Aim: How do we solve problems leading to rational equation?

DO NOW:

Exercise 1

1. Anne and Maria play tennis almost every weekend. So far, Anne has won 12 out of 20 matches.
   a. How many matches will Anne have to win in a row to improve her winning percentage to 75%?

   b. How many matches will Anne have to win in a row to improve her winning percentage to 90%?

   c. Can Anne reach a winning percentage of 100%?

   d. After Anne has reached a winning percentage of 90% by winning consecutive matches as in part (b), how many matches can she now lose in a row to have a winning percentage of 50%?

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Example

Working together, it takes Sam, Jenna, and Francisco two hours to paint one room. When Sam works alone, he can paint one room in 6 hours. When Jenna works alone, she can paint one room in 4 hours. Determine how long it would take Francisco to paint one room on his own.

Exercises 2–4

2. Melissa walks 3 miles to the house of a friend and returns home on a bike. She averages 4 miles per hour faster when cycling than when walking, and the total time for both trips is two hours. Find her walking speed.

3. You have 10 liters of a juice blend that is 60% juice.
   a. How many liters of pure juice need to be added in order to make a blend that is 75% juice?

   b. How many liters of pure juice need to be added in order to make a blend that is 90% juice?

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c. Write a rational equation that relates the desired percentage p to the amount A of pure juice that needs to be added to make a blend that is $p\%$ juice, where $0 < p < 100$. What is a reasonable restriction on the set of possible values of p? Explain your answer.

d. Suppose that you have added 15 liters of juice to the original 10 liters. What is the percentage of juice in this blend?

e. Solve your equation in part (c) for the amount A. Are there any excluded values of the variable p? Does this make sense in the context of the problem?

4. You have a solution containing 10% acid and a solution containing 30% acid.
   a. How much of the 30% solution must you add to 1 liter of the 10% solution to create a mixture that is 22% acid?

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b. Write a rational equation that relates the desired percentage p to the amount A of 30% acid solution that needs to be added to 1 liter of 10% acid solution to make a blend that is $p\%$ acid, where $0 < p < 100$. What is a reasonable restriction on the set of possible values of p? Explain your answer.

c. Solve your equation in part (b) for A. Are there any excluded values of p? Does this make sense in the context of the problem?

d. If you have added some 30% acid solution to 1 liter of 10% acid solution to make a 26% acid solution, how much of the stronger acid did you add?

COMPLETE EXERCISES: 1,2,3,5,7,8,9

Problem Set

1. If two inlet pipes can fill a pool in one hour and 30 minutes, and one pipe can fill the pool in two hours and 30 minutes on its own, how long would the other pipe take to fill the pool on its own?

2. If one inlet pipe can fill the pool in 2 hours with the outlet drain closed, and the same inlet pipe can fill the pool in 2.5 hours with the drain open, how long does it take the drain to empty the pool if there is no water entering the pool?

3. It takes 36 minutes less time to travel 120 miles by car at night than by day because the lack of traffic allows the average speed at night to be 10 miles per hour faster than in the daytime. Find the average speed in the daytime.

4. The difference in the average speed of two trains is 16 miles per hour. The slower train takes 2 hours longer to travel 170 miles than the faster train takes to travel 150 miles. Find the speed of the slower train.

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5. A school library spends $\$80$ a month on magazines. The average price for magazines bought in January was 70 cents more than the average price in December. Because of the price increase, the school library was forced to subscribe to 7 fewer magazines. How many magazines did the school library subscribe to in December?

6. An investor bought a number of shares of stock for $\$1{,}600$. After the price dropped by $\$10$ per share, the investor sold all but 4 of her shares for $\$1{,}120$. How many shares did she originally buy?

7. Newton's law of universal gravitation, $F = \frac{Gm_1m_2}{r^2}$, measures the force of gravity between two masses m_1 and m_2, where r is the distance between the centers of the masses, and G is the universal gravitational constant. Solve this equation for G.

8. Suppose that $t = \frac{x+y}{1-xy}$.

   a. Show that when $x = \frac{1}{a}$ and $y = \frac{2a-1}{a+2}$, the value of t does not depend on the value of a.

   b. For which values of a do these relationships have no meaning?

9. Consider the rational equation $\frac{1}{R} = \frac{1}{x} + \frac{1}{y}$.

   a. Find the value of R when $x = \frac{2}{5}$ and $y = \frac{3}{4}$.

   b. Solve this equation for R, and write R as a single rational expression in lowest terms.

10. Consider an ecosystem of rabbits in a park that starts with 10 rabbits and can sustain up to 60 rabbits. An equation that roughly models this scenario is

$$P = \frac{60}{1 + \frac{5}{t+1}},$$

where P represents the rabbit population in year t of the study.

   a. What is the rabbit population in year 10? Round your answer to the nearest whole rabbit.

   b. Solve this equation for t. Describe what this equation represents in the context of this problem.

   c. At what time does the population reach 50 rabbits?