

MAT-120 – HW #12– Answers

Please select the correct answer number of each question. There are more answers than questions. Answers may be repeated.

1) No; requirement #3 is not met. There are more than 2 possible outcomes.

2) 46%

3)

X is a binomial RV, because the experiment has a fixed number of trials ( $n=3$ ); each trial results in 2 complimentary possibilities: a ball is green (success) and a ball is red (failure); and the probability of success  $p=4/7$  for any trial (trials are independent because we choose with replacement)

4) Yes; all four requirements are met.

5)

$$E(X)=1.7143.$$

$$6) P(x) = \left[ \frac{n!}{x!(n-x)!} \right] \cdot p^x \cdot (1-p)^{n-x}$$

$$P(X = 2) = \left[ \frac{6!}{4!2!} \right] \cdot (0.45)^2 \cdot (0.55)^4$$
$$= [15] \cdot (0.45)^2(0.55)^4 = 0.2780$$

$$7) P(x) = \left[ \frac{n!}{x!(n-x)!} \right] \cdot p^x \cdot (1-p)^{n-x}$$

$$P(X = 15) = \left[ \frac{15!}{15!0!} \right] \cdot (0.85)^{15} \cdot (0.15)^0$$
$$= [1] \cdot (0.874) \cdot (1) = 0.0874$$

8)

Yes, the results of the survey suggest that gender of a child, aged 6-11, has an effect on the probability of reading books for fun every day. In this survey, girls were more likely to read for fun than boys (28.1% chance vs. 20.9%). However, to draw the conclusion about population (all children ages 6-11), the claim must be tested using Hypothesis Testing (Chi-square test for independence).

$$9) P(x \leq 2) = P(x = 0 \text{ or } x = 1 \text{ or } x = 2) = P(x = 0) + P(x = 1) + P(x = 2) = 0.0005 + 0.0071 + 0.0462 = 0.0538$$

10)

$$E(X) = np = 0.2$$

$$11) P(x) = \frac{n!}{x!(n-x)!} \cdot p^x \cdot (1-p)^{n-x}$$

$$P(X=4) = \frac{6!}{4!2!} \cdot (0.55)^4 \cdot (0.45)^2$$
$$= [15] \cdot (0.55)^4 \cdot (0.45)^2 = [15] \cdot (0.915) \cdot (0.2025) = 0.2779$$

12)

Profit (\$)	Probability
8	1/6
-2	5/6

13) 20.9%

14)

X # of green	P(X)	XP(X)
0	${}_3C_0(4/7)^0(3/7)^3=0.0787$	0
1	${}_3C_1(4/7)^1(3/7)^2=0.3149$	0.3149
2	${}_3C_2(4/7)^2(3/7)^1=0.4198$	0.8396
3	${}_3C_3(4/7)^3(3/7)^0=0.1866$	0.5598
		E(X)=1.7143

15)

Lose \$44

$$16) P(x) = \frac{n!}{x!(n-x)!} \cdot p^x \cdot (1-p)^{n-x}$$

$$P(X=8) = \frac{8!}{3!5!} \cdot \left(\frac{1}{4}\right)^4 \cdot \left(\frac{3}{4}\right)^6 = [56] \cdot (0.25)^3 \cdot (0.75)^5 = 0.208$$

$$17) P(x) = \frac{n!}{x!(n-x)!} \cdot p^x \cdot (1-p)^{n-x}$$

$$P(X=2) = \frac{6!}{2!4!} \cdot (0.09)^2 \cdot (0.91)^4 = [15] \cdot (0.0081) \cdot (0.6857) = 0.0833$$

18)

$$P(X \leq 1) = P(X=0) + P(X=1) = 0.8171 + 0.1667 = 0.9838$$

19)

The events are not independent. For independent events,  $P(A \text{ and } B) = P(A) \cdot P(B)$ .  
 $P(\text{"fun"} \text{ and } \text{"boy"}) = 48/500$ ;  $P(\text{"fun"}) \cdot P(\text{"boy"}) = 124/500 \cdot 230/500$ .  $P(\text{"fun"} \text{ and } \text{"boy"}) \neq P(\text{"fun"}) \cdot P(\text{"boy"})$ . Thus they are not independent.

20)  $P(x) = \left[ \frac{n!}{x!} (n-x)! \right] \cdot p^x \cdot (1-p)^{n-x}$

$$P(x \leq 1) = P(X = 0) + P(X = 1) = \left[ \frac{24!}{0! 24!} \right] \cdot (0.04)^0 \cdot (0.96)^{24} + \left[ \frac{24!}{1! 23!} \right] \cdot (0.04)^1 \cdot (0.96)^{23}$$

$$= [1] \cdot (1) \cdot (0.3754) = 0.7508$$

21)

-\$0.33 expected loss of 33 cents per game

22)

$$P(X \geq 5) = P(X = 5 \text{ or } X = 6) = P(x = 5) + P(x = 6) = 0.3283 + 0.1428 = 0.4711$$

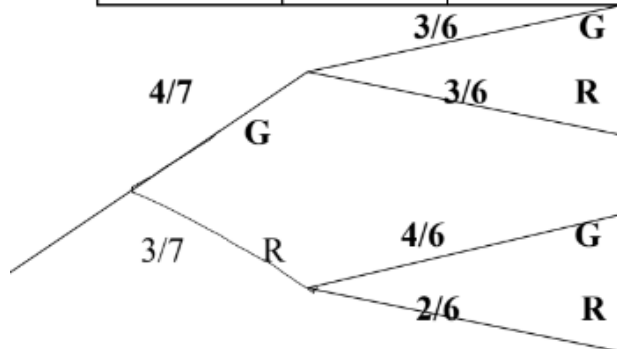
23) Yes; since  $P(\text{not having more than one on time}) = 1 - 0.9924 = 0.0076 < 0.05$ , not having more than one on time would be an unusual occurrence.

24)

X is not binomial RV, because trials are not independent. Probability of choosing a green ball changes from trial to trial, and depends on the outcome of previous trials, because we choose without replacement.

25)

event	X # of green	P(X)
RR	0	1/7
RG or GR	1	4/7
GG	2	2/7



26) 28.1%

27)

X is a **binomial** random variable:

- There is a fixed number of trials,  $n=10$
- Each trial has 2 outcomes. "Success": "a TV set is defective"
- The trials are independent, because the population we choose from is very large.
- Probability of success,  $p=0.02$

28)

If the described experiment is repeated a lot of times, the average number of green balls per each attempt is about 1.7

29) No; since  $0.544 > 0.05$ , "at most one wrong number" is not an unusual occurrence when the error rate is 15%

30)

0.055

31)

mean = 5, std. dev = 1.581

32)

the usual # of correct answers is 2 to 8. So passing by guessing is not unusual.

33) 10, 0.2