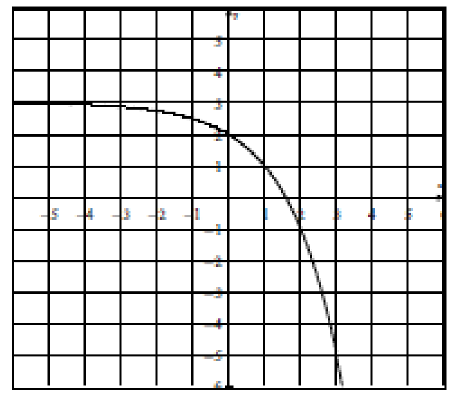
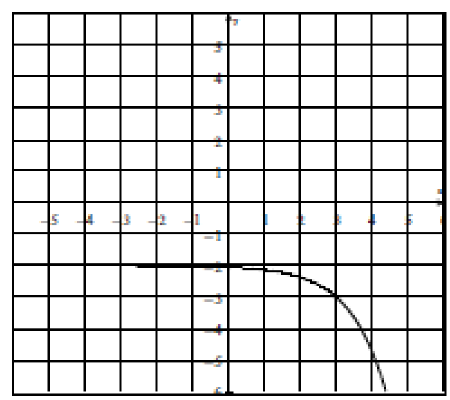
4.2 Applications of Exponential Functions

In this section you will learn to:
- find exponential equations using graphs
- solve exponential growth and decay problems
- use logistic growth models

Example 1: The graph of g is the transformation of $f(x) = 2^x$.
Find the equation of the graph of g.
HINTS:
1. There are no stretches or shrinks.
2. Look at the general graph and asymptote to determine any reflections and/or vertical shifts.
3. Follow the point (0, 1) on f through the transformations to help determine any vertical and/or horizontal shifts.



Example 2: The graph of g is the transformation of $f(x) = e^x$.
Find the equation of the graph of g.



Example 3: In 1969, the world population was approximately 3.6 billion, with a growth rate of 1.7% per year. The function $f(x) = 3.6e^{0.017x}$ describes the world population, $f(x)$, in billions, ***x* years after 1969**. Use this function to estimate the world population in

1969 ____________________ 2000 ____________________ 2012 ____________________

Example 4: The exponential function $f(x) = 84.5(1.012)^x$ models the population of Mexico, $f(x)$, in millions, x years after 1986.

(a) Without using a calculator, substitute 0 for x and find Mexico's population in 1986.

(b) Estimate Mexico's population, to the nearest million in the year 2000.

(c) Estimate Mexico's population, to the nearest million, this year.

Example 5: College students study a large volume of information. Unfortunately, people do not retain information for very long. The function $f(x) = 80e^{-0.5x} + 20$ describes the percentage of information, $f(x)$, that a particular person remembers x weeks after learning the information (without repetition).

(a) Substitute 0 for x and find the percentage of information remembered at the moment it is first learned.

(b) What percentage of information is retained after 1 week? ______ 4 weeks? _______ 1 year? _______

Radioactive Decay Formula:

The amount A of radioactive material present at time t is given by $A = A_0(2)^{-\frac{t}{h}}$ where A_0 is the amount that was present initially (at $t = 0$) and h is the material's half-life.

Example 6: The half-life of radioactive carbon-14 is 5700 years. How much of an initial sample will remain after 3000 years?

Example 7: The half-life of Arsenic-74 is 17.5 days. If 4 grams of Arsenic-74 are present in a body initially, how many grams are presents 90 days later?

Logistic growth models are used in the study of conservation biology, learning curves, spread of an epidemic or disease, carrying capacity, etc. The mathematical model for limited logistic growth is given by: $f(t)=\dfrac{c}{1+ae^{-bt}}$ *or* $A=\dfrac{c}{1+ae^{-bt}}$, where *a, b,* and *c* are constants, $c > 0$ and $b > 0$.

As time increases ($t \to \infty$), the expression $ae^{-bt} \to$ _______ and $A \to$ _______ .

Therefore **$y = c$ is a horizontal asymptote** for the graph of the function. Thus *c* represents the limiting size.

Example 8: The function $f(t)=\dfrac{200{,}000}{1+1999e^{-0.06t}}$ describes the number of people, $f(t)$, who have become ill with influenza *t* weeks after its initial outbreak in a town with 200,000 inhabitants.

(a) How many people became ill with the flu when the epidemic began? __________

(b) How many people were ill by the end of the 4th week? __________

(c) What is the limiting size of $f(t)$, the population that becomes ill? __________

(d) What is the horizontal asymptote for this function? __________

Example 9: The function $f(t)=\dfrac{0.8}{1+e^{-0.2t}}$ is a model for describing the proportion of correct responses, $f(t)$, after *t* learning trials.

