

Aim: How can we determine if two events are independent or not independent?**DO NOW****Exercise 1**

In previous lessons, conditional probabilities were used to investigate whether or not there is a connection between two events. This lesson formalizes this idea and introduces the concept of *independence*.

1. Several questions are posed below. Each question is about a possible connection between two events. For each question, identify the two events, and indicate whether or not you think that there would be a connection. Explain your reasoning.
 - a. Are high school students whose parents or guardians set a midnight curfew less likely to have a traffic violation than students whose parents or guardians have not set such a curfew?

 - b. Are left-handed people more likely than right-handed people to be interested in the arts?

 - c. Are students who regularly listen to classical music more likely to be interested in mathematics than students who do not regularly listen to classical music?

 - d. Are people who play video games more than 10 hours per week more likely to select football as their favorite sport than people who do not play video games more than 10 hours per week?

Two events are independent when knowing that one event has occurred does not change the likelihood that the second event has occurred. How can conditional probabilities be used to tell if two events are independent or not independent?

Aim: How can we determine if two events are independent or not independent?**Exercises 2–6**

1000 two-way frequency table that was used to classify students at Rufus King High School according to gender and whether or not they participated in an after-school athletics program.

Table 1: Participation of Male and Female Students in an After-School Athletics Program

	Participate in an After-School Athletics Program	Do Not Participate in an After-School Athletics Program	Total
Female	232	348	580
Male	168	252	420
Total	400	600	1,000

2. For each of the following, indicate whether the probability described is one that can be calculated using the values in Table 1. Also indicate whether or not it is a conditional probability.
 - a. The probability that a randomly selected student participates in an after-school athletics program

 - b. The probability that a randomly selected student who is female participates in an after-school athletics program

 - c. The probability that a randomly selected student who is male participates in an after-school athletics program

3. Use Table 1 to calculate each of the probabilities described in Exercise 2.
 - a. The probability that a randomly selected student participates in an after-school athletics program

 - b. The probability that a randomly selected student who is female participates in an after-school athletics program

Aim: How can we determine if two events are independent or not independent?

- c. The probability that a randomly selected student who is male participates in an after-school athletics program
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4. Would your prediction of whether or not a student participates in an after-school athletics program change if you knew the gender of the student? Explain your answer.

Two events are *independent* if knowing that one event has occurred does not change the probability that the other event has occurred. For example, consider the following two events:

F : The event that a randomly selected student is female

S : The event that a randomly selected student participates in an after-school athletics program

Events F and S would be independent if the probability that a randomly selected student participates in an after-school athletics program is equal to the probability that a randomly selected student who is female participates in an after-school athletics program. If this was the case, knowing that a randomly selected student is female does not change the probability that the selected student participates in an after-school athletics program. Then, F and S would be independent.

5. Based on the definition of independence, are the events “randomly selected student is female” and “randomly selected student participates in an after-school athletics program” independent? Explain.
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6. A randomly selected student participates in an after-school athletics program.
 - a. What is the probability this student is female?

Aim: How can we determine if two events are independent or not independent?

- b. Using only your answer from part (a), what is the probability that this student is male? Explain how you arrived at your answer.

Exercises 7–11

Consider the data below.

	No Household Member Smokes	At Least One Household Member Smokes	Total
Student Has Asthma	69	113	182
Student Does Not Have Asthma	473	282	755
Total	542	395	937

7. You are asked to determine if the two events “a randomly selected student has asthma” and “a randomly selected student has a household member who smokes” are independent. What probabilities could you calculate to answer this question?
8. Calculate the probabilities you described in Exercise 7.
9. Based on the probabilities you calculated in Exercise 8, are these two events independent or not independent? Explain.
10. Is the probability that a randomly selected student who has asthma and who has a household member who smokes the same as or different from the probability that a randomly selected student who does not have asthma but does

Aim: How can we determine if two events are independent or not independent?

have a household member who smokes? Explain your answer.

11. A student is selected at random. The selected student indicates that he has a household member who smokes. What is the probability that the selected student has asthma?

Aim: How can we determine if two events are independent or not independent?**Lesson Summary**

Data organized in a two-way frequency table can be used to calculate conditional probabilities.

Two events are independent if knowing that one event has occurred does not change the probability that the second event has occurred.

Probabilities calculated from two-way frequency tables can be used to determine if two events are independent or not independent.

Homework – Complete 1, 3, 4

1. Consider the following questions:

a. A survey of the students at a Midwest high school asked the following questions:

“Do you use a computer at least 3 times a week to complete your schoolwork?”

“Are you taking a mathematics class?”

Do you think the events “a randomly selected student uses a computer at least 3 times a week” and “a randomly selected student is taking a mathematics class” are independent or not independent? Explain your reasoning.

b. The same survey also asked students the following:

“Do you participate in any extracurricular activities at your school?”

