## 4.1 Exponential Functions and Their Graphs

**In this section you will learn to:**

- evaluate exponential functions
- graph exponential functions
- use transformations to graph exponential functions
- use compound interest formulas

An **exponential function *f*** with base $b$ is defined by

$$f(x) = b^x \text{ or } y = b^x, \text{ where } b > 0, b \neq 1, \text{ and } x \text{ is any real number.}$$

Note: Any transformation of $y = b^x$ is also an exponential function.

**Example 1:** Determine which functions are exponential functions. For those that are not, explain why they are not exponential functions.

(a) $f(x) = 2^x + 7$ Yes No _____

(b) $g(x) = x^2$ Yes No _____

(c) $h(x) = 1^x$ Yes No _____

(d) $f(x) = x^x$ Yes No _____

(e) $h(x) = 3 \cdot 10^{-x}$ Yes No _____

(f) $f(x) = -3^{x+1} + 5$ Yes No _____

(g) $g(x) = (-3)^{x+1} + 5$ Yes No _____

(h) $h(x) = 2x - 1$ Yes No _____

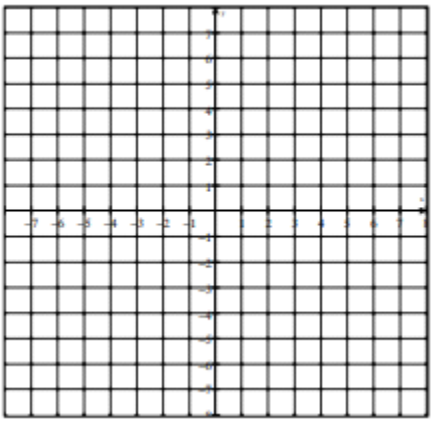
**Example 2:** Graph each of the following and find the domain and range for each function.

(a) $f(x) = 2^x$ domain: _____

range: _____

(b) $g(x) = \left(\frac{1}{2}\right)^x$ domain: _____

range: _____



| Characteristics of Exponential Functions $f(x) = b^x$ | |
|---|---|
| $b > 1$ | $0 < b < 1$ |
| **Domain:**<br>**Range:** | |
| | |
| | |
| | |
| | |

## Transformations of $g(x) = b^x$ $(c > 0)$: (Order of transformations is H S R V.)

**Horizontal:** $g(x) = b^{x+c}$ (graph moves $c$ units left)
$g(x) = b^{x-c}$ (graph moves $c$ units right)

**Stretch/Shrink: (Vertical)** $g(x) = cb^x$ (graph stretches if $c > 1$)
(graph shrinks if $0 < c < 1$)

**Stretch/Shrink: (Horizontal)** $g(x) = b^{cx}$ (graph shrinks if $c > 1$)
(graph stretches if $0 < c < 1$)

**Reflection:** $g(x) = -b^x$ (graph reflects over the $x$-axis)
$g(x) = b^{-x}$ (graph reflects over the $y$-axis)

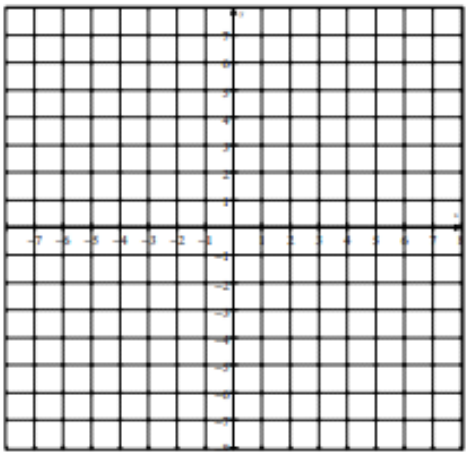
**Vertical:** $g(x) = b^x + c$ (graph moves up $c$ units)
$g(x) = b^x - c$ (graph moves down $c$ units)