(a) Find the proportion of correct responses prior to learning trials taking place. __________

(b) Find the proportion of correct responses after 10 learning trials. __________

(c) What is the limiting size of $f(t)$ as continued trials take place? __________

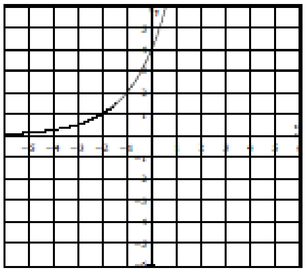
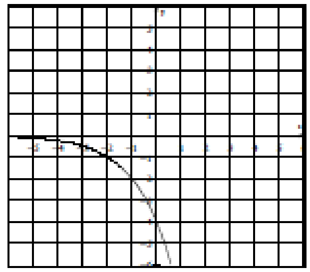
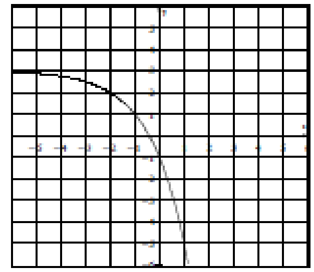
(d) What is the horizontal asymptote for this function? __________

(e) Sketch a graph of this function.

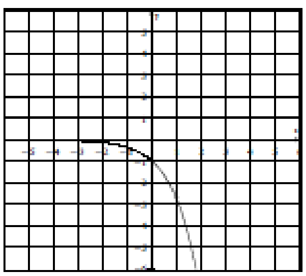
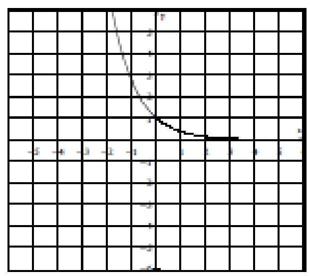
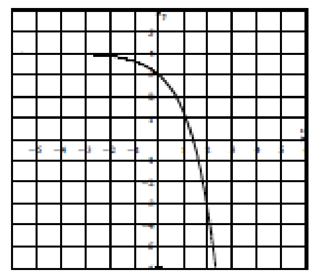
MPS21-07 **Mr. Pineda**
Lesson #19

Problems:

1. Find the equation of each exponential function, $g(x)$, whose graph is shown. Each graph involves one or more transformation of the graph of $f(x) = 2^x$.

(a)  (b)  (c) 

2. Find the equation of each exponential function, $g(x)$, whose graph is shown. Each graph involves one or more transformation of the graph of $f(x) = e^x$.

(a)  (b)  (c) 

3. In 1970, the U. S. population was approximately 203.3 million, with a growth rate of 1.1% per year. The function $f(x) = 203.3e^{0.011x}$ describes the U. S. population, $f(x)$, in millions, x years after 1970. Use this function to estimate the U. S. population in the year 2012.

4. A common bacterium with an initial population of 1000 triples every day. This is modeled by the formula $P(t) = 1000(3)^t$, where $P(t)$ is the total population after t days. Find the total population after

   (a) 12 hours (b) 1 day (c) 1½ days (d) 2 days

5. Assuming the rate of inflation is 5% per year, the predicted price of an item can be modeled by the function $P(t) = P_0(1.05)^t$, where P_0 represents the initial price of the item and t is in years.

   (a) Based on this information, what will the price of a new car be in the year 2012, if it cost $20,000 in the year 2000?

   (b) Estimate the price of a gallon of milk be in the year 2012, if it cost $2.95 in the year 2000? Round your estimate to the nearest cent.

6. The 1986 explosion at the Chernobyl nuclear power plant in the former Soviet Union sent about 1000 kilograms of radioactive cesium-137 into the atmosphere. The function $f(x) = 1000(0.5)^{\frac{x}{30}}$ describes the amount, $f(x)$, in kilograms, of cesium-137 remaining in Chernobyl x years after 1986. If even 100 kilograms of cesium-137 remain in Chernobyl's atmosphere, the area is considered unsafe for human habitation. Find $f(60)$ and determine if Chernobyl will be safe for human habitation by 2046.

7. The logistic growth function $f(t) = \dfrac{100{,}000}{1 + 5000e^{-t}}$ describes the number of people, $f(t)$, who have become ill with influenza t weeks after its initial outbreak in a particular community.

   (a) How many people became ill with the flu when the epidemic began?

   (b) How many people were ill by the end of the fifth week?

   (c) What is the limiting size of the population that becomes ill?

8. The logistic growth function $P(x) = \dfrac{90}{1 + 271e^{-0.122x}}$ models the percentage, $P(x)$ of Americans who are x years old with some coronary heart disease.

   (a) What percentage of 20-year-olds have some coronary heart disease?

   (b) What percentage of 80-year-olds have some coronary heart disease?