## Autumn Block 4

Fractions B

## Small steps

| Step 1 | Multiply fractions by integers |
| :--- | :--- |
| Step 2 | Multiply fractions by fractions |
| Step 3 | Divide a fraction by an integer |
| Step 4 | Divide any fraction by an integer |
|  |  |
| Step 5 | Mixed questions with fractions |
| Step 6 | Fraction of an amount |

## Notes and guidance

Building on their learning in Year 5, this small step provides practice in multiplying fractions and mixed numbers by integers.
A variety of representations can show that multiplying fractions by integers is the same as repeated addition of a fraction. As when adding and subtracting fractions, the denominator does not change. Children recognise that they need to multiply the numerator by the integer. When multiplying mixed numbers, children can either partition them into wholes and parts, multiplying each of them by the integer, or convert the mixed number to an improper fraction and then multiply the numerator by the integer.

## Things to look out for

- Children may multiply both the denominator and numerator by the integer, or only multiply the numerator of the part in a mixed number and not the whole.
- Children may make mistakes when converting between mixed numbers and improper fractions.
- Children should be encouraged to give their answers in their simplest form and convert any improper fractions to mixed numbers.


## Key questions

- How is multiplying fractions by integers similar to addition of fractions? How is it different?
- What happens to the denominator when you multiply a fraction by an integer?
- Do you find it easier to partition the mixed number first or to convert it to an improper fraction?
- Is $\frac{2}{3} \times 7$ equal to $7 \times \frac{2}{3}$ ? Why?


## Possible sentence stems

- To multiply a fraction by an integer, I need to multiply the numerator by $\qquad$
- To multiply a mixed number by an integer, I can partition it into $\qquad$ and $\qquad$ and then multiply them both by the integer.
- To multiply a mixed number by an integer, I can convert the mixed number to an $\qquad$ and then ...


## National Curriculum links

- Multiply proper fractions and mixed numbers by whole numbers, supported by materials and diagrams (Y5)


## Multiply fractions by integers

## Key learning

- Use the diagrams to work out the multiplications.

- Complete the calculations.
$-\frac{3}{5} \times$ $\qquad$ $=\frac{9}{5}=$ $\qquad$

$$
\stackrel{2}{7} \times-=\frac{\square}{7}=1 \frac{1}{7}
$$

- Huan works out $4 \times \frac{7}{8}$

$$
4 \times \frac{7}{8}=\frac{28}{8}=3 \frac{4}{8}
$$

How can you improve Huan's answer?

- Eva partitions $2 \frac{3}{5}$ to help her work out $2 \frac{3}{5} \times 3$


$$
\begin{aligned}
& 2 \times 3=6 \\
& \frac{3}{5} \times 3=\frac{9}{5}=1 \frac{4}{5} \\
& 6+1 \frac{4}{5}=7 \frac{4}{5}
\end{aligned}
$$

Use Eva's method to work out the multiplications.
$2 \frac{5}{6} \times 3$

$$
1 \frac{3}{7} \times 5
$$

$$
3 \times 2 \frac{2}{3}
$$

$$
1 \frac{1}{6} \times 4
$$

- Tommy works out $2 \frac{3}{5} \times 3$ by converting the mixed number to an improper fraction first.

$$
2 \frac{3}{5}=\frac{13}{5} \quad \frac{13}{5} \times 3=\frac{39}{5} \quad \frac{39}{5}=7 \frac{4}{5}
$$

Use Tommy's method to work out the multiplications.


## Multiply fractions by integers

## Reasoning and problem solving

There are 12 children in a class.
The teacher has 4 litres of orange juice.


Each child gets $\frac{1}{5}$ litre of orange juice.
How much orange juice will be left over?

A classroom desk is $1 \frac{1}{3} \mathrm{~m}$ long.
The classroom is 6 m wide.
Will 5 desks fit side by side in the classroom?

No
$5 \times 1 \frac{1}{3}=6 \frac{2}{3}$

Tiny is working out $4 \times 3 \frac{2}{5}$


Is Tiny correct?
Explain your reasoning.


No
multiple possible answers, e.g. $\mathrm{A}=5$ and $B=16$

## Notes and guidance

Building on the previous step, children multiply a fraction by another fraction.

