



Module Code: BUSN8022

Module Title: Quantitative Methods (2022/2023)

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Assessment Type: Coursework

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INTRODUCTION

Econometric analysis of equity returns time series in the US market, with a focus on two models. The CAPM and FAMA-FRENCH models were developed with the goal of identifying the optimal match.

DESCRIPTIVE STATISTICS

Descriptive statistics are brief informational coefficients that summarize a given data set.

<i>DESCRIPTIVE STATISTICS</i>	
Mean	0.012556
Standard Error	0.002423838
Median	0.01458
Standard	
Deviation	0.059371675
Sample Variance	0.003524996
Kurtosis	4.643102239
<u>Skewness</u>	0.188649778
Range	0.657606
Minimum	-0.292344
Maximum	0.365262
Sum	7.5336
Count	600

Let's discuss some of the descriptive statistics from above table, the monthly average return for the portfolio is 0.0125. Median is 0.014, indicate the middle number from total samples. The standard deviation is 0.059 and we can observe that it is higher than mean value, which indicates data are more spread out. Sample variance 0.003 is the average degree to which each point differs from the mean. Kurtosis here is 4.64, which tell us that the dataset has heavier tails than a normal distribution. Skewness is 0.188 (skewed-right) that means mean is greater than the median. Range mentioned here is 0.657, indicates difference between the lowest and highest returns. Minimum is -0.2923, which indicates the lowest value in time series and maximum 0.365 is the highest value in time series

REGRESSION MODEL

A regression model is a statistical model that estimates the relationship between one dependent variable and one or more independent variables using a line. Models use to estimate are

- 1) CAPM (Capital Assets Pricing Model)
- 2) FAMA-FRENCH 3 Factor Model

CAPITAL ASSET PRICING MODEL(CAPM)

financial model that calculates the expected rate of return for an asset or investment.

SUMMARY OUTPUT		CAPM							
<i>Regression Statistics</i>									
Multiple R	0.727344191								
R Square	0.529029573								
Adjusted R Square	0.528241997								
Standard Error	0.040779223								
Observations	600								
<i>ANOVA</i>									
	<i>df</i>	<i>SS</i>	<i>MS</i>	<i>F</i>	<i>Significance F</i>				
Regression	1	1.117031394	1.117031394	671.7187874	7.5184E-100				
Residual	598	0.994441105	0.001662945						
Total	599	2.111472499							
	<i>Coefficients</i>	<i>Standard Error</i>	<i>t Stat</i>	<i>P-value</i>	<i>Lower 95%</i>	<i>Upper 95%</i>	<i>Lower 95.0%</i>	<i>Upper 95.0%</i>	
Intercept	0.007161881	0.001677764	4.268706282	2.28605E-05	0.003866855	0.010456907	0.003866855	0.010456907	
X Variable 1	0.957961039	0.036961884	25.91753822	7.5184E-100	0.885370158	1.030551921	0.885370158	1.030551921	

