

## Lesson #16

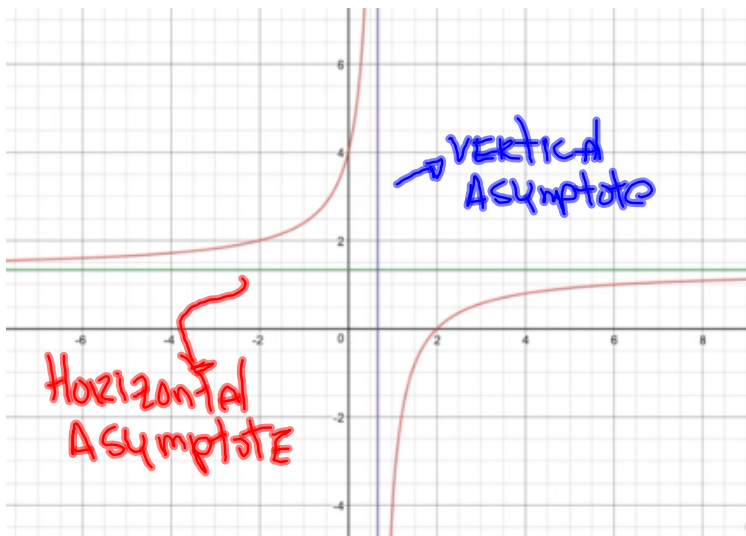
Aim: What ARE other types of Asymptotes for rational equations?

Do Now: Graph (USE CALCULATOR)

$$1) f(x) = \frac{-4x+8}{2-3x}$$

$$2) h(x) = \frac{3x^3-x}{x^3-1}$$

Find any asymptote; Describe any other behavior?



coefficient      Greatest exponent:  
 numerator: Degree

$$y = \frac{-4x+8}{2-3x} = \frac{4(-x+2)}{(2-3x)}$$

① Degree Numerator = Denominator

- a) Find the greatest exponent Numerator  
 $\frac{-4}{-3}$  take its coefficient
- b) same as a but in the denominator

∴ Horizontal Asymptote

$$\text{at } x = \frac{-4}{-3} = \frac{4}{3} = 1.\bar{3}$$

$$2) h(x) = \frac{3x^2 - x}{x^3 - 1} = \frac{x(3x^2 - 1)}{x^3 - 1}$$

Vertical Asymptotes ✓

$$x^3 - 1 = 0$$

$$x = 1$$

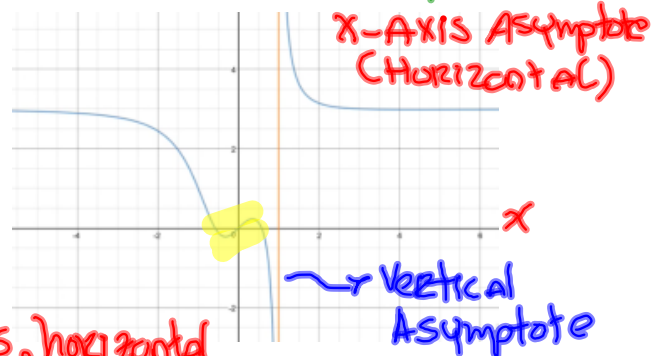
Horizontal Asymptotes ✓

Degrees

$$\frac{2}{3}$$

$$2 < 3$$

~~No Horizontal Asymptotes~~



Note: unlike vertical asymptotes, horizontal or other type of asymptote can be crossed at some point

Note Degree of Numerator ( $m$ )  
Degree of Denominator ( $n$ )

\*  $m < n$  Horizontal Asymptote at  $y = 0$  (X-axis)

$m > n$  No Horizontal Asymptote

$m = n$  Horizontal Asymptote at  $y = \frac{a}{b}$

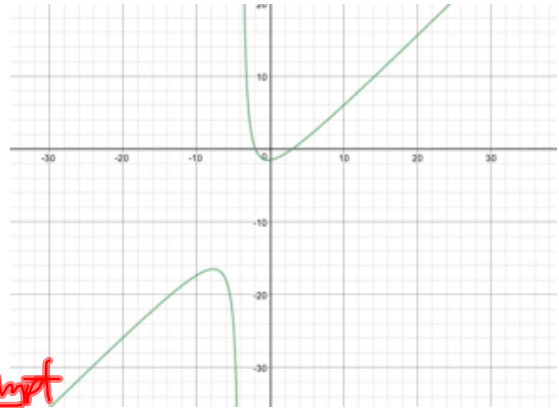
where "a" is the leading coefficient of numerator ÷ "b" is the leading coefficient of denominator.

\* Note: Even when the graph does not show them.

find the Asymptote(s)

$$f(x) = \frac{x^2 - x - 6}{x + 4}$$

$$\frac{x^2 - x - 6}{x + 4} = \frac{(x - 3)(x + 2)}{(x + 4)}$$



Vertical Asympt.

$$\begin{aligned} x + 4 &= 0 \\ x &= -4 \end{aligned}$$

Horizontal Asympt.

Degree Numerator > Degree of Denominator  
 $\therefore$  No Horizontal Asympt.

MORE ASYMP.

DIVIDE  $x^2 - x - 6 \div x + 4$

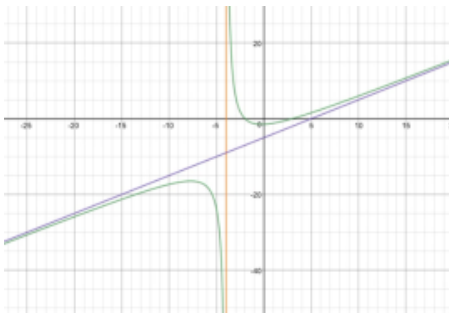
USE synthetic DIVISION

a)  $x + 4 = 0$   
 $x = -4$

$$\begin{array}{r|rrrr} -4 & 1 & -1 & -6 & \\ & & -4 & 20 & \\ \hline & 1 & -5 & +14 & \end{array}$$

c) write your answer (quotient)

$$\underbrace{x - 5}_{\text{Quotient}} + \frac{14}{x + 4} \left. \vphantom{\frac{14}{x + 4}} \right\} \text{Remainder}$$



d) A New Asymptote at  
 the Quotient:  $x - 5$