Children use concrete and pictorial representations to support them, including folding paper, diagrams and bar models.
By exploring the pictorial representations, children identify the fact that fractions can be multiplied by multiplying both the numerators and denominators. They may need to be reminded that answers should be given in their simplest form.

As the fractions children multiply in this step are all proper, they could be stretched to explain why their answer is always smaller than the fractions given in the question.

## Things to look out for

- Children may believe that "multiplication always makes numbers bigger", but should realise that this is not the case when multiplying by a number less than 1
- The processes for different operations could get mixed up and children may unnecessarily convert to a common denominator as if they are adding or subtracting fractions.


## Key questions

- How can you show the calculation as a diagram?
- What is the same and what is different about "half of" a number and " $\frac{1}{2} \times$ " a number?
- When you multiply two fractions, is the product greater than or smaller than each of the fractions? Why?
- Why are all of your answers less than 1 ?


## Possible sentence stems

- To show $\qquad$ I have split my diagram into $\qquad$ equal sections.
- To find the product, I need to ...
- When multiplying a pair of fractions, I need to multiply the
$\qquad$ and multiply the $\qquad$


## National Curriculum links

- Multiply simple pairs of proper fractions, writing the answer in its simplest form


## Multiply fractions by fractions

## Key learning

- Alex is using a piece of paper to work out $\frac{1}{2} \times \frac{1}{3}$

First, she folds the piece of paper in half.
Then she folds the half into thirds.
Alex shades the fraction that she has created.


I have shaded $\frac{1}{6}$,

$$
\text { so } \frac{1}{2} \times \frac{1}{3}=\frac{1}{6}
$$

Use Alex's method to work out the multiplications.

$$
\frac{1}{4} \times \frac{1}{2}
$$



$$
\frac{1}{4} \times \frac{1}{4}
$$

- Whitney is using diagrams to represent multiplying fractions.

Shade the diagrams to work out the multiplications.

$\qquad$ $\frac{1}{4} \times \frac{1}{2}=$ $\qquad$ $\frac{1}{5} \times \frac{1}{4}=$ $\qquad$

- Dani is using a diagram to work out $\frac{2}{3} \times \frac{4}{5}$


Explain why the diagram shows $\frac{2}{3} \times \frac{4}{5}=\frac{8}{15}$
Use similar diagrams to work out $\frac{2}{3} \times \frac{2}{5}$ and $\frac{2}{3} \times \frac{3}{5}$

- Dexter has spotted a connection between the numerators and the denominators in the question and answer.

$$
\frac{3}{4} \times \frac{1}{5}=\frac{3}{20} \quad \frac{4}{5} \times \frac{3}{7}=\frac{12}{35} \quad \frac{4}{5} \times \frac{2}{3}=\frac{8}{15}
$$

What connection has Dexter spotted?
Use the connection to work out the multiplications.

$$
\frac{2}{5} \times \frac{1}{3}
$$

$$
\frac{3}{4} \times \frac{3}{5}
$$

$$
\frac{2}{7} \times \frac{4}{5}
$$

Can any of your answers be simplified?

## Multiply fractions by fractions

## Reasoning and problem solving

Aisha uses this diagram to work out the product of two fractions.


$$
\begin{aligned}
& \frac{3}{4} \times \frac{1}{6} \text { or } \frac{1}{6} \times \frac{3}{4} \\
& \frac{3}{24}=\frac{1}{8}
\end{aligned}
$$

What fractions has Aisha multiplied?
What is the answer?

Work out the missing numbers.

$$
\begin{aligned}
& \frac{1}{2} \times \frac{1}{\square}=\frac{1}{16} \\
& \frac{\square}{6} \times \frac{3}{5}=\frac{21}{30} \\
& \frac{3}{\square} \times \frac{4}{5}=\frac{3}{5}
\end{aligned}
$$

Find the missing numbers.


Is there more than one answer?