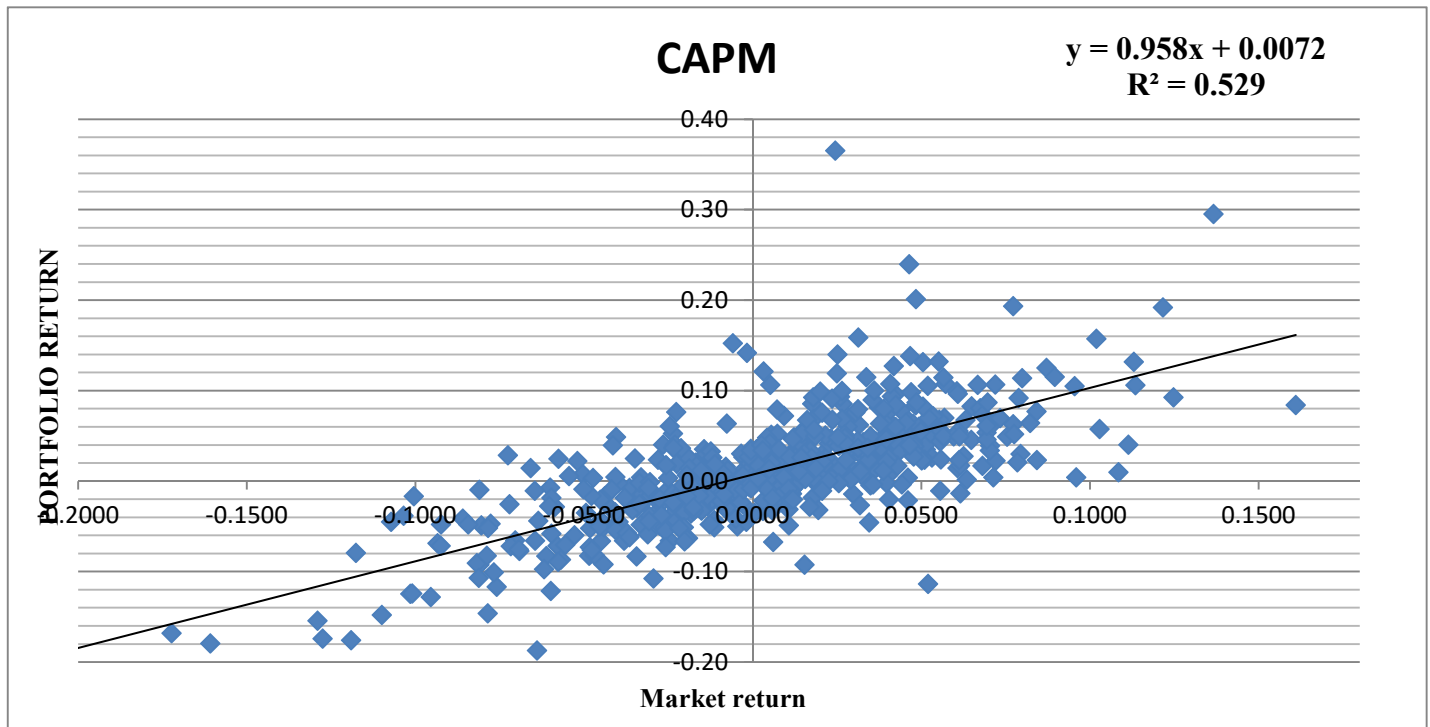
CAPM is given by the equation

$$R_t = \alpha + \beta MKT_t + \varepsilon_t$$

R_t = Portfolio Return at time t, MKT_t = Return of the market at t, ε_t = Random error term.

$$R_t = 0.0072 + 0.958x + 0$$

$$R_t = 0.0072 + 0.958x$$



Here, the Equation represents the expected rate of return of the portfolio. Intercept (or) Alpha(α) value tells you how much better the fund did than CAPM predicted; basically it's a constant return. And Beta(β) represent the portfolio volatility in relation to the market; here the beta is 0.958 which indicates change in market by 1% and change in portfolio by 0.958% and standard error should be zero

It's customary to use "a" or "alpha" for the intercept of the line, and "b" or "beta" for the slope; so linear regression gives you a formula of the form $y = bx + a$ and **R-squared (R^2)** shows how well the data fit the regression model (the goodness of fit). Here the R^2 is 0.52 which indicates moderate to good correlation.

FAMA-FRENCH(3 factor)

The asset pricing model that expands on the capital asset pricing model by adding size risk and value risk factors to the market risk factors.

SUMMARY OUTPUT		FAMA- FRENCH						
<i>Regression Statistics</i>								
Multiple R	0.931220142							
R Square	0.867170952							
Adjusted R Square	0.86650235							
Standard Error	0.021692825							
Observations	600							
<i>ANOVA</i>								
	<i>df</i>	<i>SS</i>	<i>MS</i>	<i>F</i>	<i>Significance F</i>			
Regression	3	1.831007617	0.610336	1296.99	1E-260			
Residual	596	0.280464882	0.000471					
Total	599	2.111472499						
	<i>Coefficients</i>	<i>Standard Error</i>	<i>t Stat</i>	<i>P-value</i>	<i>Lower 95%</i>	<i>Upper 95%</i>	<i>Lower 95.0%</i>	<i>Upper 95.0%</i>
Intercept	0.005603083	0.000902369	6.209303	9.98E-10	0.0038309	0.0073753	0.003830873	0.00737529
X Variable 1	0.793144421	0.02098681	37.79252	2.2E-160	0.7519273	0.8343615	0.751927329	0.83436151
X Variable 2	1.172957275	0.030355945	38.64012	1.8E-164	1.1133396	1.2325749	1.113339649	1.2325749
X Variable 3	0.323742481	0.031931294	10.13872	2.16E-22	0.2610309	0.386454	0.261030944	0.38645402