“Do you know what you want to do after high school?”

Do you think the events “a randomly selected student participates in extracurricular activities” and “a randomly selected student knows what she wants to do after completing high school” are independent or not independent? Explain your reasoning.

c. People attending a professional football game in 2013 completed a survey that included the following questions:

“Is this the first time you have attended a professional football game?”

“Do you think football is too violent?”

Do you think the events “a randomly selected person who completed the survey is attending a professional football game for the first time” and “a randomly selected person who completed the survey thinks football is too violent” are independent or not independent? Explain your reasoning.

2. Complete the table below in a way that would indicate the two events “uses a computer” and “is taking a mathematics class” are independent.

	Uses a Computer at Least 3 Times a Week for Schoolwork	Does Not Use a Computer at Least 3 Times a Week for Schoolwork	Total
In a Mathematics Class			700
Not in a Mathematics Class			
Total	600		1,000

Aim: How can we determine if two events are independent or not independent?

3. Complete the following hypothetical 1000 table. Are the events “participates in extracurricular activities” and “know what I want to do after high school” independent or not independent? Justify your answer.

	Participates in Extracurricular Activities	Does Not Participate in Extracurricular Activities	Total
Know What I Want to Do After High School			800
Do Not Know What I Want to Do After High School	50		
Total	600		1,000

4. The following hypothetical 1000 table is from Lesson 2:

	No Household Member Smokes	At Least One Household Member Smokes	Total
Student Has Asthma	73	120	193
Student Does Not Have Asthma	506	301	807
Total	579	421	1,000

The actual data from the entire population are given in the table below.

	No Household Member Smokes	At Least One Household Member Smokes	Total
Student Has Asthma	69	113	182
Student Does Not Have Asthma	473	282	755
Total	542	395	937

- Based on the hypothetical 1000 table, what is the probability that a randomly selected student who has asthma has at least one household member who smokes?
- Based on the actual data, what is the probability that a randomly selected student who has asthma has at least one household member who smokes (round your answer to 3 decimal places)?
- Based on the hypothetical 1000 table, what is the probability that a randomly selected student who has no household member who smokes has asthma?
- Based on the actual data, what is the probability that a randomly selected student who has no household member who smokes has asthma?
- What do you notice about the probabilities calculated from the actual data and the probabilities calculated from the hypothetical 1000 table?

Aim: How can we determine if two events are independent or not independent?

5. As part of the asthma research, the investigators wondered if students who have asthma are less likely to have a pet at home than students who do not have asthma. They asked the following two questions:

“Do you have asthma?”

“Do you have a pet at home?”

Based on the responses to these questions, you would like to set up a two-way table that you could use to determine if the following two events are independent or not independent:

Event 1: A randomly selected student has asthma.

Event 2: A randomly selected student has a pet at home.

- How would you label the rows of the two-way table?
- How would you label the columns of the two-way table?
- What probabilities would you calculate to determine if Event 1 and Event 2 are independent?

MRS22-HW-13

Answers will vary. Anticipate that students indicate that people who attend football more often are more likely to not think the game is too violent. It is likely the events are not independent. Again, examine the explanations students provide if they indicate the events are not independent.

1)

	Participates in Extracurricular Activities	Does Not Participate in Extracurricular Activities	Total
Know What I Want to Do After High School	550	250	800
Do Not Know What I Want to Do After High School	50	150	200
Total	600	400	1,000

The events “student participates in extracurricular activities” and “student knows what I want to do after high school” are not independent. Students could indicate that the events are not independent in several ways. For example, $\frac{50}{200}$ (the probability that a randomly selected student who does not know what he wants to do after high school participates in extracurricular activities) does not equal $\frac{550}{800}$ (the probability that a randomly selected student who does know what he wants to do after high school participates in extracurricular activities).

2)

3) $\frac{113}{182} \approx 0.621$

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The conditional probabilities differ only due to rounding in constructing the hypothetical 1000 table from probability information based on the actual data. When an actual data table is available, it can be used to calculate probabilities. When only probability information is available, constructing a hypothetical 1000 table from that information and using it to compute other probabilities will give the same answers as if the actual data were available.

4)

5)
$$\frac{69}{542} \approx 0.127$$

Answers will vary. Anticipate students indicate that students involved in extracurricular activities are often students who want to attend college. It is likely the events are not independent.

6)

7)
$$\frac{73}{579} \approx 0.126$$

8)
$$\frac{120}{193} \approx 0.622$$

Anticipate students indicate that using a computer at least 3 times per week and taking a mathematics class are not independent. However, it is also acceptable for students to make a case for independence. Examine the explanations students provide to see if they understand the meaning of independence.

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