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INTRODUCTION

Gold and silver have long been regarded as valuable commodities and have attached significant attention from investors and analysts worldwide. Understanding the dynamics of the relation between gold and silver prices are crucial for traders to seek profit from the price movement and for the policy makers who try to understand and manage market risk. In this context, Study conducted by Batten et al. (2013) investigate the bivariate relationship between gold and silver prices, with the focus on the spread between the two metal's futures contract.

The aim of this study is to look at the spread's dynamics, which is calculated as the price difference between gold and silver when they are traded as futures contracts. Statistical methods based on rescaled range analysis were also used to account for short-term autocorrelated innovations in the return process before fractal structure was quantified. (Batten et al. 2013)

To filter the series and overcome the short-term autocorrelated innovations, the authors of this study utilized Autoregressive Moving Average (ARMA) modelling. ARMA is a statistical method used to analyse and forecast time-series data. The model aims to capture the correlation between past and current values in the series, and its main components are the autoregressive (AR) and moving average (MA) terms. The AR term represents the dependence on past values in the series, while the MA term represents the dependence on past errors or residuals. The ARMA model has been widely used in finance, economics, and engineering to model and forecast time-series data.

The ARMA model is used to filter time series data in the study of dependence structures between financial variables, such as the S&P500, VIX, and Interexptile Differences. It helps to estimate and remove linear dependencies, allowing for better understanding of underlying structures. ARMA-GARCH models are effective in analysing dynamic dependence between financial markets and indices, providing valuable insights into interdependencies and flow of information (Bellini et al., 2018; Hammoudeh et al., 2014). These models are significant in forecasting and analysing time series data, particularly in finance.

The Encyclopedia of Quantitative Finance article by Brockwell (2010) highlights the significance of Autoregressive Moving Average (ARMA) processes in modelling time series data with diverse dependence structures and marginal distributions. The article emphasizes the importance of Continuous-time ARMA (CARMA) processes in continuous time modelling and high-frequency data analysis. It focuses on second-order processes with finite moments, and the article provides a comprehensive overview of their essential properties and structure. The reference for this article is: Brockwell, P.J. (2010). Autoregressive Moving Average (ARMA) Processes.

In conclusion, the ARMA model played a crucial role in this study by filtering the gold-silver spread returns and making the series stationary, enabling the authors to apply statistical techniques based on rescaled range analysis to investigate the presence of a fractal structure in the series. The outcomes of this study add to a growing body of research showing that, despite the development of better pricing and information technologies, there are still substantial arbitrage possibilities in financial markets. The study's findings give attractive trading guidelines based on the Hurst coefficient, which may be used by investors and traders in the gold and silver futures markets. The ARMA model has proven to be a powerful tool for modelling and analysing time-series data in finance and other fields, and its application in this study demonstrates its usefulness in detecting and analysing long-term dependence or fractality in financial data. (Baillie et al, 1989)

DATA DESCRIPTION

The dataset used in this study consist of historical futures data for silver and gold, Collected from Yahoo Finance. The data covers a period ranging from January 2016 to December 2022.

The line chart depicting Gold's historical prices shows a trend from January 2016 to December 2022, with a notable increase in April 2020 and a light fall is seen.



The line chart depicting silver's historical prices shows a trend from January 2016 to December 2022, with a notable increase in April 2020 and a lightly fall is seen.

Overall, it seems that gold and silver are moving together and has a same trend in long term.



Descriptive Statistics

Table 1	Gold Silver Spread $P_t = G_t - S_t$	Gold- Silver Spread Return $\Delta P_t = \log(P_t) - \log(P_{t-1})$
Mean	1499.87144	0.0002208
Median	1441.191	0.00039023
Minimum	1113.868	-0.0518217
Maximum	2040.139	0.05762756
Standard Deviation	258.352711	0.0091634
Skewness	0.26100831	-0.1909848
Kurtosis	-1.5539369	5.13302082
Jarque - Bera	196.8204	1818.819
Probability	0.00	0.0000
Observation	1791	1791

Table 2	Gold Interday Return $\Delta G_t = \log(G_t) - \log(G_{t-1})$	Silver Interday Return $\Delta S_t = \log(S_t) - \log(S_{t-1})$
Mean	0.0002214	0.00026824
Median	0.00039728	0.00038709
Minimum	-0.0511396	-0.1234543
Maximum	0.05775363	0.08896013
Standard Deviation	0.00913038	0.01805684
Skewness	-0.1848234	-0.5848674
Kurtosis	5.11376937	6.94019638
Jarque - Bera	1803.515	3477.869
Probability	0.00	0.0000
Observation	1791	1791