What is the area of the shaded region?

multiple possible answers, e.g.
$\frac{2}{3} \times \frac{3}{4}=\frac{6}{12}=\frac{1}{2}$
$\frac{2}{6} \times \frac{3}{2}=\frac{6}{12}=\frac{1}{2}$

$\frac{11}{21} \mathrm{~m}^{2}$

## Notes and guidance

In this small step, children are introduced to dividing fractions by integers for the first time. They focus on dividing fractions where the numerator is a multiple of the integer they are dividing by, for example $\frac{3}{5}$ divided by 3 , or $\frac{6}{7}$ divided by 2 Bar models are used initially to represent fractions and to explore how to divide a fraction by an integer. Children complete the number sentence alongside the representation to encourage them to notice that the denominator stays the same and the numerator is divided by the integer. The idea of unitising could be used to support children with dividing fractions by integers. For example, if they know that 6 ones shared between 2 is equal to 3 ones, and 6 eggs shared between 2 is equal to 3 eggs, then 6 sevenths shared between 2 is equal to 3 sevenths. Links can be made to previous representations when multiplying fractions, for example by looking at the equivalence of $\frac{4}{7} \div 2$ and $\frac{4}{7} \times \frac{1}{2}$

## Things to look out for

- Children may divide both the numerator and denominator by the integer.
- Children may be tempted to use an abstract procedure, rather than think carefully about what the question is asking.


## Key questions

- How could you represent the fraction?
- How could you split the fraction into $\qquad$ equal parts?
- What do you notice about the numerators in the question and the answer?
- What do you notice about the denominators in the question and the answer?
- What changes and what stays the same?
- How can you show the division as a bar model?


## Possible sentence stems

- If you divide $\qquad$ into equal groups, then each group is
$\qquad$ because $\qquad$ $\div$ $\qquad$ = $\qquad$
- $\qquad$ ones divided by $\qquad$ is equal to $\qquad$ ones, so
$\qquad$ eighths divided by $\qquad$ is equal to $\qquad$ eighths.


## National Curriculum links

- Divide proper fractions by whole numbers


## Divide a fraction by an integer

## Key learning

- Filip has $\frac{2}{5}$ of a chocolate bar.

He shares it with his friend.
What fraction of the chocolate bar do they each get?


- Use the diagrams to help you work out the divisions.

$\frac{3}{4} \div 3=$ $\qquad$

$\frac{4}{7} \div 2=$ $\qquad$
- Use the division fact $12 \div 4=3$ to work out the divisions.
- $12,000 \div 4$
- $12 \mathrm{~m} \div 4$
- $12 p \div 4$
$-\frac{12}{19} \div 4$
- Complete the divisions.
- $\frac{6}{11} \div 3$
- $\frac{15}{17} \div 5$
$-\frac{49}{50} \div 7$
$-\frac{96}{101} \div 12$
- A cake has a mass of $\frac{8}{9} \mathrm{~kg}$.
- What is the mass of each piece if the cake is cut into 8 equal pieces?
- What is the mass of each piece if the cake is cut into 4 equal pieces?
- What is the mass of each piece if the cake is cut into 2 equal pieces?
- Find the missing integers.
$-\frac{15}{16} \div$ $\qquad$ $=\frac{5}{16}$
$-\frac{20}{23} \div-=\frac{4}{23}$
$-\frac{15}{16} \div-=\frac{3}{16}$
$-\frac{20}{23} \div-=\frac{5}{23}$
- Mo works out $1 \frac{3}{5} \div 2$ using improper fractions.

$$
1 \frac{3}{5} \div 2=\frac{8}{5} \div 2=\frac{4}{5}
$$

Use Mo's method to work out the divisions.

- $1 \frac{1}{3} \div 2$
- $1 \frac{7}{9} \div 4$
- $2 \frac{5}{8} \div 3$
- $3 \frac{3}{4} \div 5$


## Divide a fraction by an integer

## Reasoning and problem solving



Amir and Alex each have a piece of ribbon that is $\frac{99}{100} \mathrm{~m}$ long.

- Amir cuts his ribbon into 9 equal pieces.
- Alex cuts her ribbon into 3 equal pieces.
Whose pieces of ribbon are longer?
By how much?
Give your answer in centimetres.
Compare methods with a partner.


Alex
22 cm

What could the missing
numbers be?

$$
\frac{\square}{21} \div 4=\frac{\square}{21}
$$

Can any of your answers be simplified?
multiple possible
answers, e.g.
$\frac{4}{21} \div 4=\frac{1}{21}$
$\frac{12}{21} \div 4=\frac{3}{21}=\frac{1}{7}$

## Notes and guidance

In this small step, children build on their learning from the previous step to divide fractions where the numerator is not a multiple of the integer they are dividing by.