Fama- French is given by following equation

$$R_t = \alpha + \beta_1 MKT_t + \beta_2 SMB_t + \beta_3 HML_t + \epsilon_t$$

SMB = R_t small stock – R_t of large stock

HML = R_t high value stock – R_t of low value stock

$$R_t = 0.0056 + 0.7931x_1 + 1.1729x_2 + 0.3237x_3$$

Here the equation represents the pricing model by adding size and value risk factor to the market risk. The alpha (α) 0.0056 denotes the access return that an active portfolio achieves above the expected return due to market, size, value, profitability and investment risk factors. And beta (β) calculated based on assets in the portfolio being measured compared against small-cap/large-cap returns in the market at large. **R-squared(R^2)** 0.8671 represent the ratio of systematic risk to total risk.

ASSUMPTIONS

1) The expected value of the error term is equal to zero $E(\epsilon_i) = 0$

DO NOT PERFORM

2) The variance of the error term is constant and finite $Var(\epsilon_i) = \sigma^2 < \infty$

This assumption is typically referred to as homoscedasticity

✓ GOLDFELD-QUANDT TEST

The Goldfeld Quandt Test is a test used in regression analysis to test for homoscedasticity.

1 FACTOR(CAPM)

SUMMARY OUTPUT		GQ - CAPM						
<i>Regression Statistics</i>								
Multiple R	0.680997							
R Square	0.463757							
Adjusted R Square	0.461957							
Standard Error	0.04327							
Observations	300							
<i>ANOVA</i>								
	<i>df</i>	<i>SS</i>	<i>MS</i>	<i>F</i>	<i>Significance F</i>			
Regression	1	0.482515	0.482515	257.7181	3.2E-42			
Residual	298	0.557934	0.001872					
Total	299	1.040449						
	<i>Coefficients</i>	<i>Standard Error</i>	<i>t Stat</i>	<i>P-value</i>	<i>Lower 95%</i>	<i>Upper 95%</i>	<i>Lower 95.0%</i>	<i>Upper 95.0%</i>
Intercept	0.005019	0.002533	1.981782	0.048422	3.5E-05	0.010004	3.5E-05	0.010004
X Variable 1	0.92689	0.057737	16.0536	3.2E-42	0.813265	1.040514	0.813265	1.040514

SUMMARY OUTPUT		GQ - CAPM						
<i>Regression Statistics</i>								
Multiple R	0.772321							
R Square	0.59648							
Adjusted R Square	0.595126							
Standard Error	0.038075							
Observations	300							
<i>ANOVA</i>								
	<i>df</i>	<i>SS</i>	<i>MS</i>	<i>F</i>	<i>Significance F</i>			
Regression	1	0.638592	0.638592	440.5011	1.12E-60			
Residual	298	0.432009	0.00145					
Total	299	1.070601						
	<i>Coefficients</i>	<i>Standard Error</i>	<i>t Stat</i>	<i>P-value</i>	<i>Lower 95%</i>	<i>Upper 95%</i>	<i>Lower 95.0%</i>	<i>Upper 95.0%</i>
Intercept	0.009407	0.002206	4.263524	2.7E-05	0.005065	0.013749	0.005065	0.013749
X Variable 1	0.988106	0.047079	20.98812	1.12E-60	0.895456	1.080756	0.895456	1.080756

The test statistics can be computed as

$$GQ = \frac{s_1^2}{s_2^2} \sim F(n_1 - k, n_2 - k) \quad GQ = \frac{0.00187}{0.00144} = 1.29$$

Followed by F distribution,

- P-value is equal to 0.0137 which is below the critical value 0.05, so reject the Null hypothesis
- REJECT $H_0: \sigma_1^2 = \sigma_2^2$ at a meaningful Significance level.
- Residuals are heteroscedastic.
- The coefficients' standard errors will be biased