Table 1 shows descriptive statistics for gold-silver spread return. The mean return is 0.0002208 with a standard deviation of 0.0091634. The minimum and maximum returns are -0.0518217 and 0.05762756, respectively. The skewness and kurtosis suggest a slightly skewed distribution with a tail extending to the right. The kurtosis of the return was negative for Gold and positive for Silver, indicating that the distribution was flatter than the normal distribution for Gold and more peaked than the normal distribution for Silver. The Jarque-Bera test shows that both gold and silver returns were not normally distributed. The findings suggest that there is a significant variation in gold and silver returns, indicating that they may be useful for diversification purposes.

Table 2 shows the descriptive statistics for the interday returns of gold and silver. The mean interday return for gold is 0.0002214 with a standard deviation of 0.00913038, while the mean interday return for silver is 0.00026824 with a higher standard deviation of 0.01805684. Both gold and silver have positive mean returns, indicating a potential for profit over time. The skewness values are negative for both gold and silver, indicating that the data is slightly skewed to the left. The kurtosis values for both gold and silver are greater than 3, indicating that the distributions are leptokurtic and have heavy tails. The Jarque-Bera test results for both gold and silver indicate that the distributions are not normally distributed. Overall, the descriptive statistics suggest that both gold and silver exhibit high volatility and non-normality in their interday returns.

Overall, the descriptive statistics presented in Table 1 and Table 2 suggest that gold and silver exhibit significant variation and non-normality in their returns, making them potentially useful for diversification purposes. Both metals have positive mean returns, with silver exhibiting higher volatility than gold. However, the non-normality of the distributions indicates that traditional statistical methods may not be appropriate for analyzing these returns. Further research is needed to develop alternative models and methods for analyzing the behaviour of gold and silver returns.

METHODOLOGY

The first step of the methodology is data collection. This involves collecting time-series data for gold and silver futures contracts from January 2016 to December 2022. The data obtained from Yahoo Finance. The data collected include daily prices for gold and silver futures contracts.

After data collection, the next step is data pre-processing. This step involves checking for outliers and missing values in the data and addressing them appropriately. Pre-processing has been done by the Statistical Software "E-Views". It also involves examining the data for seasonality, trends, and stationarity using statistical tests such as the Augmented Dickey-Fuller (ADF) test. Stationarity achieved through differencing the interday logarithmic returns. If the ADF test p-value is less than the significance level (often 0.05), we may reject the null hypothesis and conclude that the time series is stationary. If the p-value is larger than the significance threshold, we cannot reject the null hypothesis and infer that the time series is non-stationary. (Liu Et.al. 2003)

Model identification is the next step after data pre-processing. This includes figuring out whether Autoregressive (AR) and Moving Average (MA) processes suit the data the best fit the data. Statistical tests like the Akaike Information Criterion (AIC) and the Bayesian Information Criterion (BIC) are frequently used for this. Based on the lowest AIC and BIC values, the AR and MA procedures are ranked in that order. A lower AIC or BIC value, said simply, denotes a better model fit. It is preferable to use a model with a lower AIC or BIC value than one with a larger value. However, it is important to note that both AIC and BIC values should be compared for different models estimated on the same data, as they are relative measures of model fit.

Next step is model estimation is the process of estimating the parameters of the selected ARMA model using the maximum likelihood estimation method. This involves calculating the likelihood of the model given the observed data and finding the values of the parameters that maximize this likelihood. This process is done by using EViews. Then the model must be diagnosed to check goodness of fit for the estimated model. Forecasting involves using the estimated ARMA model to predict future values of the variable of interest. This is typically done by specifying the number of periods to forecast and the starting values for the dependent variable. The forecasted values can then be used to make informed decisions or to form the basis for further analysis.

If the model diagnostics indicate problems with the model, the next step is model refinement. This involves changing the order of the AR and MA processes or including additional explanatory variables to improve the model's performance. The final model is validated using out-of-sample testing, such as comparing the forecasted values with the actual values.

