Lesson #10 Sect. 3.54

Aim: What ARE iterations and dynamic systems?

Do Now: Rewrite the following system in any other way that you like

Initial Amount: # 100

Amount After one year $1.08(100) = 108$ ← \$ After 1 year (Rate, Initial Amount)

Amount After 2 years $1.08(108) = 116.64$

Amount After 3 years $1.08(116.64) = 125.97$

Re-write = $100(1.08)^t$

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I- Iterations - Repetitions

1) Iteration of a function ARE repeated compositions of a function with itself.

2) ex: $f(f(f(x))$ or $f^3(x)$

$f(x) \cdot f(x) \cdot f(x) \neq f^3(x)$

3) Write the 1st 4 iteration of the function
$f(x) = \sqrt{x}$ begin $x = 0.75$

No calculator

1) $\sqrt{0.75}$
2) $\sqrt{0.8660}$
3) $\sqrt{0.9306}$
4) $\sqrt{0.9646} = 0.9821$

Calculator (casio)

1) Menu 3 (Graph)
2) $Y1 = \sqrt{x}$
3) Menu 1
4) $0.75 \rightarrow x$
5) VARS, f4
6) $Y1 \rightarrow x$ 4 times

II- Orbits

1) The sequence of output values produced by starting with c.

ex) $c, f(c), f^2(c), f^3(c) \ldots$

2) Orbits tend to
   a) converge
   b) approach infinity
   c) go to a fixed point
   d) go eventually to a fixed value (point)
   e) be periodic

3) Let $f(x) = \sqrt{x}$ and compute the 1st 12 terms of the orbit $x = 100$. To what number does the orbit appear to approach.

Mar 19-2:49 PM

III – Exercises – Explain, what's happening to the orbit

$|x|>1$

9
81
6561
43046721
$1.853020189 \times 10^{15}$
$3.43368382 \times 10^{30}$
$1.179018458 \times 10^{61}$

$f^n(x) \to +\infty$ as $n \to \infty$

a) $x=3$ for $f(x)=x^2$

b) $x=0.6$ for $f(x)=x^2$ → $f^n(x) \to 0$ (Approached zero)

c) $x=0$ for $f(x)=\sqrt{x}$ → $f^n(x)=0$ (Goto a fix)

d) $x=-1$ for $f(x)=|x|$ → $f^n(x)=1$ (Goto a fix)

e) $x=3$ for $f(x)=1/x$ → Periodic (pattern) → $0.3\overline{3}$, 3

f) $x=-1$ for $f(x)=|x-3|$ → Periodic after 4

Reading/Study Example 5 on page 202

Example 5 Orbit Analysis

Analyze all the orbits of $f(x) = x^2$ and illustrate them graphically.

Solution

Example 3 and the Calculator Exploration after it suggest two possibilities.
When $|x| < 1$, the orbit of x converges to 0.
When $|x| > 1$, the orbit of x approaches infinity.

The orbit of 1 is 1, 1, 1, . . ., so 1 is a fixed point.
The orbit of 0 is 0, 0, 0, . . ., so 0 is a fixed point.
The orbit of -1 is -1, 1, 1, . . ., so -1 is an eventually fixed point.

Figure 3.5.A-4 illustrates the orbits of $f(x) = x^2$.

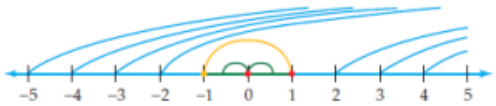

Figure 3.5.A-4

Red: Fixed points
Orange: Eventually fixed point
Green: Orbits that converge to 0
Blue: Orbits that approach ∞

For a function f, if $f(a) = a$, for some value a, then a is called a fixed point of the function.