Children continue to use models and draw diagrams to divide fractions into equal parts. There are two methods that children could use throughout this step. They could use their prior knowledge of equivalent fractions combined with learning from the previous step to find an equivalent fraction where the numerator is a multiple of the integer they are dividing by. Alternatively, through the use of diagrams, children could explore the link between multiplying by a unit fraction and dividing by an integer. When using this method, children should be encouraged to spot the pattern that the numerator stays the same and the denominator is multiplied by the integer.
Encourage children to compare methods and decide which is more efficient, and why.

## Things to look out for

- Following on from the previous step, children may try to divide the numerator by the integer anyway even when it is not a multiple, for example $\frac{3}{5} \div 2=\frac{1.5}{5}$
- Children may become over-reliant on quick tricks.


## Key questions

- How can you split a fraction into equal parts? What is each part of the fraction worth?
- How can you show the division as a bar model?
- How is $\frac{1}{3} \div 2$ similar to $\frac{1}{3} \times \frac{1}{2}$ ?
- What fractions are equivalent to $\qquad$ ?
- Why does finding an equivalent fraction help you to divide a fraction by an integer?
- What multiplication can you use to work out
$\qquad$ $\div$ $\qquad$ ?


## Possible sentence stems

- The bar is split into $\qquad$ equal parts.
- I am dividing each $\qquad$ by $\qquad$ , so I must split each part into $\qquad$ equal parts.
- $\qquad$ is equivalent to $\qquad$ , so $\qquad$ $\div$ $\qquad$ is equal
to $\qquad$ $\div$ $\qquad$


## National Curriculum links

- Divide proper fractions by whole numbers


## Divide any fraction by an integer

## Key learning

- Teddy divides one third into 2 equal parts.


Draw diagrams to work out the divisions.
$\frac{1}{3} \div 3$

$$
\frac{2}{3} \div 3
$$


$\square$

- Annie is dividing $\frac{2}{3}$ by 4


Use equivalent fractions to work out the divisions.


$$
\frac{4}{5} \div 8
$$

- Jack is dividing fractions by integers.

$$
\begin{aligned}
& \frac{2}{5} \div 3=\frac{2}{15} \\
& \frac{3}{4} \div 5=\frac{3}{20} \\
& \frac{5}{7} \div 6=\frac{5}{42}
\end{aligned}
$$

I've noticed something!

What has Jack noticed?

- Work out the divisions.
$\frac{1}{8} \div 3$


$$
\frac{2}{7} \div 3
$$



- Use the diagram to explain why $\frac{1}{4} \div 2=\frac{1}{4} \times \frac{1}{2}=\frac{1}{8}$

- Work out the missing numbers.
$-\frac{1}{3} \div 2=\frac{3}{4} \times \frac{\square}{\square}=\frac{\square}{\square}$
$-\frac{3}{5} \div 2=\frac{3}{5} \times \frac{\square}{\square}=\frac{\square}{\square}$


## Divide any fraction by an integer

## Reasoning and problem solving



## Notes and guidance

Children have now used all four operations with fractions in isolation. In this small step, children identify the appropriate operation(s) to use in a given situation.

Bar models are used to explore word problems and to support children in selecting the correct operation(s). Children start by choosing the correct single operation to solve a problem and move on to explore multi-step problems using all four operations. This step provides a good opportunity to revisit learning from earlier in the year. They can consolidate their knowledge of the order of operations, and also topics such as measure from earlier years.

## Things to look out for

- Children may find it difficult to identify the different steps within a problem.
- Children may perform the operations in the wrong order.
- If there are a lot of steps, children may get confused about where they are in the solution to the problem.
- The presence of a fraction in the question may make it feel harder for children, and they could be prompted by considering a similar question with integer values.


## Key questions

- Do you need to find the whole or a part? Where can you show this on the bar model?
- What type of calculation do you need to do? How can you tell?
- Does it matter in which order you perform the calculations? Why/why not?
- Which operation should you perform first/second?
- What happens when you insert brackets into the calculation?


## Possible sentence stems

- In this calculation, first I need to do $\qquad$ and then ...
- To solve the problem, I need to find the $\qquad$ of the two fractions.


