Lesson#15 Section 4.4

Aim: What ARE Rational functions?

Note: Rational = Fraction

Do Now: Find the domain, x-intercepts and y-intercepts

a) $f(x)=\frac{1}{x^2}$

Domain: all real numbers except zero

x-intercept: No

b) $g(x)=\frac{x^2+3x+1}{x^2-x-6}$ $x^2-x-6=(x+2)(x-3)=0$

All real numbers except $x=-2$ and $x=+3$

x-intercept $(y=0)$

$(-2.618, 0)$

$(-0.382, 0)$

$\frac{0}{1}=\frac{x^2+3x+1}{x^2-x-6}$

$0=x^2+3x+1$

$x^2+3x+\frac{9}{4}-\frac{9}{4}+\frac{4}{4}=-\frac{5}{4}$

$0=(x+\frac{3}{2})^2-\frac{5}{4}$

$\sqrt{\frac{5}{4}}=x+\frac{3}{2}$

$\pm\frac{\sqrt{5}}{2}-\frac{3}{2}=x$

$-0.382\approx\frac{+\sqrt{5}-3}{2}$

$-2.618=\frac{-\sqrt{5}-3}{2}$

I- Rational functions ?

1) Rational function is a quotient (Division/fraction) of two polynomials

2) Domain: set of all Real numbers that are not a zero for its Denominator

3) Y-interrept is at $f(0)$

4) X-intercept: zeros for the numerator but not zeros for the Denominator

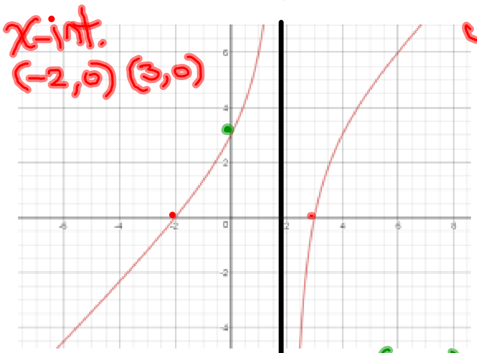
<u>examples</u>| Find the Domain/ x / y intercepts

a) $f(x) = \frac{x^2-x-6}{x-2}$

Domain: All Real #'s except $x=2$

x-int. $(-2,0)$ $(3,0)$

y-int. $(0,3)$



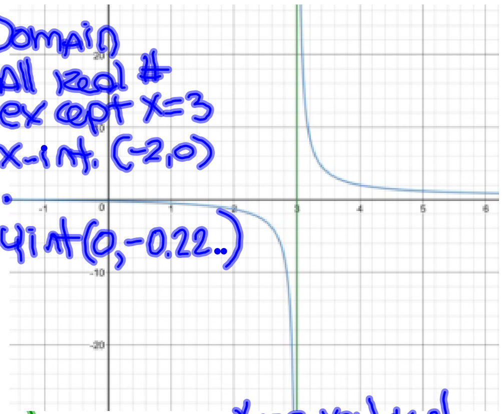
$x=2$ (vertical Asymptote)

<u>notation</u> (Limit)

$g(x) \xrightarrow[+\infty]{} x=3$

b) $g(x) = \frac{x+2}{3x-9}$

Domain
All Real #
except $x=3$
x-int. $(-2,0)$
y-int $(0,-0.22..)$



$x=3$ vertical Asympt.

<u>Note:</u>

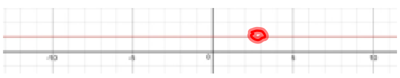
a) $g(x)$ increases without a bound as x approaches $x=3$ from the right side

b) $g(x)$ decreases w/o a bound as x approaches $x=3$ from the left side

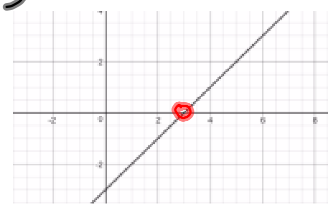
example 2 please indicate the differences with their graphs

a) $f_1(x) = \frac{x-3}{x-3}$

b) $f_2(x) = \frac{\cancel{(x-3)}(x-3)}{\cancel{x-3}}$



common hole at $x=3$

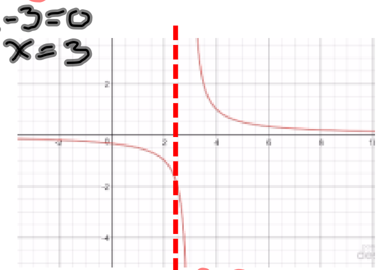


c) $f_3(x) = \frac{\cancel{x-3}}{\cancel{(x-3)}(x-3)}$

$x=c$

$x-3=0$

$x=3$



Note 1: At $x=c$ if c is a zero of the denominator but not of the numerator $\Rightarrow$ vertical asymptote

Note 2: At $x=c$ if c is a zero for both denominator and numerator then c is a hole

$$f(x) = \frac{g(x)}{h(x)} = \frac{(x+3)(x+3)}{(x+3)}$$

multiplicity

$\frac{2}{1}$ $\quad 2 \geq 1$

hole at c

begin H.W #15