Aim: How do we model situations of exponential growth and decay using geometric sequences?

DO NOW:

Opening Exercise

Suppose a ball is dropped from an initial height h_0 and that each time it rebounds, its new height is 60% of its previous height.

a. What are the first four rebound heightsh_1,h_2, h_3, and h_4 after being dropped from a height of $h_0 = 10$ ft.?

b. Suppose the initial height is A ft. What are the first four rebound heights? Fill in the following table:

Rebound	Height (ft.)
1	
2	
3	
4	

c. How is each term in the sequence related to the one that came before it?

d. Suppose the initial height is A ft. and that each rebound, rather than being 60% of the previous height, is r times the previous height, where $0 < r < 1$. What are the first four rebound heights? What is the n^{th} rebound height?

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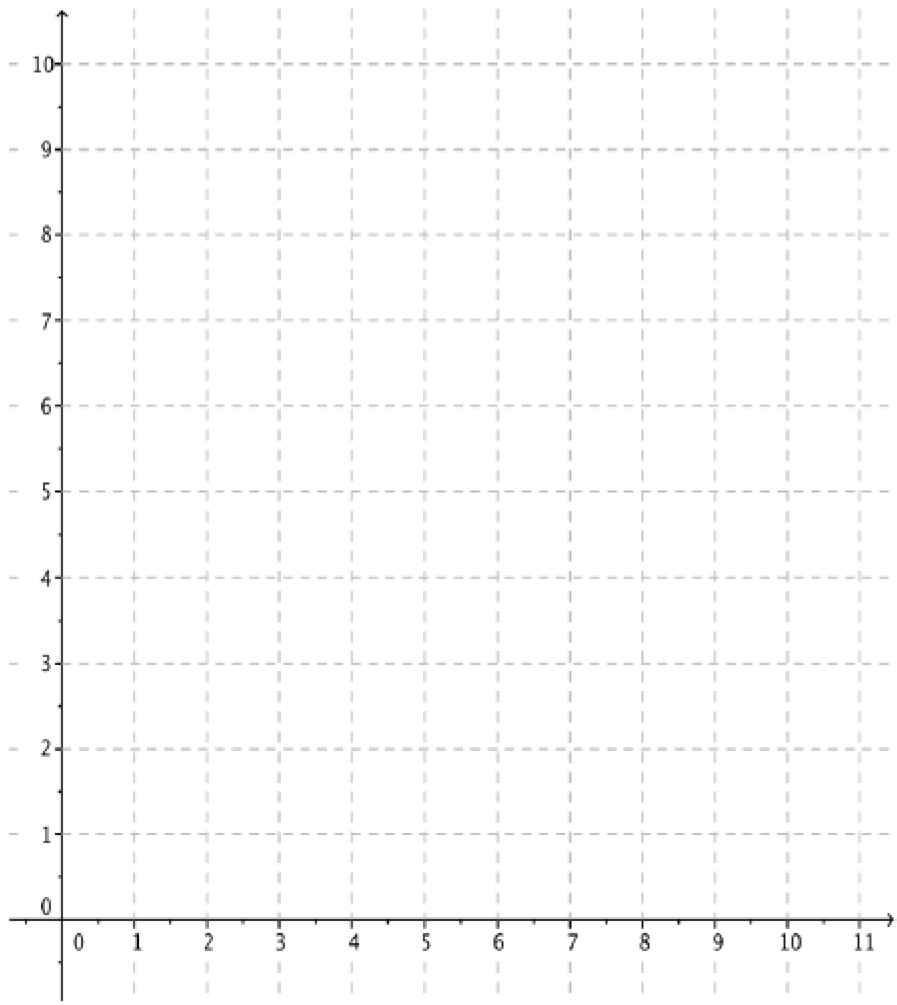
e. What kind of sequence is the sequence of rebound heights?

f. Suppose that we define a function f with domain the positive integers so that $f(1)$ is the first rebound height, $f(2)$ is the second rebound height, and continuing so that $f(k)$ is the k^{th} rebound height for positive integers k. What type of function would you expect f to be?

g. On the coordinate plane below, sketch the height of the bouncing ball when $A = 10$ and $r = 0.60$, assuming that the highest points occur at $x = 1, 2, 3, 4, \ldots$.



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h. Does the exponential function $f(x) = 10(0.60)^x$ for real numbers x model the height of the bouncing ball? Explain how you know.

i. What does the function $f(n) = 10(0.60)^n$ for integers $n \geq 0$ model?

Exercises

1. Jane works for a videogame development company that pays her a starting salary of $\$100$ per day, and each day she works she earns $\$100$ more than the day before.

   a. How much does she earn on day 5?

   b. If you were to graph the growth of her salary for the first 10 days she worked, what would the graph look like?

   c. What kind of sequence is the sequence of Jane's earnings each day?