In Conclusion, the methodology involves collecting data for gold and silver prices, checking for issues like missing values and outliers, and testing for seasonality and stationarity. Then, the best ARMA model is identified based on statistical tests, and its parameters are estimated using maximum likelihood estimation. The model is checked for its goodness-of-fit and refined if necessary. Finally, the model is used to forecast future prices and validated using out-of-sample testing. Overall, this methodology provides a systematic approach to analyse and forecast gold and silver prices. (Baillie et al 1989, Batten et al. 2013)

ANALYSIS OF RESULT

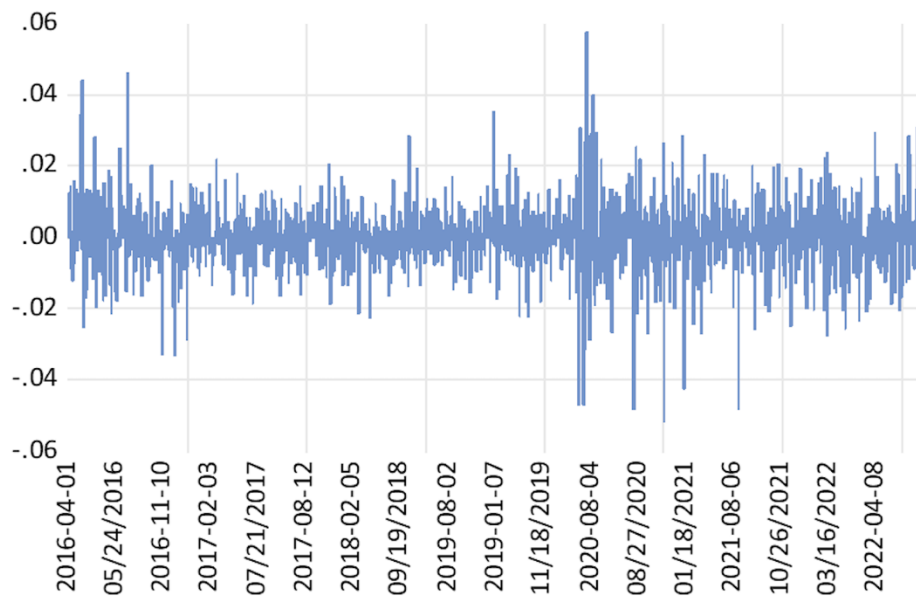
Stationarity Test (ADF)

The Augmented Dickey-Fuller (ADF) test checks if a time series is stationary by testing for the presence of a unit root, which indicates non-stationarity.

ADF	Test Statistics	P-value	1%	5%	10%
SPREAD(Pt)	-42.85627	0.0000	-3.433803	-2.862952	-2.567569
GOLD(Gt), 1 st Diff	-20.62103	0.0000	-3.433832	-2.862965	-2.567575
SILVER(St) 1 st Diff	-22.63158	0.0000	-3.433828	-2.862963	-2.567575

Test results indicated small ADF test statistic and a p-value close to zero indicate strong evidence for stationarity. Here, ADF test results show that the three-time series (SPREAD(Pt), GOLD(Gt), and SILVER(St)) are stationary after taking their first differences, meaning that their statistical properties remain constant over time.

Correlogram



Analyzing the relationship between two time series, such as the prices of gold and silver, a correlogram can be used to explore whether there is any evidence of a correlation between the two series at different lags. Specifically, a correlogram for this ARMA model of the spread between the two series would show the autocorrelation of the spread at different lags.

