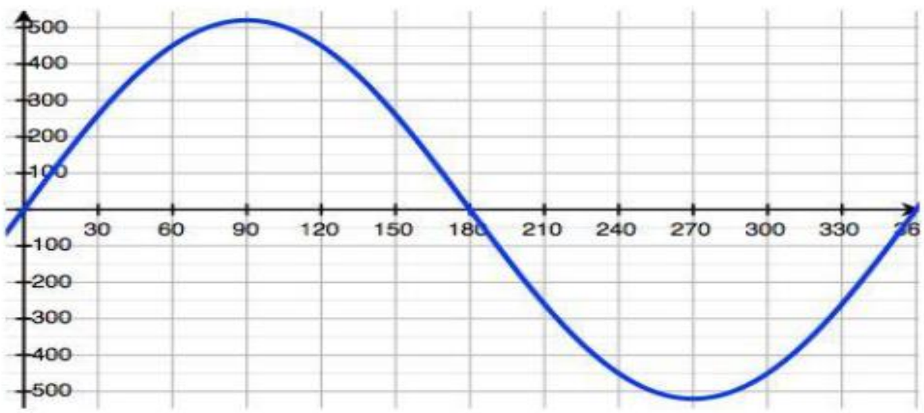
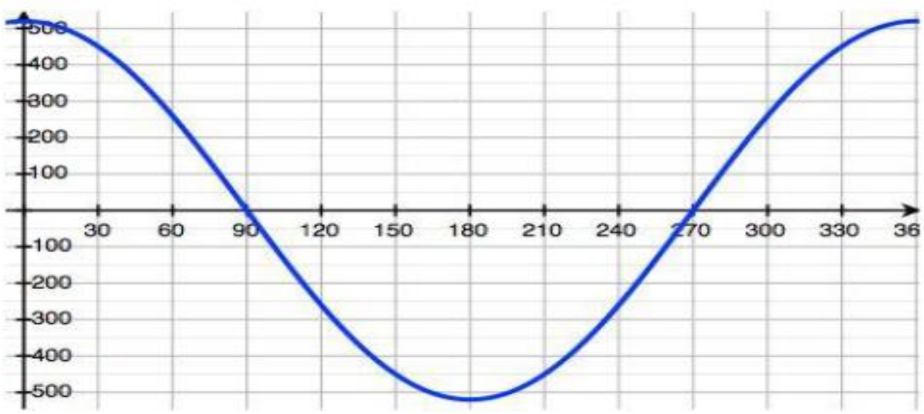
MRS21-HW-15

1)

Height:



Co-Height:



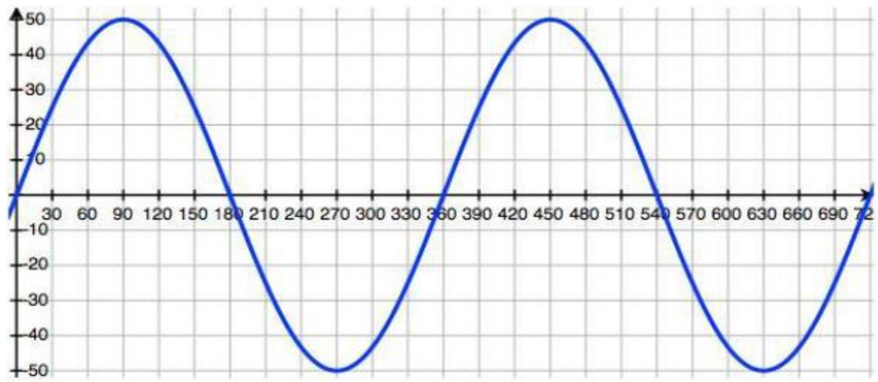
2)

The radius of the wheel is the distance from the center of the wheel to a point on the wheel. We can easily measure this at one of the four points when the car is at the top or bottom of the wheel or at the far left or the far right. Thus, the radius is the difference between the maximum value of either function and zero, so the radius is the maximum value of either the height or the co-height function.

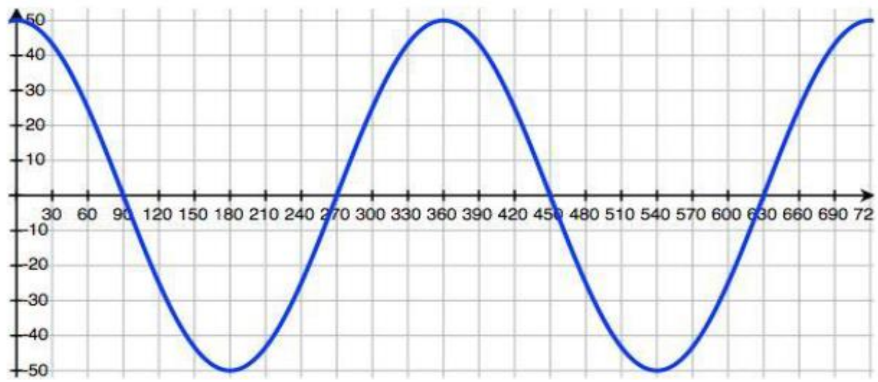
3)

Both the height and co-height functions have the same domain, $[0, 360]$, and range, $[-87.5, 87.5]$. Both functions have the same maximum value of 87.5 and minimum value of -87.5, but they occur at different amounts of rotation. When one function takes on a value of zero, the other either takes on its maximum value of 87.5 or its minimum value of -87.5. The co-height function starts at its maximum value, and the height function starts at zero. The graph of the co-height function is the graph of the height function translated horizontally to the left by 90.