## National Curriculum links

- Add and subtract fractions with different denominators and mixed numbers, using the concept of equivalent fractions
- Multiply simple pairs of proper fractions, writing the answer in its simplest form
- Divide proper fractions by whole numbers
- Solve problems involving addition, subtraction, multiplication and division


## Mixed questions with fractions

## Key learning

- Match the bar models to the correct problems.

A piece of ribbon
is 4 m long. Tom cuts $\frac{3}{5}$ off. How much ribbon is left?


Nijah has 4 pieces of ribbon.
Each piece is $\frac{3}{5} \mathrm{~m}$ long.
How much ribbon does
Nijah have altogether?


A piece of ribbon is
$\frac{3}{5}$ m long. Brett cuts it into 4 equal parts.

How long is each part?


Work out the answer to each problem.

- Find the total length of the bar. Is there more than one way to find the answer?

- Find the difference between $\frac{3}{4} \times 3$ and $\frac{3}{4}+3$
- Match the bar models to the calculations.


$$
\left(\frac{4}{5}+\frac{5}{6}\right) \times 4
$$



- Work out the calculations.
$3 \frac{1}{3}+\frac{3}{4} \times 2 \quad 3 \frac{1}{3}+\frac{3}{4} \div 2 \quad 3 \frac{1}{3}+\frac{3}{4} \div 2$
- Scott has one-quarter of a bag of sweets.

Kim has two-thirds of a bag of sweets.
They combine their sweets and share them between themselves and Esther.

What fraction of a bag of sweets does each child get?

## Mixed questions with fractions

## Reasoning and problem solving

Square A and rectangle B have the same area.
Find the difference between their perimeters.


$$
2 \frac{1}{4} \mathrm{~m}
$$

Add two sets of brackets to make the calculation correct.


$$
\frac{1}{2}+\frac{1}{4} \times 8+\frac{1}{6} \div 2+1=6 \frac{1}{18}
$$

$$
\left(\frac{1}{2}+\frac{1}{4}\right) \times 8+\frac{1}{6} \div(2+1)
$$





Using each digit once only, find as many solutions to the calculation that are between 1 and 2 as you can.


Compare answers with a partner.
multiple possible answers, e.g.
$\frac{1}{3}+2 \times \frac{4}{5}$
$\frac{1}{4}+2 \times \frac{3}{5}$
$\frac{1}{5}+2 \times \frac{3}{4}$
$\frac{1}{3}+4 \times \frac{2}{5}$
$\frac{1}{4}+3 \times \frac{2}{5}$
$\frac{1}{5}+3 \times \frac{2}{4}$

## Notes and guidance

In Year 5, children used bar models to pictorially represent unit and non-unit fractions of an amount. The main focus of this small step is on understanding that the denominator is the number of parts the whole is divided into, and the numerator represents the number of those parts that are selected.

Bar models are a useful way for children to realise the connection between parts and wholes of an amount. By the end of this step, children should be able to find fractions of an amount in different contexts. Encourage them to divide by the denominator and multiply by the numerator, understanding why they are doing this and what they are finding in each step.

## Things to look out for

- Children may divide by the numerator instead of the denominator.
- Support may be needed for children who are not fluent with times-tables facts.
- Children may only find the value of the unit fraction and not multiply by the numerator to find the value of the whole fraction.


## Key questions

- How do multiplication and division help us when finding fractions of an amount?
- What does dividing the whole amount by the denominator work out?
- How are the parts and wholes represented in a fraction?
- What bar model could you draw to represent the calculation?
- What is the difference between a unit fraction and a non-unit fraction?


## Possible sentence stems

- The whole is divided into $\qquad$ equal parts. Each part is worth $\qquad$
- The numerator is $\qquad$ so the fraction is worth $\qquad$
- If one fifth is equal to $\qquad$ , then $\qquad$ fifths are equal to $\qquad$


## National Curriculum links

- Associate a fraction with division and calculate decimal fraction equivalents


## Fraction of an amount

## Key learning

- Complete the sentences.
- To find one-half of an amount, divide the amount by $\qquad$
- To find one-third of an amount, divide the amount by $\qquad$
- To find one-quarter of an amount, divide the amount by $\qquad$ -
- To find one-tenth of an amount, divide the amount by $\qquad$
- To find one-eighteenth of an amount, divide the amount by $\qquad$
- Work out the fractions of the amounts.


$$
\frac{1}{10} \text { of } £ 20
$$

$$
\frac{1}{8} \text { of } 40 \mathrm{~m}
$$

$$
\frac{1}{10} \text { of } 90 \mathrm{~g}
$$

- Use the bar model to find the missing numbers.