ARMA Model Estimation

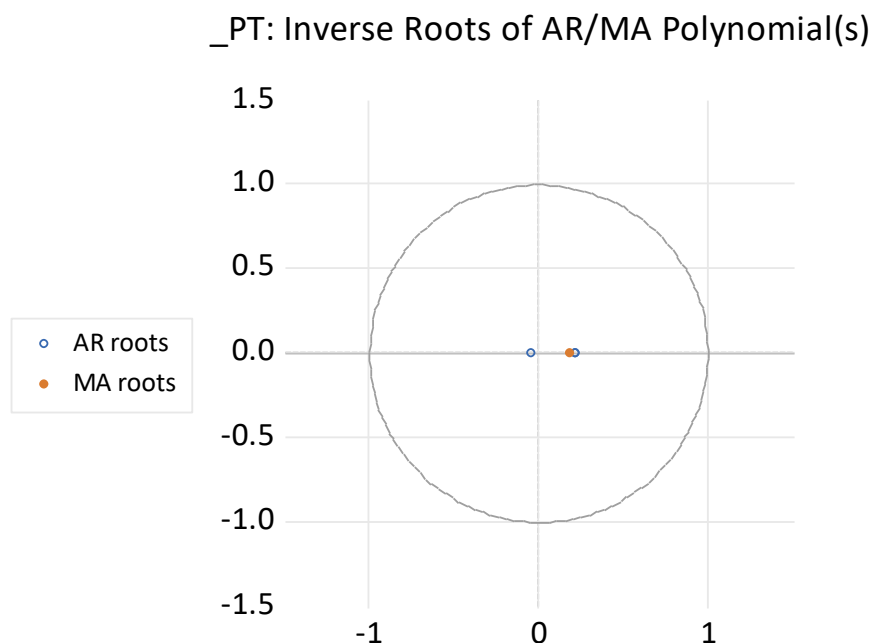
Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.000288	0.000220	1.310943	0.1900
AR (1)	0.176009	2.579971	0.068221	0.9456
AR (2)	0.009476	0.035522	0.266772	0.7897
MA (1)	-0.189608	2.578961	-0.073521	0.9414
SIGMASQ	8.61E-05	1.59E-06	54.17615	0.0000
R-squared	0.000237		Mean dependent var	0.000288
Adjusted R-squared	-0.002004		S.D. dependent var	0.009284
S.E. of regression	0.009293		Akaike info criterion	-6.516226
Sum squared resid	0.154167		Schwarz criterion	-6.500891
Log likelihood	5837.022		Hannan-Quinn criter.	-6.510564
F-statistic	0.105621		Durbin-Watson stat	2.000124
Prob(F-statistic)	0.980579			

The ARMA model output provides a comprehensive view of the estimated coefficients for the model parameters and various tests and statistics to evaluate the goodness of fit of the model. The constant term (C) in the model has a coefficient of 0.000288, which is not statistically significant (p-value > 0.05). This implies that the mean of the dependent variable is not significantly different from zero. The autoregressive (AR) and moving average (MA)

terms represent the relationship between the current value of the dependent variable and its past values and the past values of the error term, respectively. However, the coefficients for the lag 1 and lag 2 terms of the AR term and the lag 1 term of the MA term are not statistically significant, indicating that the model may not be a good fit for the data. AR (0) term was not estimated in the ARMA model because it represents the constant term, which is already included in the model.

Furthermore, the low R-squared value suggests that the model explains very little of the variance in the dependent variable, and the negative adjusted R-squared value indicates that the model is not a good fit for the data. The SIGMASQ parameter, representing the estimated variance of the error term, is significant, and the Durbin-Watson statistic, which tests for autocorrelation in the residuals, suggests that there is no significant autocorrelation. Although the AIC and Schwarz criterion are both negative, indicating a good fit, they cannot be used to compare models with different numbers of parameters. Overall, the ARMA model may not be a good fit for the data as many of the coefficients are not statistically significant, and the model does not explain much of the variance in the dependent variable.

In conclusion, the ARMA model output suggests that further investigation is needed to determine if the model is a good fit for the data. The model's low R-squared value and insignificant coefficients indicate that the model may not accurately explain the relationship between the dependent variable and its past values and the past values of the error term. However, the significant SIGMASQ parameter and lack of significant autocorrelation in the residuals indicate that the model may have some utility. Therefore, refining the ARMA model to achieve a better fit for the data is suggested.