4) *Height:*

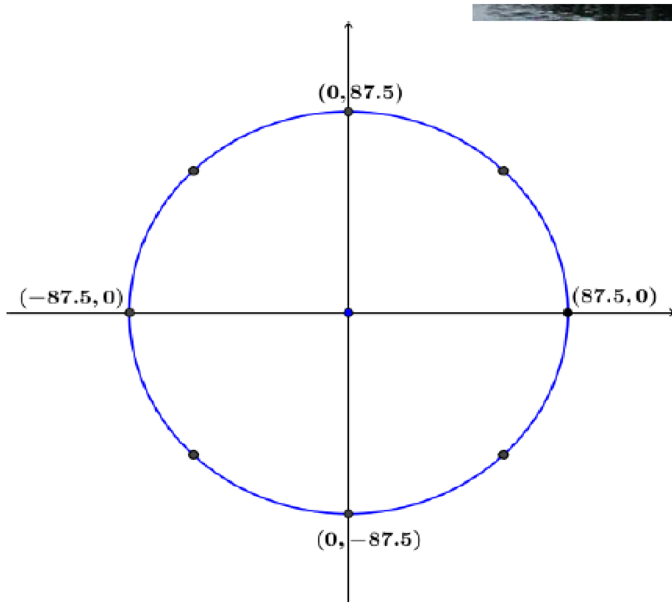


Co-Height:



5)

The Great Wheel has a diameter of 175 *feet, so the radius is* 87.5 *feet.*



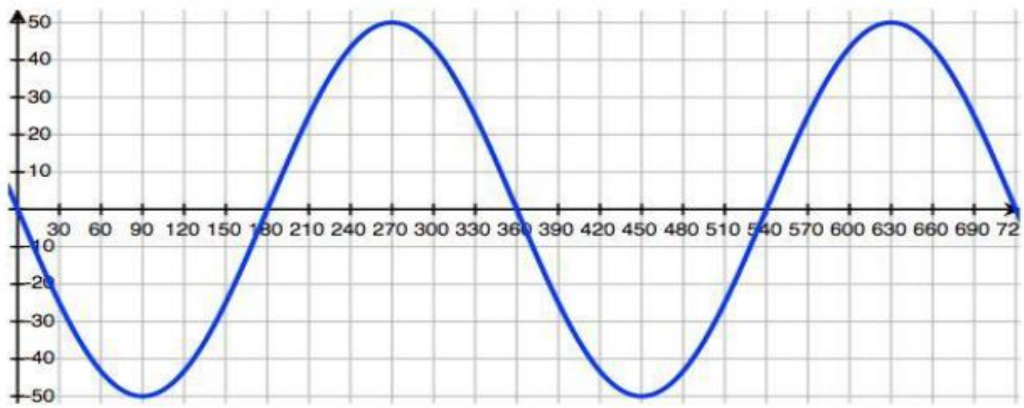
6)

Consider the right triangle formed by the spoke of the wheel connecting the car to the center, the horizontal axis, and the perpendicular line dropped from the car's position to the horizontal axis. If the value of the height and co-height functions are equal, then the legs of this triangle have the same length, meaning that it is an isosceles right triangle. There are four locations for such a triangle, with the passenger car being located in the first, second, third, or fourth quadrant. However, in the second and fourth quadrants, either the co-height takes on a negative value or the height takes on a negative value, but not both. Thus, for the co-height and height to take on the same value, the passenger car must be in either the first or the third quadrant. In the first quadrant, the car has rotated through $45°$, and in the third quadrant, the car has rotated through $180° + 45° = 225°$.

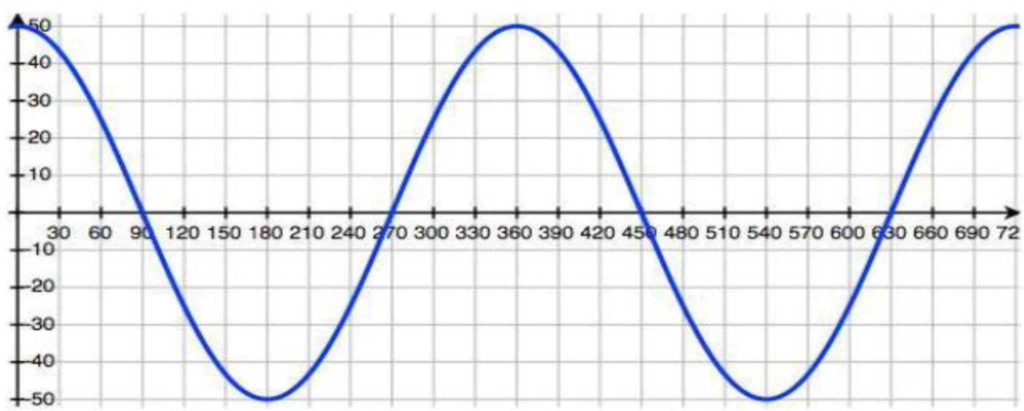
The same result holds for a Ferris wheel of any radius.