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2. A laboratory culture begins with $1{,}000$ bacteria at the beginning of the experiment, which we denote by time 0 hours. By time 2 hours, there are $2{,}890$ bacteria.

   a. If the number of bacteria is increasing by a common factor each hour, how many bacteria are there at time 1 hour? At time 3 hours?

   b. Find the explicit formula for term P_n of the sequence in this case.

   c. How would you find term P_{n+1} if you know term P_n? Write a recursive formula for P_{n+1} in terms of P_n.

   d. If P_0 is the initial population, the growth of the population P_n at time n hours can be modeled by the sequence $P_n = P(n)$, where P is an exponential function with the following form:

   $$P(n) = P_0 2^{kn}, \text{ where } k > 0.$$

   Find the value of k and write the function P in this form. Approximate k to four decimal places.

   e. Use the function in part (d) to determine the value of t when the population of bacteria has doubled.

   f. If P_0 is the initial population, the growth of the population P at time t can be expressed in the following form:

   $$P(n) = P_0 e^{kn}, \text{ where } k > 0.$$

   Find the value of k, and write the function P in this form. Approximate k to four decimal places.

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Lesson Summary

ARITHMETIC SEQUENCE: A sequence is called *arithmetic* if there is a real number d such that each term in the sequence is the sum of the previous term and d.

- *Explicit formula:* Term a_n of an arithmetic sequence with first term a_0 and common difference d is given by $a_n = a_0 + nd$, for $n \geq 0$.
- *Recursive formula:* Term a_{n+1} of an arithmetic sequence with first term a_0 and common difference d is given by $a_{n+1} = a_n + d$, for $n \geq 0$.

GEOMETRIC SEQUENCE: A sequence is called *geometric* if there is a real number r such that each term in the sequence is a product of the previous term and r.

- *Explicit formula:* Term a_n of a geometric sequence with first term a_0 and common ratio r is given by $a_n = a_0 r^n$, for $n \geq 0$.
- *Recursive formula:* Term a_{n+1} of a geometric sequence with first term a_0 and common ratio r is given by $a_{n+1} = a_n r$.

g. Use the formula in part (d) to determine the value of t when the population of bacteria has doubled.

Homework # 11 (new) -Complete exercises: 1a,2a,3,7,10,13

Problem Set

1. Convert the following recursive formulas for sequences to explicit formulas.
   a. $a_{n+1} = 4.2 + a_n$ with $a_0 = 12$
   b. $a_{n+1} = 4.2a_n$ with $a_0 = 12$
   c. $a_{n+1} = \sqrt{5}\, a_n$ with $a_0 = 2$
   d. $a_{n+1} = \sqrt{5} + a_n$ with $a_0 = 2$
   e. $a_{n+1} = \pi\, a_n$ with $a_0 = \pi$

2. Convert the following explicit formulas for sequences to recursive formulas.
   a. $a_n = \frac{1}{5}(3^n)$ for $n \geq 0$
   b. $a_n = 16 - 2n$ for $n \geq 0$
   c. $a_n = 16\left(\frac{1}{2}\right)^n$ for $n \geq 0$
   d. $a_n = 71 - \frac{6}{7}n$ for $n \geq 0$
   e. $a_n = 190(1.03)^n$ for $n \geq 0$

3. If a geometric sequence has $a_1 = 256$ and $a_8 = 512$, find the exact value of the common ratio r.

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4. If a geometric sequence has $a_2 = 495$ and $a_6 = 311$, approximate the value of the common ratio r to four decimal places.

5. Find the difference between the terms a_{10} of an arithmetic sequence and a geometric sequence, both of which begin at term a_0 and have $a_2 = 4$ and $a_4 = 12$.

6. Given the geometric sequence defined by the following values of a_0 and r, find the value of n so that a_n has the specified value.
   a. $a_0 = 64, r = \frac{1}{2}, a_n = 2$
   b. $a_0 = 13, r = 3, a_n = 85293$
   c. $a_0 = 6.7, r = 1.9, a_n = 7804.8$
   d. $a_0 = 10958, r = 0.7, a_n = 25.5$

7. Jenny planted a sunflower seedling that started out 5 cm tall, and she finds that the average daily growth is 3.5 cm.
   a. Find a recursive formula for the height of the sunflower plant on day n.
   b. Find an explicit formula for the height of the sunflower plant on day $n \geq 0$.

8. Kevin modeled the height of his son (in inches) at age n years for $n = 2, 3, \dots, 8$ by the sequence $h_n = 34 + 3.2(n - 2)$. Interpret the meaning of the constants 34 and 3.2 in his model.

9. Astrid sells art prints through an online retailer. She charges a flat rate per order for an order processing fee, sales tax, and the same price for each print. The formula for the cost of buying n prints is given by $P_n = 4.5 + 12.6n$.
   a. Interpret the number 4.5 in the context of this problem.
   b. Interpret the number 12.6 in the context of this problem.
   c. Find a recursive formula for the cost of buying n prints.

10. A bouncy ball rebounds to 90% of the height of the preceding bounce. Craig drops a bouncy ball from a height of 20 feet.
   a. Write out the sequence of the heights h_1, h_2, h_3, and h_4 of the first four bounces, counting the initial height as $h_0 = 20$.
   b. Write a recursive formula for the rebound height of a bouncy ball dropped from an initial height of 20 feet.
   c. Write an explicit formula for the rebound height of a bouncy ball dropped from an initial height of 20 feet.
   d. How many bounces will it take until the rebound height is under 6 feet?
   e. Extension: Find a formula for the minimum number of bounces needed for the rebound height to be under y feet, for a real number $0 < y < 20$.