- $\frac{1}{8}$ of $160=$ $\qquad$ - $\frac{5}{8}$ of $160=$ $\qquad$ - $\qquad$ of $160=60$
- A cook has 48 kg of potatoes.

He uses $\frac{5}{6}$ of the potatoes.
How many kilograms of the potatoes does he have left?


- Use the bar model to complete the calculations.

- Work out the fractions of the amounts.

| $\frac{3}{8}$ of 40 | $\frac{5}{6}$ of 18 |
| :--- | :--- |$\frac{3}{4}$ of $160 \quad \frac{4}{7}$ of 35

## Fraction of an amount

## Reasoning and problem solving



Find the values of $\mathrm{A}, \mathrm{B}$ and C .


Compare methods with a partner.
$A=648$
$B=540$
$C=180$

## Fraction of an amount - find the whole

## Notes and guidance

In the previous step, children found a fraction of an amount. In this small step, they find the whole amount given a fraction of it.

Using a bar model to represent the parts and the whole is a useful support to children when working through this step. When finding the whole from a unit fraction, a pictorial representation helps children to understand why they simply need to multiply the given amount by the denominator. They then find a unit fraction from a given non-unit fraction and use this to find the whole.

Draw attention to the fact that, when calculating the whole, their answer will be greater than the number in the question. This will help children to sense check their answer.

Fluency with times-tables facts is very helpful here; some children may need a times-table square as support.

## Things to look out for

- Children may misinterpret $\frac{3}{4}$ of $-\quad=24$ as "Find $\frac{3}{4}$ of 24 "
- Without pictorial support, children may find it difficult to work out whether to divide or multiply by the numerator/ denominator.


## Key questions

- How many equal parts are there altogether?
- How many equal parts do you know the value of?
- What is the value of each equal part?
- How can you find the whole?
- Should the whole be greater than or less than the value you are given? Why?


## Possible sentence stems

- If one-sixth is equal to $\qquad$ , then the whole is equal to $\qquad$
- If five-sixths is equal to $\qquad$ then one-sixth is equal to $\qquad$ and the whole is equal to $\qquad$
- The whole is split into $\qquad$ equal parts.
- To find one part, I need to divide by $\qquad$ To find the whole, I need to multiply by $\qquad$


## National Curriculum links

- Associate a fraction with division and calculate decimal fraction equivalents


## Fraction of an amount - find the whole

## Key learning

- Complete the calculations.
$\frac{1}{4}$ of $20=$ $\qquad$ $\frac{1}{4}$ of $\qquad$ $=20$


What is the same about the calculations? What is different?

- Tommy runs $\frac{2}{5}$ of a race for his running club. He runs 6 km.


How far is $\frac{1}{5}$ of the race?
How far is the race altogether?

- Work out the missing wholes.

- Find the missing numbers.
$>\frac{3}{5}$ of $\qquad$ $=21$
- $£ 180=\frac{3}{7}$ of
$\frac{\square}{3}$ of $60=40$
$\frac{2}{\square}$ of $80=32$


## Fraction of an amount - find the whole

## Reasoning and problem solving

Miss Rose lights a candle before
she has a bath.
After her bath, $\frac{2}{5}$ of the candle
is left.
This part of the candle measures 13 cm .
Is Miss Rose correct?

Class 6 voted for their favourite ice cream flavour.

The table shows the fraction of the class that voted for each flavour.

| Strawberry | $\frac{1}{4}$ |
| :---: | :---: |
| Raspberry | $\frac{1}{6}$ |
| Vanilla | $\frac{1}{12}$ |
| Chocolate | $\frac{3}{8}$ |
| Bubblegum | $\frac{1}{8}$ |

6 children in the class voted for strawberry.
How many children are there in Class 6? How many children voted for chocolate?