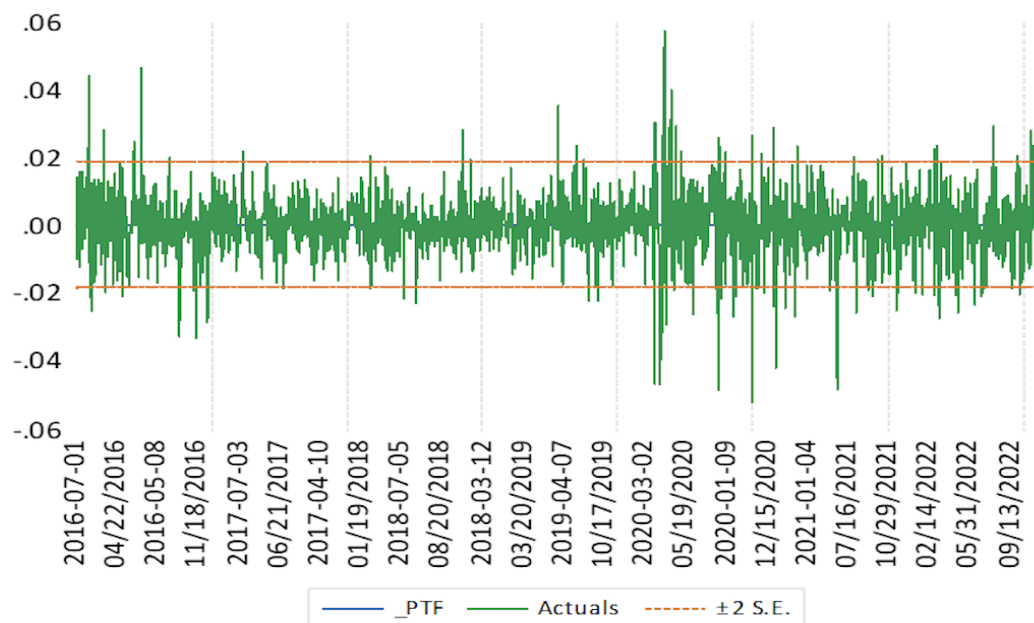
The inverse roots of the AR and MA polynomials provide information about the stability of the ARMA model. If all the roots lie within the unit circle, the model is stable, and the process is stationary. The inverse roots are given in the output as the reciprocal of the roots of the AR and MA polynomials.

In this case, the inverse root of the AR polynomial is 0.22 and -0.04, while the inverse root of the MA polynomial is 0.19. Since all the roots have a magnitude less than one, the model is stable, and the process is stationary. The stability of the ARMA model is important because an unstable model can produce unreliable forecasts and can cause problems in econometric analysis. Therefore, it is essential to check the inverse roots of the ARMA model to ensure that the model is stable, and the process is stationary.

Forecasting

Root Mean Squared Error	0.009282
Mean Absolute Error	0.006444
Mean Absolute Percentage Error	120.5253
Theil Inequality Coef.	0.969481
Bias Proportion	0.000001
Variance Proportion	0.999068
Covariance Proportion	0.000932
Theil U2 Coefficient	1.053982
Symmetric MAPE	179.1150

The forecast accuracy of the ARMA model was assessed using many measures. The RMSE was 0.009282, indicating that the model's predictions were, on average, 0.009282 units away from the actual values. The MAE was 0.006444, indicating that the model's predictions were, on average, 0.006444 units away from the actual values. The MAPE was quite high at 120.5253, indicating that the model's predictions were, on average, 120.5253% away from the actual values. The Theil Inequality Coefficient was 0.969481, indicating a slight bias towards underestimation. The Theil U2 Coefficient was 1.053982, suggesting that the model's predictions were somewhat less accurate than a naive forecast based on the historical mean. Finally, the Symmetric MAPE was 179.1150, which indicates a high degree of forecasting error. Overall, the model's performance was mixed, with some metrics indicating reasonable accuracy, while others suggesting poor performance.



The ARMA model's forecast graph shows how well the predicted values match the actual values. Here, significant deviations indicate that the model is not accurately capturing the underlying patterns in the data. Overall, the combination of the forecast graph and the various metrics provides a comprehensive evaluation of the ARMA model's performance, highlighting both its strengths and weaknesses.

Reference:

Baillie, R.T. and Bollerslev, T., 1989. Common stochastic trends in a system of exchange rates. *the Journal of Finance*, 44(1), pp.167-181.

Batten, J.A., Ciner, C., Lucey, B.M. and Szilagyi, P.G., 2013. The structure of gold and silver spread returns. *Quantitative finance*, 13(4), pp.561-570.

Bellini, F., Mercuri, L. and Rroji, E., 2020. On the dependence structure between S&P500, VIX and implicit Interexpectile Differences. *Quantitative Finance*, 20(11), pp.1839-1848.

