Lesson #17 Section 4.4

Aim: How do we graph a complete graph of a rational function?

Do Now: Complete the sentences

Let $f(x) = \frac{ax^n + ...}{cx^k ...}$ be a rational function whose numerator has a degree of n and denominator has a degree of K.

- If $n < K$, then the x-Axis is a horizontal Asymptote
  ex) $\frac{-7x^2 ...}{2x^3 ...}$
- If $n = K$, then the line $y = \frac{a}{c}$ is a horizontal Asymp.
- If $n > K$, then divide the Numerator by Denominator

  the quotient polynomial is diagonal Asymptote.

I- Complete Graph Analysis.

1) $f(x) = \frac{x+2}{x^2+x-12}$ find a complete graph

Step1 (Vertical Asympt.)

zeros in the Denominator but Not in the Numerator

$x^2+x-12 = (x+4)(x-3)=0$

$x=-4$ $x=3$

Step 2 (Horizontal Asymp)

Degrees

Numerator = 1

Denominator = 2

$1 < 2$

x-Axis

Step3 x-int.

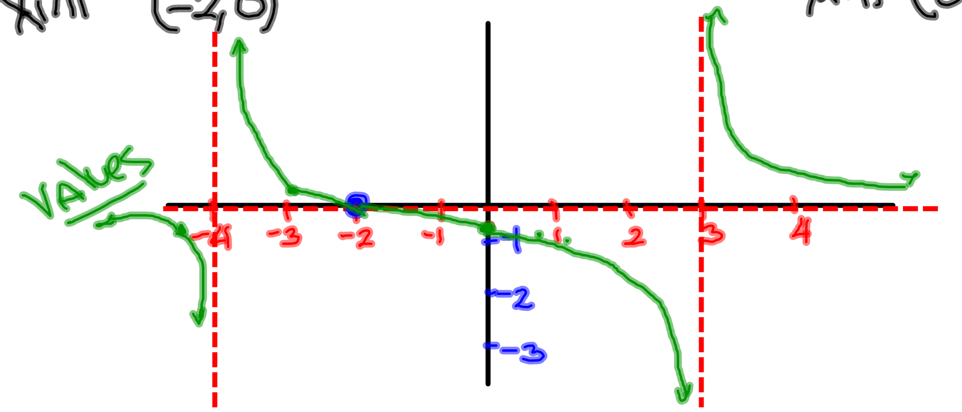
$\frac{0}{1} = \frac{x+2}{x^2+x-12}$

xint $(-2,0)$

Step4 yint. $(x=0)$

$\frac{0+2}{0+0-12} = -\frac{1}{6}$

yint $(0,-1/6)$



2) find a Complete graph for

$$f(x) = \frac{\overset{a_n}{x^3} - x^2 - 10x - \overset{a_0}{8}}{x^2 - 7x + 12} = \frac{(x-4)(x+2)(x+1)}{(x-3)(x-4)}$$

Solution

a) Factor both the numerator and denominator

$a_0 = \pm 1, \pm 2, \pm 4, \pm 8$

$a_n = \pm 1$

$> \frac{a_0}{a_n} = \pm 1, \pm 2, \pm 4, \pm 8$

not a factor

$$\begin{array}{r|rrrr} 1 & 1 & -1 & -10 & -8 \\ & & 1 & 0 & -10 \\ \hline & 1 & 0 & -10 & -18 \end{array} \rightarrow \text{not zero}$$

Factor

$$\begin{array}{r|rrrr} -1 & 1 & -1 & -10 & -8 \\ & & -1 & +2 & +8 \\ \hline & 1 & -2 & -8 & 0 \end{array}$$

$x^2 - 2x - 8$

$(x-4)(x+2)$

b) To Find Vertical Asymptotes

$$\frac{(x+2)(x+1)\cancel{(x-4)}}{\cancel{(x-4)}(x-3)}$$

only have one vertical asymptotes (zeros in the Denominator.