3 FACTORS(FAMA-FRENCH)

SUMMARY OUTPUT		GQ (S1)										
Regression Statistics												
Multiple R	0.9558											
R Square	0.9136											
Adjusted R Square	0.9127											
Standard Error	0.0177											
Observations	300											
ANOVA												
	df	SS	MS	F	Significance F							
Regression	3	0.978128334	0.3260428	1043.641692	5.064E-157							
Residual	296	0.092472985	0.0003124									
Total	299	1.070601318										
	Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%	Lower 95.0%	Upper 95.0%				
Intercept	0.0066	0.0011	6.2658	0.0000	0.0045	0.0087	0.0045	0.0087				
X Variable 1	0.8216	0.0251	32.7819	0.0000	0.7723	0.8709	0.7723	0.8709				
X Variable 2	1.2137	0.0382	31.8098	0.0000	1.1386	1.2888	1.1386	1.2888				
X Variable 3	0.3282	0.0411	7.9958	0.0000	0.2474	0.4090	0.2474	0.4090				

SUMMARY OUTPUT		GQ (S2)						
Regression Statistics								
Multiple R	0.9069							
R Square	0.8224							
Adjusted R Square	0.8206							
Standard Error	0.0250							
Observations	300							
ANOVA								
	df	SS	MS	F	Significance F			
Regression	3	0.85569596	0.285231985	456.9805798	1.0081E-110			
Residual	296	0.18475329	0.000624167					
Total	299	1.04044925						
	Coefficients	tandard Error	t Stat	P-value	Lower 95%	Upper 95%	Lower 95.0%	Upper 95.0%
Intercept	0.00469	0.00147	3.20036	0.00152	0.00181	0.00757	0.00181	0.00757
X Variable 1	0.75902	0.03438	22.07717	0.00000	0.69136	0.82668	0.69136	0.82668
X Variable 2	1.13984	0.04663	24.44549	0.00000	1.04808	1.23161	1.04808	1.23161
X Variable 3	0.31799	0.04887	6.50618	0.00000	0.22180	0.41417	0.22180	0.41417

The T-stat compute as

$$GQ = \frac{s_1^2}{s_2^2} \sim F(n_1 - k, n_2 - k)$$

$$GQ = \frac{0.000624}{0.000312} = 1.998$$

- P-value is equal to 0.0 which is below the critical value 0.05, so reject the Null hypothesis
- REJECT $H_0: \sigma_1^2 = \sigma_2^2$ at a meaningful Significance level.
- Residuals are heteroscedastic.
- Coefficients' standard errors will be biased

✓ WHITE'S TEST

1-FACTOR(CAPM)

Test statistics can be compute as

$$\begin{aligned} w &= nR^2 \sim \chi^2(m) \\ &= 600 * 0.01314 \\ &= 7.885 \end{aligned}$$

Regression Statistics	
Multiple R	0.114634
R Square	0.013141
Adjusted R Square	0.009835
Standard Error	0.005506
Observations	600

- Auxiliary regression produced an R^2 of 1.3%
- Critical value from the $\chi^2(3)$ distribution at the 5% significance level p value is equal to 0.0484
- T-statistic exceed the critical value
- Reject the null hypothesis at the 5% significance level

3 FACTORS(FAMA-FRENCH)

$$KB = \frac{\hat{\gamma}_1}{SE(\hat{\gamma}_1)} \sim t_{n-2}$$

Test-statistics can be compute as

$$\begin{aligned} w &= nR^2 \sim \chi^2(m) \\ &= 600 * 0.3409 \\ &= 204.54 \end{aligned}$$

Regression Statistics	
Multiple R	0.583860654
R Square	0.340893264
Adjusted R Square	0.330839093
Standard Error	0.000796701
Observations	600

- Auxiliary regression produced an R^2 of 34.08%
- Critical value from the $\chi^2(9)$ distribution at the 5% significance level p value is equal to 0.00
- Test statistic exceed the critical value
- Reject the null hypothesis at the 5% significance level.