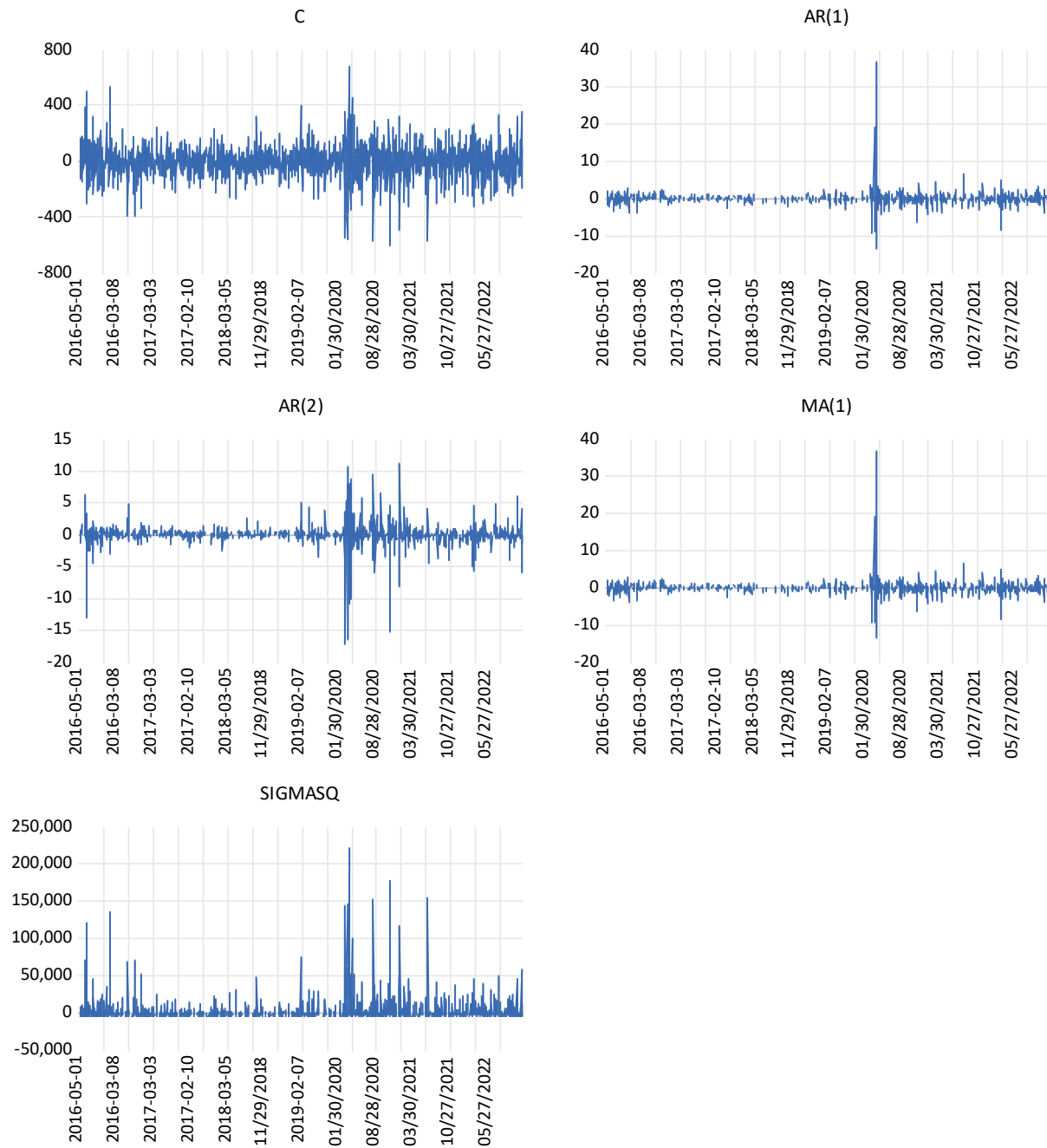
Hammoudeh, S., Mensi, W., Reboredo, J.C. and Nguyen, D.K., 2014. Dynamic dependence of the global Islamic equity index with global conventional equity market indices and risk factors. *Pacific-Basin Finance Journal*, 30, pp.189-206.

Liu, S.M. and Chou, C.H., 2003. Parities and spread trading in gold and silver markets: A fractional cointegration analysis. *Applied Financial Economics*, 13(12), pp.899-911.

Zhang, D., Yan, M. and Tsopanakis, A., 2018. Financial stress relationships among Euro area countries: an R-vine copula approach. *The European Journal of Finance*, 24(17), pp.1587-1608.

Appendix

Gradients of the Objective Function



ARMA Frequency Spectrum