$x = 3$

c) X-intercept

When y = 0

$$0=\frac{\cancel{(x-4)}(x+1)(x+2)}{\cancel{(x-4)}(x-3)}$$

$$0=(x+1)(x+2)$$

$x=-1$ $x=-2$

d) y intercept

When x = 0

$$y=\frac{\cancel{(0-4)}(0+1)(0+2)}{\cancel{(0-4)}(0-3)}$$

$$y=\frac{2}{-3}=-\frac{2}{3}$$

d) check the values of (n) and (k)

Degree

n: Degree of Numerator

k: Degree of Denominator

$$\frac{x^3-x^2-10x-8}{x^2-7x+12}$$

$$\frac{3}{2} \quad 3>2$$

conclusion: ① No Horizontal Asymptote

* ② Diagonal Asymptote (Parabolic Asymptotes)

* Divide the Numerator by the Denominator (Long Division)

$$\begin{array}{r} x+6 \\ x^2-7x+12\,\overline{)\,x^3-x^2-10x-8} \\ \vdots \end{array}$$

③ there is a parabolic Asymptote at $y=x+6$

e) Holes

$$\frac{\cancel{(x-4)}(x+1)(x+2)}{\cancel{(x-4)}(x-3)}$$

(4, 30)

→ $x-4=0$

$x=4$ → Replace $\dfrac{(4+1)(4+2)}{(4-3)}=30$

$y=$

f) Graph

✓ - vertica Asympt.

$X=3$

✓ - Parobolic Asympt

$y=x+6$

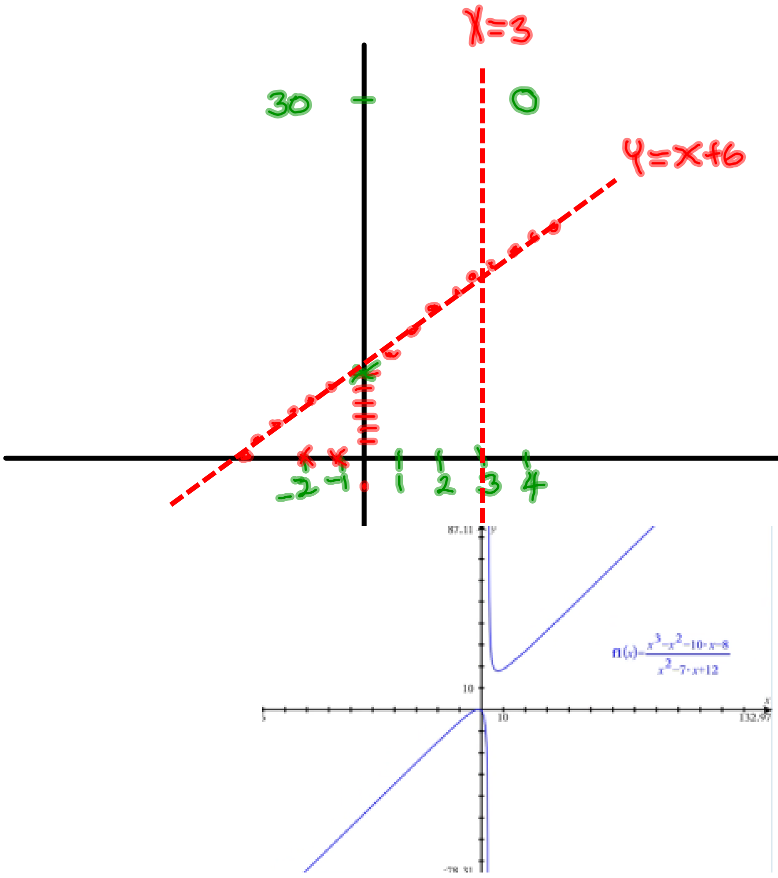
↳ slope → y int.

- x int. $x=-1$

$x=-2$

- y int. $y=-\frac{2}{3}$

- hole at $(4,30)$



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