✓ KOENKER-BASSETT TEST

1 FACTOR (CAPM)

CAPM									
SUMMARY OUTPUT									
Regression Statistics									
Multiple R	0.099317								
R Square	0.009864								
Adjusted R Square	0.008208								
Standard Error	0.005511								
Observations	600								
ANOVA									
	df	SS	MS	F	gnificance F				
Regression	1	0.000180913	0.000181	5.957383	0.014945				
Residual	598	0.018160018	3.04E-05						
Total	599	0.018340932							
Coefficients									
Intercept	0.001343	0.000259217	5.181442	3.01E-07	0.000834	0.001852	0.000834	0.001852	
X Variable 1	0.155635	0.063764454	2.440775	0.0149	0.030405	0.280864	0.030405	0.280864	

Test Statistics can be computed as

$$KB = \frac{\hat{\gamma}_1}{SE(\hat{\gamma}_1)} \sim t_{n-2} = 2.44$$

- Regressing $(\varepsilon_t)^2$ against $(y_t)^2$ produces a slope coefficient with a test statistic equal to 2.440
- p-value under the t_{600} , distribution is equal to 0.0149
- Reject the null hypothesis of homoscedasticity at the 5% significance level and t-stat exceed the critical value

3 FACTORS(FAMA-FRENCH)

SUMMARY OUTPUT		FAMA-FRENCH					
Regression Statistics							
Multiple F	0.33884						
R Square	0.114813						
Adjusted R Square	0.113332						
Standard Error	0.000917						
Observations	600						
ANOVA							
	df	SS	MS	F	Significance F		
Regression	1	6.52E-05	6.52E-05	77.56308787	1.39E-17		
Residual	598	0.000503	8.41E-07				
Total	599	0.000568					
Coefficients		Standard Error	t Stat	P-value	Lower 95%	Upper 95%	Lower 95.0%
Intercept	0.000308	4.16E-05	7.391526	4.91702E-13	0.000226	0.000389	0.000226
X Variable	0.049819	0.005657	8.806991	0.00	0.038709	0.060928	0.038709

Test Statistics can be computed as

$$KB = \frac{\hat{\gamma}_1}{SE(\hat{\gamma}_1)} \sim t_{n-2}$$

- Regressing $(\varepsilon_t)^2$ against $(y_t)^2$ produces a slope coefficient with a test statistic equal to 8.806
- p-value under the t_{600} , distribution is equal to 0.0

- Reject the null hypothesis of homoscedasticity at the 5% significance level and t-stat exceed the critical value

Presence of heteroscedasticity, the coefficients' standard errors will be biased. One way of addressing this problem is to compute **robust standard errors**

	Coef.	ORIGINAL			ROBUST		
		SE	T-STAT	CONC	SE	T-STAT	CONC
β_1	0.793	0.0209	37.79	Reject	0.023173	34.32	Reject
β_2	1.172	0.0303	38.64	Reject	0.059047	19.84	Reject
β_3	0.323	0.0319	10.13	Reject	0.043705	7.39	Reject

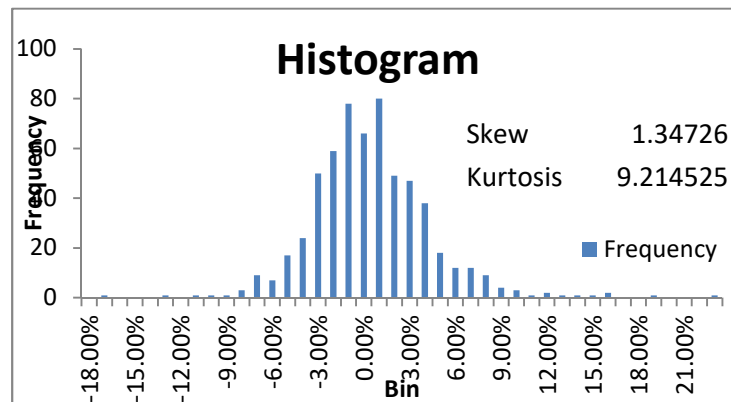
All rejected, t-statistics exceed the critical value

3) The errors are normally distributed $\varepsilon_i \sim N(0, \sigma^2)$

Errors are normally distributed, A quick way of examining the normality assumption is by plotting the residuals' histogram

