

Aim: How do we use the special role of zero in factoring to solve polynomial equations? (Chapter 5-5)

Do now: To use your calculator to find the roots (zeros) of the function

$$F(x) = 21x^2 + 29x + 10$$

I- Rational Root Theorem

**Theorem Rational Root Theorem**

Let $P(x) = a_nx^n + a_{n-1}x^{n-1} + \dots + a_1x + a_0$ be a polynomial with integer coefficients.

There are a limited number of possible roots of $P(x) = 0$:

- Integer roots must be factors of a_0 .
- Rational roots must have reduced form $\frac{p}{q}$ where p is an integer factor of a_0 and q is an integer factor of a_n .

1) Use the Rational Root Theorem to list all possible rational roots for each equation.

Then find any actual rational roots.

a. $x^3 - 5x^2 + 17x - 13$

To start, list the constant term's factors
and the leading coefficient's factors.

constant term factors: $\pm 1, \pm 13$
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leading coefficient factors: ± 1

b. Use a table to test each possible root into the equation. One or more values will be the root(s) of the equation.

x	1	-1	13	-13
P(x)				

c. What are the root(s)?

2) Use the Rational Root Theorem to list all possible rational roots for each equation.

Then find any actual rational roots.

a. $2x^3 - 5x^2 + x - 7$

b. $x^3 - 4x^2 - 15x + 18$

II- Using the Conjugate Root Theorem to Identify Roots

**Theorem Conjugate Root Theorem**

If $P(x)$ is a polynomial with *rational* coefficients, then irrational roots of $P(x) = 0$ that have the form $a + \sqrt{b}$ occur in conjugate pairs. That is, if $a + \sqrt{b}$ is an irrational root with a and b rational, then $a - \sqrt{b}$ is also a root.

If $P(x)$ is a polynomial with *real* coefficients, then the complex roots of $P(x) = 0$ occur in **conjugate** pairs. That is, if $a + bi$ is a complex root with a and b real, then $a - bi$ is also a root.

1) A polynomial function $P(x)$ with rational coefficients has the given roots. Find two additional roots of $P(x) = 0$.

a. $1 + 4i$ and $\sqrt{3}$

b. $3 - \sqrt{2}$ and $1 + \sqrt{3}$

2) Write a polynomial function with rational coefficients so that $P(x) = 0$ has the given roots.

a. $3i$

To start, use the Conjugate Root Theorem to identify a second root.

Since $3i$ is a root, $-3i$ is also a root.

b. -2 and -8