) . What is the graph of the rational function $y = \frac{x + 3}{x^2 - 6x + 5}$?