✓ JARQUE-BERA TEST

1 FACTOR(CAPM)



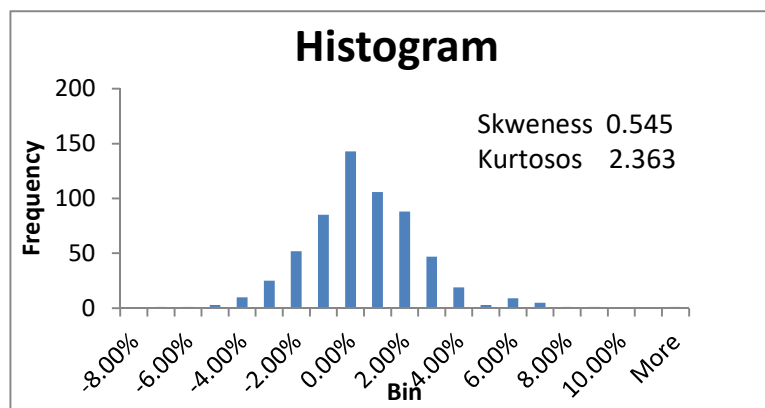
Test computed as

$$JB = n \left[\frac{(skew)^2}{6} + \frac{(kurt - 3)^2}{24} \right]$$

$$JB = 1147.01$$

- p-value of this JB test statistic under the chi-square distribution with 2 degrees of freedom is effectively equal to zero and t-statistics exceed the critical value.
- Reject the null at any meaningful significance level
- Residuals deviate significantly from normality

3 FACTORS (FAMA-FRENCH)



Test computed as

$$JB = n \left[\frac{(skew)^2}{6} + \frac{(kurt - 3)^2}{24} \right]$$

$$JB = 39.83$$

- p-value of this JB test statistic under the chi-square distribution with 2 degrees of freedom is effectively equal to zero and t-statistics exceed the critical value.
- Reject the null at any meaningful significance level
- Residuals deviate significantly from normality

4) Errors are uncorrelated with one another $cov \varepsilon_i, \varepsilon_j = 0 \forall i \neq j$

errors are uncorrelated with one another

✓ RUNS TEST

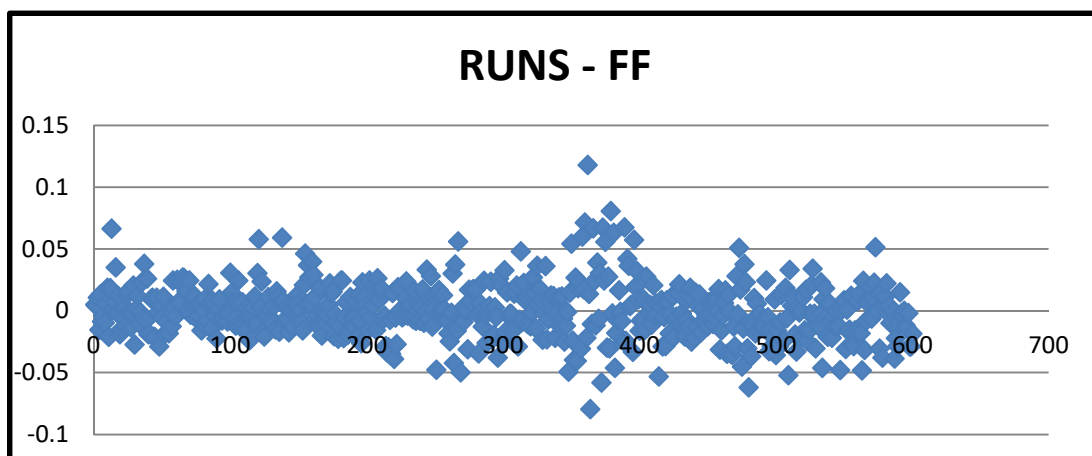
1 FACTOR (CAPM)

SUMMARY OUTPUT		CAPM				N		600	
						N(-)		282	
						N(+)		318	
						R		300	
						E[R]		298.42	
						VAR®		148.67	
						T STAT		0.12937	
						P VALUE		0.8971	

TEST BASES ON 3 FACTORS (FAMA-FRENCH)

