

Aim: How do we use the properties of exponents to rewrite expressions with integral exponents?

Do Now: **Simplify. To start, factor each radicand.**

$$1. \sqrt{18} + \sqrt{32}$$

$$= \sqrt{9 \cdot 2} + \sqrt{16 \cdot 2}$$

$$2. \sqrt[4]{324} - \sqrt[4]{2500}$$

$$3. \sqrt[3]{192} + \sqrt[3]{24}$$

**Multiply.**

$$4. (3 - \sqrt{6})(2 - \sqrt{6})$$

$$5. (5 + \sqrt{5})(1 - \sqrt{5})$$

$$6. (4 + 7)^2$$

## I – Rational Exponent

1) Solve for x:

$$a^x = \sqrt[4]{a^3}$$

2) In General,

$$\sqrt[n]{x} = x^{\frac{1}{n}}$$

3) **Simplify each expression.**

$$1. 16^{\frac{1}{4}}$$

$$\sqrt[4]{16}$$

$$2. (-3)^{\frac{1}{3}} \cdot (-3)^{\frac{1}{3}} \cdot (-3)^{\frac{1}{3}}$$

$$3. 5^{\frac{1}{2}} \cdot 45^{\frac{1}{2}}$$

## II- Converting between exponential and radical form

1)

All the properties of integer exponents apply to rational exponents.

Take note

### Properties Properties of Rational Exponents

Let  $m$  and  $n$  represent rational numbers. Assume that no denominator equals 0.

**Property**

$$a^m \cdot a^n = a^{m+n}$$

**Example**

$$8^{\frac{1}{3}} \cdot 8^{\frac{2}{3}} = 8^{\frac{1}{3} + \frac{2}{3}} = 8^1 = 8$$

**Property**

$$a^{-m} = \frac{1}{a^m}$$

**Example**

$$9^{-\frac{1}{2}} = \frac{1}{9^{\frac{1}{2}}} = \frac{1}{3}$$

$$(a^m)^n = a^{mn}$$

$$(5^{\frac{1}{2}})^4 = 5^{\frac{1}{2} \cdot 4} = 5^2 = 25$$

$$\frac{a^m}{a^n} = a^{m-n}$$

$$\frac{7^{\frac{3}{2}}}{7^{\frac{1}{2}}} = 7^{\frac{3}{2} - \frac{1}{2}} = 7^1 = 7$$

$$(ab)^m = a^m b^m$$

$$(4 \cdot 5)^{\frac{1}{2}} = 4^{\frac{1}{2}} \cdot 5^{\frac{1}{2}} = 2 \cdot 5^{\frac{1}{2}}$$

$$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$$

$$\left(\frac{5}{27}\right)^{\frac{1}{3}} = \frac{5^{\frac{1}{3}}}{27^{\frac{1}{3}}} = \frac{5^{\frac{1}{3}}}{3}$$

2)

a. What are the expressions  $w^{-\frac{5}{8}}$  and  $w^{0.2}$  in radical form?

b. What are the expressions  $\sqrt[4]{x^3}$  and  $(\sqrt[5]{y})^4$  in exponential form?

c. **Reasoning** Refer to the definition of rational exponent. Explain the need for the restriction that  $a \neq 0$  if  $m$  is negative.

## III- Combining radical expressions and simplifying

1) Simplify each number.

a.  $(-216)^{\frac{1}{3}}$

b.  $243^{1.2}$

c.  $32^{-0.4}$

$$\sqrt[3]{-216}$$

2) Find each product or quotient. To start, rewrite the expression using exponents.

a.  $(\sqrt[4]{6})(\sqrt[3]{6})$

b.  $\frac{\sqrt[5]{x^2}}{\sqrt[10]{x^2}}$

c.  $\sqrt{20} \cdot \sqrt[3]{135}$

$$= \left(6^{\frac{1}{4}}\right)\left(6^{\frac{1}{3}}\right)$$

3) Simplify each number.

a.  $(125)^{\frac{2}{3}}$

b.  $(216)^{\frac{2}{3}}(216)^{\frac{2}{3}}$

c.  $(-243)^{\frac{2}{5}}$

4) Write each expression in simplest form. Assume that all variables are positive.

a.  $(16x^{-8})^{\frac{3}{4}}$

b.  $(8x^{15})^{-\frac{1}{3}}$

c.  $\left(\frac{x^2}{x^{-10}}\right)^{\frac{1}{3}}$