**Example 3:** Use $f(x) = 2^x$ to obtain the graph $g(x) = -2^{x+3} - 1$.

Domain of $g$: _____

Range of $g$: _____

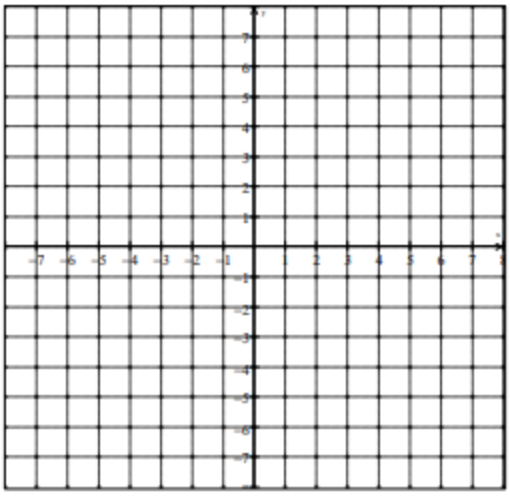
Equation of any asymptote(s) of $g$: _____



$f(x) = e^x$ is called the **natural exponential function,**
where the irrational number $e$ (approximately 2.718282) is called the **natural base**.

(The number $e$ is defined as the value that $\left(1+\frac{1}{n}\right)^n$ approaches as $n$ gets larger and larger.)

**Example 4:** Graph $f(x) = e^x$, $g(x) = e^{x-3}$, and $h(x) = -e^x$
on the same set of axes.



| Periodic Interest Formula | Continuous Interest Formula |
|---|---|
| $A = P\left(1+\frac{r}{n}\right)^{nt}$ | $A = Pe^{rt}$ |

$A$ = balance in the account (Amount after $t$ years)

$P$ = principal (beginning amount in the account)

$r$ = annual interest rate (as a decimal)

$n$ = number of times interest is compounded per year

$t$ = time (in years)

**Example 5:** Find the accumulated value of a $5000 investment which is invested for 8 years at an interest rate of 12% compounded:

(a) annually

(b) semi-annually

(c) quarterly

(d) monthly

(e) continuously

## 4.1 Homework Problems

1. Use a calculator to find each value to four decimal places.

   (a) $5^{\sqrt{3}}$ (b) $7^{\pi}$ (c) $2^{-5.3}$ (d) $e^{2}$ (e) $e^{-2}$ (f) $-e^{0.25}$ (g) $\pi^{-1}$

2. Simplify each expression without using a calculator. (Recall: $b^n \cdot b^m = b^{n+m}$ and $(b^m)^n = b^{mn}$ )

   (a) $6^{\sqrt{2}}6^{\sqrt{2}}$ (b) $\left(3^{\sqrt{2}}\right)^{\sqrt{2}}$ (c) $\left(b^{\sqrt{2}}\right)^{\sqrt{8}}$ (d) $\left(5^{\sqrt{3}}\right)^{\sqrt{3}}$ (e) $4^{\frac{1}{2}}4^{\frac{1}{2}}$ (f) $b^{\sqrt{12}}b^{\sqrt{3}}$

For Problems 3 – 14, graph each exponential function. State the domain and range for each along with the equation of any asymptotes. Check your graph using a graphing calculator.

3. $f(x) = 3^x$ 4. $f(x) = -(3^x)$ 5. $f(x) = 3^{-x}$ 6. $f(x) = \left(\frac{1}{3}\right)^x$

7. $f(x) = 2^x - 3$ 8. $f(x) = 2^{x-3}$ 9. $f(x) = 2^{x+5} - 5$ 10. $f(x) = -2^{-x}$

11. $f(x) = -2^{x+3} + 1$ 12. $f(x) = \left(\frac{1}{2}\right)^{x-3} - 4$ 13. $f(x) = e^{-x} + 2$ 14. $f(x) = -e^{x+2}$

15. $10,000 is invested for 5 years at an interest rate of 5.5%. Find the accumulated value if the money is (a) compounded semiannually; (b) compounded quarterly; (c) compounded monthly; (d) compounded continuously.

16. Sam won $150,000 in the Michigan lottery and decides to invest the money for retirement in 20 years. Find the accumulated value for Sam's retirement for each of his options:
    (a) a certificate of deposit paying 5.4% compounded yearly
    (b) a money market certificate paying 5.35% compounded semiannually
    (c) a bank account paying 5.25% compounded quarterly
    (d) a bond issue paying 5.2% compounded daily
    (e) a saving account paying 5.19% compounded continuously