SUMMARY OUTPUT									
						N	600		
						n(-)	320		
						n(+)	280		
						R	300		
						E[R]	298.1697		
						VAR	148.4192		
						t stat	0.150236		
						p value	0.880579		
ANOVA									
	df	SS	MS	F	gnificance F				
Regression	3	1.83100762	0.610335872	1296.99012	1E-260				
Residual	596	0.28046488	0.000470579						
Total	599	2.1114725							
	Coefficients	tandard Error	t Stat	P-value	Lower 95%	Upper 95%	lower 95.0%	Upper 95.0%	
Intercept	0.005603083	0.00090237	6.209302716	9.9816E-10	0.003831	0.007375	0.003831	0.0073753	
X Variable	0.793144421	0.02098681	37.79252013	2.211E-160	0.751927	0.834362	0.751927	0.8343615	
X Variable	1.172957275	0.03035594	38.64011768	1.84E-164	1.11334	1.232575	1.11334	1.2325749	
X Variable	0.323742481	0.03193129	10.13872091	2.1558E-22	0.261031	0.386454	0.261031	0.386454	

- Model explains 87 % of the variation in portfolio return
- T-statistic is lower than the absolute critical value under the normal distribution at the 5% significance level (± 1.96)
- p-value of the test statistic (89%) exceeds 5%
- Do not reject the null of no autocorrelation at 5%
- Residuals do not seem to be changing sign too frequently



✓ **DURBIN-WATSON TEST**

1 FACTOR(CAPM)

- The sample correlation coefficient between ε_t and ε_{t-1} is equal to 0.0533%
- The test statistic can be approximated by $DW \approx 2 [1-0.0533] = 1.89$
- Reject the null hypothesis of no autocorrelation
- $H_0: \text{corr}(\varepsilon_t, \varepsilon_{t-1}) = 0$
- Negative Autocorrelation

3 FACTORS(FAMA-FRENCH)

- Sample correlation coefficient between ε_t and ε_{t-1} is equal to -0.0538%
- Test statistic can be approximated by $DW \approx 2 [1-(-0.0538)] = 2.106$
- Reject the null hypothesis of no autocorrelation
- $H_0: \text{corr}(\varepsilon_t, \varepsilon_{t-1}) = 0$
- Negative Autocorrelation

✓ **BREUSCH-GODFREY TEST**

1 FACTOR(CAPM)

- test for autocorrelation at the first 5 lags ($r = 5$)
- The auxiliary regression produced an $R^2 = 0.0113$
- We can compute the test statistic as

$$BG = (n - r) R^2 = 600 - 5 \times 0.0113 = 6.667$$

- p-value of the BG test statistic is 0.753
- Reject the null hypothesis at the 5% significance level, as the test statistic exceeds the critical value and p value less than significance level

3 FACTORS(FAMA-FRENCH)

- test for autocorrelation at the first 5 lags ($r = 5$)
- Auxiliary regression produced an $R^2 = 0.0176$
- We can compute the test statistic as

$$BG = (n - r)R^2 = 600 - 5 \times 0.01768 = 10.4322$$

- p-value of the BG test statistic is 0.06387
- Reject the null hypothesis at the 5% significance level and t-statistic exceed the critical value

5) Errors are uncorrelated with the independent variables $cov(\epsilon_t, x_{i,t}) = 0$

- For the portfolio The $E(\epsilon) = 0$ (sum of product of independent and dependent variable is not zero) Orthogonal regressors
- $E[\beta] = \beta$

6) Independent variables are uncorrelated with one another $cov(x_{i,t}, x_{j,t}) = 0 \forall i \neq j$

R^2 for independent Variables and R^2 for dependent variable is 0.8671

R^2

X1 0.122252

X2 0.093345

X3 0.087732

- Under the multicollinearity, R^2 for multi correlated or dependent should be higher
- Variable(x1)<Dependent
- Variable(x2)<dependent
- Variable(X3)<dependent

There is no violation

CORRELATION

	MKT	SMB	HML
MKT		0.275	-0.265
SMB	0.275		-0.199
HML	-0.26543	-0.199	

- Two market factor are strongly correlated with each other, but HML is Negatively correlated

CONCLUSION

After performing two Regression model Fama-French is better predicting variation in excess return over R_f than CAPM. Moreover FF has 3-variable which can be more precise and robust standard error always gives better assessment. The Fama-French model significantly outperforms CAPM in all portfolios and FF has a lower mean square-error and higher correlation between actual and predicted value