7)

Height:

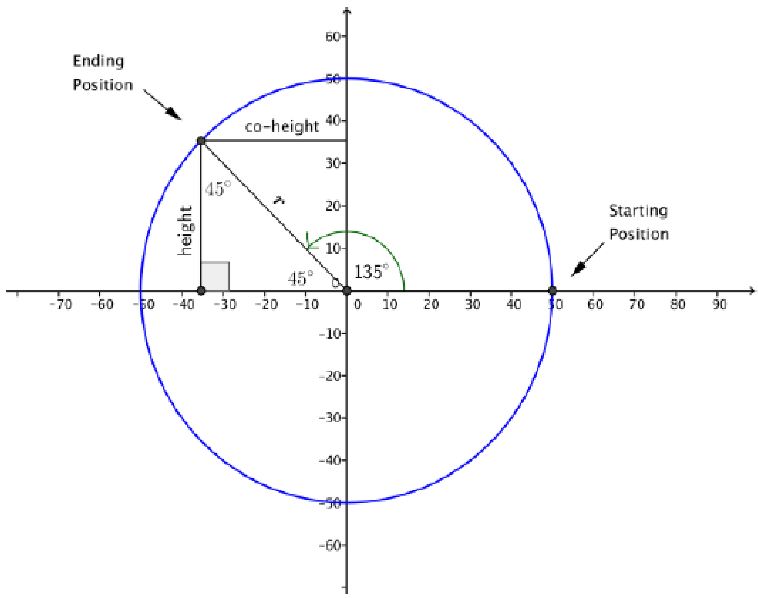


Co-Height:



8)

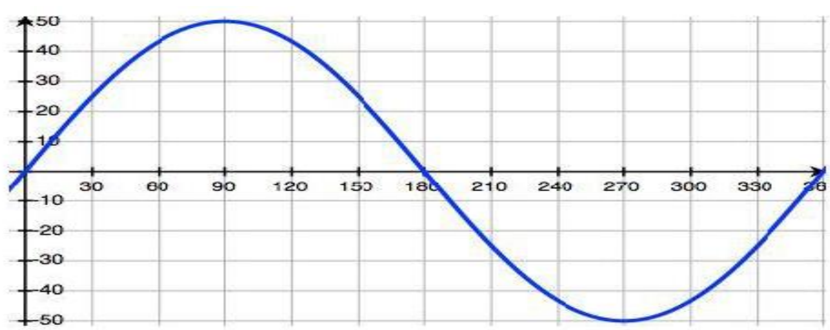
Lee is correct. Since Yuki's car started at the 3 o'clock position and rotated $135°$, then the ending position is in the second quadrant. The spoke of the Ferris wheel connecting her car to the center of the wheel makes a $45°$ angle with the horizontal, which creates a $45°$–$45°$–$90°$ triangle as shown in the diagram below. Then the height and the co-height at this position are equal, since the legs of an isosceles right triangle are congruent. Thus, if Lee knows the value of the height function of Yuki's car, then she knows the value of the co-height at this position.



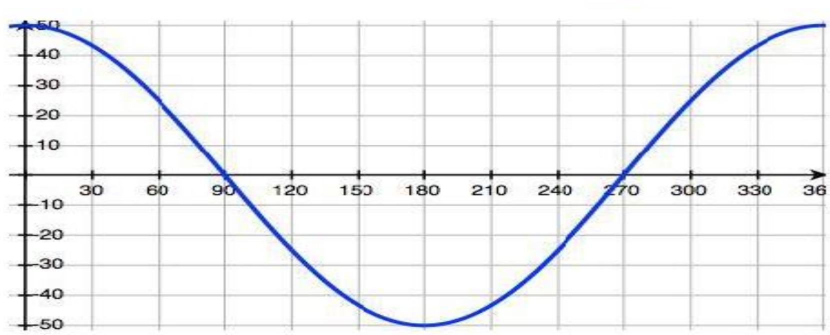
If Lee knows the diameter of the wheel, then she knows the radius, r, which is half of the diameter. Then she knows the length of a leg of an isosceles right triangle with hypotenuse of length r is $\frac{\sqrt{2}}{2}r$. Thus, if Lee knows the length of the diameter of the wheel, then she can calculate Yuki's co-height.

9)

Height:



Co-Height:



10)

Below, the blue curve represents the height function, and the red curve represents the co-height function.