For a function $f(x)$ and a real number a, $\lim_{x \to a} f(x)$ exists if and only if

1. $\lim_{x \to a^-} f(x)$ exists. In other words, there must be a limit from the left.
2. $\lim_{x \to a^+} f(x)$ exists. There must be a limit from the right.
3. $\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x)$ The limit from the left must equal the limit from the right.

(Note that this definition does not apply to limits as x approaches infinity or negative infinity.)

And here's the definition of continuity: A function $f(x)$ is continuous at a point $x = a$ if three conditions are satisfied:

1. $f(a)$ is defined.
2. $\lim_{x \to a} f(x)$ exists.
3. $f(a) = \lim_{x \to a} f(x)$.

Using these definitions and Figure 3-1, answer problems 1 through 4.

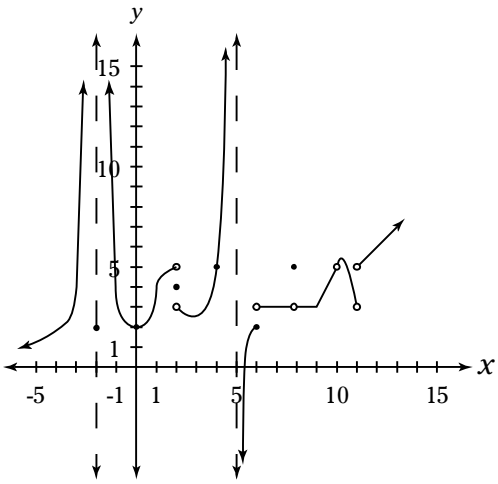


Figure 3-1: Graphus interruptus: A function with many discontinuities.

1. At which of the following x-values are all three requirements for the existence of a limit satisfied, and what is the limit at those x-values? $x = -2, 0, 2, 4, 5, 6, 8, 10,$ and 11.

Solve It

2. For the x-values at which all three limit requirements are not met, state which of the three requirements are not satisfied. If one or both one-sided limits exist at any of these x-values, give the value of the one-sided limit.

Solve It

3. At which of the x-values are all three requirements for continuity satisfied?

Solve It

4. For the rest of the x-values, state which of the three continuity requirements are not satisfied.

Solve It

Taking a Closer Look: Limit and Continuity Graphs

In this section, you get more practice at solving limit and continuity problems visually. Then in Chapter 4, you solve limit problems numerically (with your calculator) and symbolically (with algebra).

Use Figure 3-2 to answer problems 5 through 10.

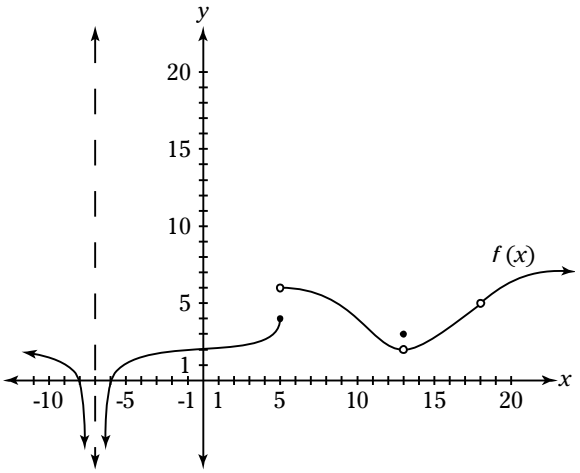


Figure 3-2: Another bizarre graph.

Q. $\lim_{x \to 0} f(x) = ?$

A. $\lim_{x \to 0} f(x) = 2$ Because $f(0) = 2$ and because f is continuous there, the limit must equal the function value. Whenever a function passes through a point and there's no discontinuity at the point, the limit equals the function value.

Q. $\lim_{x \to 13} f(x) = ?$

A. $\lim_{x \to 13} f(x) = 2$ because there's a hole at $(13, 2)$. The limit at a hole is the height of the hole.

5. $\lim_{x \to -7} f(x) = ?$

Solve It

6. $\lim_{x \to 5} f(x) = ?$

Solve It

7. $\lim_{x \to 5^-} f(x) = ?$

Solve It

8. $\lim_{x \to 18} f(x) = ?$

Solve It

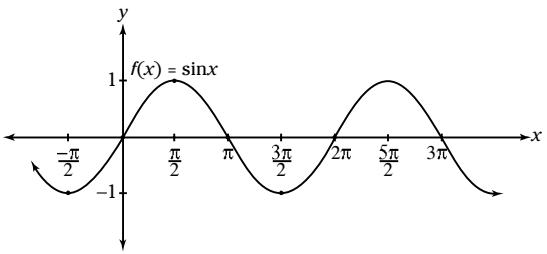
9. $\lim_{x \to 5^+} f(x) = ?$

Solve It

10. List the x-coordinates of all points of discontinuity of f and state the type of discontinuity — removable, jump, or infinite.

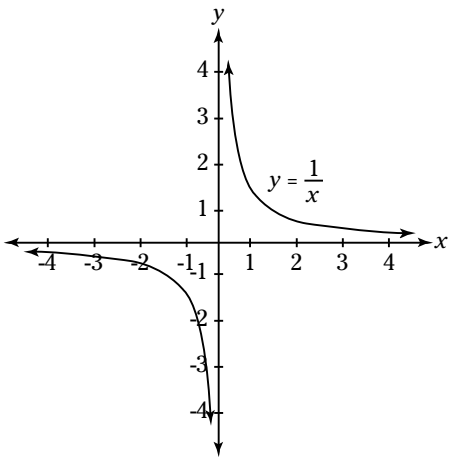
Solve It

11. $\lim_{x \to \infty} \sin x = ?$ See the following graph.



Solve It

12. $\lim_{x \to \infty} \frac{1}{x} = ?$ See the following graph of $y = \frac{1}{x}$.



Solve It